

BULLETIN 300

**SYNOPSIS OF LIFT, DRAG, AND
VORTEX FREQUENCY DATA FOR
RIGID CIRCULAR CYLINDERS**

by

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TABLE OF CONTENTS

	page
NOMENCLATURE	iii
INTRODUCTION	1
DESCRIPTION OF FLOW REGIMES.	1
THE STROUHAL-REYNOLDS NUMBER RELATIONSHIP.	7
LIFT AND DRAG FORCES	11
ROSHKO'S SPLITTER PLATE AND OTHER VORTEX INHIBITORS.	23
LITERATURE CITED	30

NOMENCLATURE

A	area
C_D	drag coefficient, $F_D / \frac{1}{2} \rho A U^2$
$C_{D_{max}}, C_{D_0}, C_{D_{rms}}, C_{D_f}, C_{D_p}, C_K, C_{K_D}, C_L, C_{L_{rms}}$:	lift and drag coefficients defined in the first five paragraphs of the section on Lift and Drag Forces.
C_p	pressure coefficient defined in Eqn. (8)
c	distance between aft end of a cylinder and aft end of a splitter plate
D	diameter of a circular cylinder
F	force
$F_D, F_{D_f}, F_{D_p}, F_L, \bar{F}$:	total drag force, frictional drag force, pressure drag force, lift force, and time average drag force, respectively
f_v	vortex shedding frequency
f_r	frequency of cylinder response
$fn(x)$	any unspecified function of x
L	a characteristic length (usually = D)
p	pressure
Re	Reynolds number, UL/ν
St_v, St_r :	Strouhal numbers, $f_v L/U$ or $f_r L/U$, respectively
t	time
U	free stream velocity
α	angle between free stream and any cylinder radius, measured from the forward stagnation point
ν	kinematic viscosity
τ_0	fluid shear stress on the surface of an immersed body

INTRODUCTION

The fluid mechanics literature is not entirely clear in defining the correct values and the experimental uncertainties of lift, drag, and vortex frequency measurements for rigid circular cylinders. Certain sets of data have traditionally been accepted, although improved data exist. Many sets of data conflict, and the variability of results is often overlooked. This bulletin seeks to summarize and evaluate existing data in an attempt to improve this situation and to provide ready access to previously scattered information.

Most of this account was first assembled in the fall of 1964 in connection with a two year investigation of the vortex shedding behind transmission line conductors. This study was under the joint support of the Bonneville Power Administration and the College of Engineering Research Division.

DESCRIPTION OF FLOW REGIMES

The frequency, f_v , of vortex shedding behind any bluff body should be completely determined by three additional kinematic variables: U , the free stream velocity; L , a characteristic length (e.g., the diameter, D , of a circular cylinder); and ν , the kinematic viscosity of the fluid. Since these four variables involve only two basic units, length and time, they can be arranged into a universal relation between two dimensionless groups:

$$St_v = fn(Re) \quad , \quad (1)$$

where the vortex Strouhal number, $St_v \equiv f_v L / U$, and the Reynolds number, $Re \equiv UL / \nu$, can be thought of as a dimensionless frequency and a dimen-

sionless velocity, respectively.

While equation (1) is widely used, it is a little unfortunate that U appears in both dimensionless groups. Another kind of dimensionless frequency can be obtained by multiplying both sides of equation (1) by Re . The result is:

$$\frac{f_v L^2}{2S} = f_n(Re) \quad . \quad (1a)$$

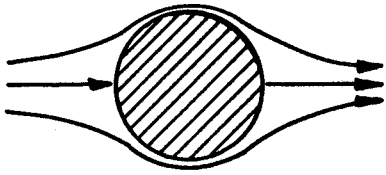
Another form of equation (1) that will be of interest to us employs the reciprocal of the Strouhal number:

$$\frac{U}{f_v L} = f_n(Re) \quad . \quad (1b)$$

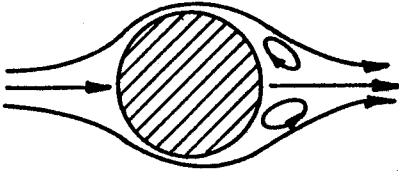
Our present understanding of the vortex-shedding action and the way in which it controls the Strouhal-Reynolds number relationship has been summarized by Marris [1]* for the lower Reynolds number regimes. Marris explains the development of the wake of a cylinder with increasing Reynolds number on the basis of earlier work by Föppl [2], Kovaszny [3], Tritton [4], Schaeffer and Eskinazi [5], Roshko [6,7], and others. The major regimes of interest for the entire range of Reynolds numbers are sketched in Fig. 1. Marris' description of the earlier regimes is roughly as follows:

A pair of fixed "Föppl" vortices first appears in the wake of the cylinder when $Re \approx 5$. When $Re \approx 40$ these vortices, which by now have stretched and elongated, become unstable and the first periodic driving forces begin. Tritton's observations reveal a small transition at $Re \approx 90$. This is probably the point at which vortex shedding takes control of the vortex spacing and frequency. Up to $Re \approx 150$, the trail of shedding vortices is laminar and ex-

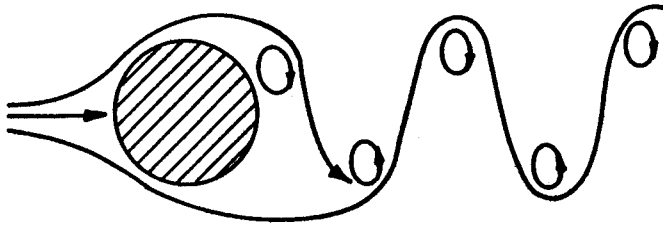
*Numbers in square brackets denote entries in the Literature Cited section of this report.



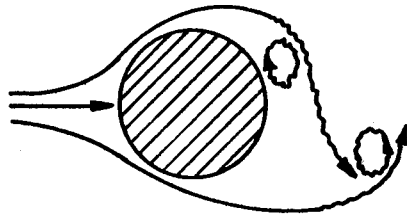
$Re < 5$ REGIME OF UNSEPARATED FLOW.



$5 \text{ TO } 15 \leq Re < 40$ A FIXED PAIR OF FÖPPL VORTICES IN THE WAKE.

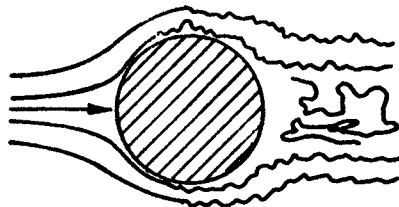


$40 \leq Re < 90$ AND $90 \leq Re < 150$
 TWO REGIMES IN WHICH VORTEX STREET IS LAMINAR:
 PERIODICITY GOVERNED IN LOW Re RANGE BY WAKE INSTABILITY
 PERIODICITY GOVERNED IN HIGH Re RANGE BY VORTEX SHEDDING.



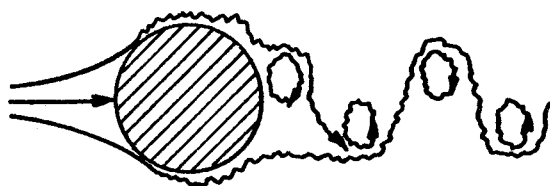
$150 \leq Re < 300$ TRANSITION RANGE TO TURBULENCE IN VORTEX.

$300 \leq Re \approx 3 \times 10^5$ VORTEX STREET IS FULLY TURBULENT.



$3 \times 10^5 \approx Re < 3.5 \times 10^6$

LAMINAR BOUNDARY LAYER HAS UNDERGONE TURBULENT TRANSITION. THE WAKE IS NARROWER AND DISORGANIZED. NO VORTEX STREET IS APPARENT.



$3.5 \times 10^6 \leq Re < \infty (?)$

RE-ESTABLISHMENT OF THE TURBULENT VORTEX STREET THAT WAS EVIDENT IN $300 \leq Re \approx 3 \times 10^5$. THIS TIME THE BOUNDARY LAYER IS TURBULENT AND THE WAKE IS THINNER.

Fig. 1. Regimes of Fluid Flow across Circular Cylinders.

tends far behind the cylinder. As the Reynolds number is increased, a laminar to turbulent transition begins in the free vortex layers leaving the cylinder. The resulting vortices in the wake are composed of turbulent fluid and, by the time $Re \approx 300$, vortex periodicity has completely vanished at a distance of about 48 diameters downstream from the cylinder.

The next important transition occurs somewhere in the range $10^5 < Re < 5 \times 10^5$, depending upon cylinder roughness and the free-stream turbulence level. This is the transition which occurs when the laminar boundary layer on the cylinder becomes turbulent. Although the vortex shedding frequency in this range is often evaluated on the rough assumption that the Strouhal number has the constant value of 0.21, the regime actually exhibits some important complexities.

The chief complication in this regime appears to be an evolving three-dimensionality of the wake. Roshko [6] provided some hot-wire anemometer observations at $Re = 500$, which bear upon this. He placed two anemometers 6 diameters downstream from the cylinder and spaced them 3, 10 and 105 diameters apart in the spanwise direction. The correlation between the outputs of the two anemometers was excellent at three diameters, poor at 10, and non-existent at 105. The implication is that at 3 diameters both anemometers observed the same eddy, but that spanwise transitions occurred when the spacing was increased.

More recent experimental studies by Macovsky in 1958 [8] and by Humphreys in 1960 [9] have shed more light on the question of spanwise three-dimensionality. Both investigators were concerned with the Reynolds number range from 10^4 to transition. Macovsky marked the flow with both ink, and strings attached to the cylinder. Both markings revealed that spanwise irregularities did exist. Humphreys used both strings and correlation of hot-wire anemometer readings. He found that the wake becomes highly three-dimensional just before transition. In

particular he found that the small strings that he attached to the cylinder caused the wake to form into alternating "cells" at $Re \approx 10^5$. These cells, spaced at 1.4 to 1.7 diameters, consisted of regions along the cylinder that were subtended by alternate laminar and turbulent boundary layers.

Apparently the extent to which spanwise nonuniformities are organized (i.e., periodic along the span) is not well understood, even at lower Reynolds numbers, however. An experimental and analytical study of aeolian tones at low Reynolds numbers by Phillips [10] revealed spanwise uniformity up to 30 diameters in the range $40 \leq Re \leq 80$. However, in the range $80 \leq Re \leq 100$ --very nearly Tritton's transition regime--he observed that ink trails spaced at 30 diameters tended to go out of phase if any disturbances were present. In the range $100 \leq Re \leq 160$ he found good spanwise correlation only for spacings less than 20 diameters. A more recent experimental study by Bloor [11] gives hot-wire anemometer measurements of cylinder wakes for $200 \leq Re \leq 10^4$. Both she and Phillips note the important point that turbulence is inherently three dimensional as well as disorganized. Accordingly, she measured even stronger spanwise irregularities in her turbulent Reynolds number range.

In all likelihood such hydro-elastic systems as transmission lines lend spanwise uniformity to their own wakes. Although we seriously lack information on this point, it is probable that only a small transverse motion will require that the vortex shedding must relate in the same way at each point to the spanwise-regular cylinder motion.

When the boundary layer becomes turbulent, the vortex shedding action is diffused and "eddy frequency" assumes an entirely different physical meaning. We can no longer speak of the eddy-shedding frequency, but now must report the dominant frequency in a spectrum of component frequencies. Delany and Sorensen [12] report such frequencies in the narrow range $1.2 \times 10^6 \leq Re \leq 2 \times 10^6$. They

employed a pressure probe at a distance of about 1-1/2 diameters downstream, and observed frequencies in the range $.33 \leq St_v \leq 47$. A later study by Fung [13] presents frequency spectra for lift and drag forces on a cylinder--not for the actual eddy-shedding--in the early supercritical Reynolds number regime. A typical spectrum is shown in Fig. 2.

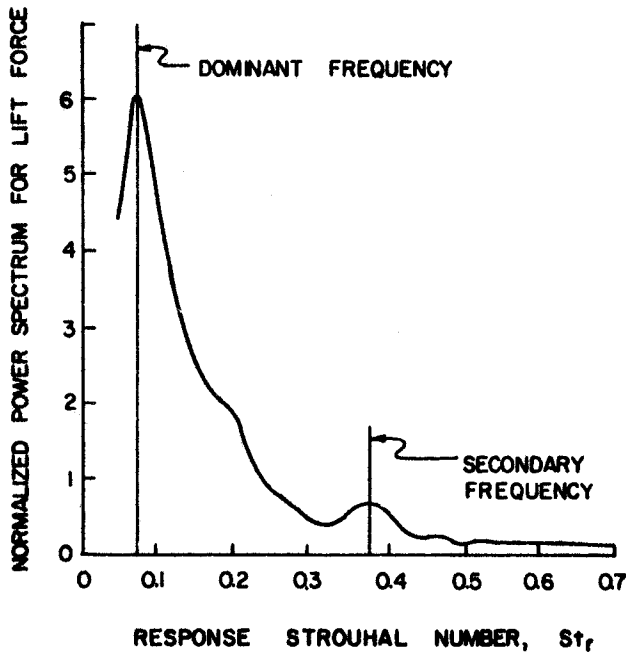


Fig. 2. Typical Power Spectrum for Lift Force on a Cylinder when $Re = 1.3 \times 10^6$ (after Fung)

While Fung's drag forces exhibit a somewhat more pronounced secondary peak in the range of Delany and Sorensen's St_v data than do his lift forces shown in Fig. 2, it is clear that the dominant response frequency is much lower. We are left to ponder whether these dominant low frequencies evolve out of three-dimensional effects, elasticity in the mounting system of the cylinder, end-effects, or some other complication.

In 1961 Roshko [14] studied a still higher Re range. He placed a probe 7 diameters aft of an 18 in. diam. cylinder in a wind tunnel, and 0.7 diameters above it. At this position--far back in the wake--he observed the clear re-establishment of turbulent vortex shedding at $Re = 3.5 \times 10^6$! This regime per-

sisted out to the upper limit of his experiments--a Reynolds number of almost 10^7 . Roshko found that Strouhal numbers in this regime were higher (about 0.267) than in the lower turbulent vortex shedding regime ($300 \leq Re \leq 2 \times 10^5$) and showed how this could result from the narrower wake that follows a turbulent boundary layer.

THE STROUHAL-REYNOLDS NUMBER RELATIONSHIP

It will be well to return to the Strouhal-Reynolds number relationship with this complicated view of the cylinder wake in mind, and with the most recent eddy-frequency data in hand. We should anticipate that the relationship will be fairly complex.

Figure 3 displays the recent hot-wire anemometer data of Roshko [6,7], Kovasznay [3], and Tritton [4] for $Re < 2 \times 10^4$. All but 7 out of about 370 data points can be enclosed in a cross-hatched area which exhibits a spread of about 10% in Strouhal number. The region is shown without data points since the individual points are prohibitively packed and overlapped.

A variety of other observations are available and some are included in Fig. 3. The older and often cited data of Tyler [14] (1931), Relf and Simmons [16], and Relf [17] (1921) are shown. These measurements were made in a variety of highly ingenious and comparatively primitive ways. Lehnert [18] (1937) reviewed the existing data up to that time. He found considerable disagreement, and advanced two sets of his own data. One, based upon a resonance technique, showed severe scatter. The other was based on the mechanical conversion of the eddy-shedding frequency into an acoustical tone. The 19 data points obtained in this manner fell on the single line shown in Fig. 3. Gerrard's [19] (1955) data are the only purely acoustical ones shown in Fig. 3.

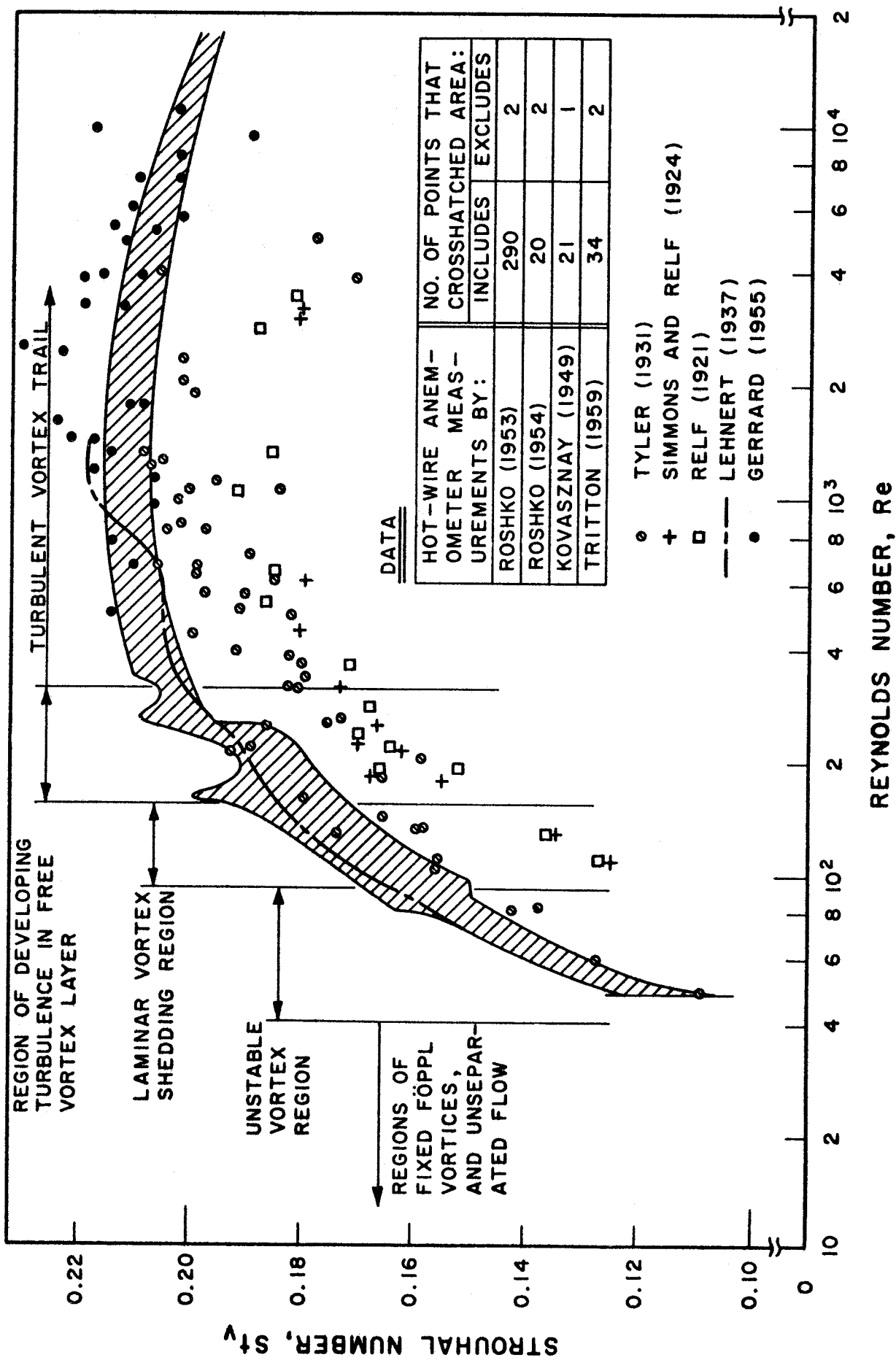


Fig. 3. Strouhal- Reynolds Number Data for Circular Cylinders; $40 < Re < 2 \times 10^4$.

Gerrard used a microphone to get spectral distributions of sound frequency in his wind tunnel, and he associated the dominant frequency of each distribution with the eddy-shedding frequency. His data are scattered, but they center about the hot-wire anemometer results.

We shall accept the envelope defined by the four relatively recent and highly refined hot-wire anemometer studies as providing our most reliable estimate of the St_v vs. Re relationship in this Reynolds number regime.

Evidence of each of the early Reynolds number transitions can be seen in Fig. 3. While the transition from fixed Föppl vortices to an unstable vortex region is usually observed at $Re \approx 40$, none of the nine sources used provide any frequency data below $Re = 48$. Tritton's minor transition point appears only as a fattening of the envelope. Tritton himself found that the transition was clear only in dimensionless f_v vs. U coordinates, while it became obscured on St_v vs. Re coordinates.

The region of developing turbulence in the free vortex layer subtends some strange irregularities in the envelope and a great randomness of data in this region is implied. In the portion of the turbulent vortex region shown in Fig. 3 the St_v vs. Re curve is not horizontal but notably convex.

Figure 4 presents St_v data in the remaining range of Reynolds numbers. Included in Fig. 4 are the data of Delany and Sorensen, Roshko (1961), Relf and Simmons, Macovsky, and a portion of the cross-hatched envelope from Fig. 3. Additional points include: data presented by Etkin et al. [20] from the work of Korbacher and Fowell [21], data of Drescher [22] given by Fung, a line by Weaver [23] as representing his wind tunnel data and some recent measurements of Gerrard [24]. Finally, four secondary points have been read off of Fung's lift and drag spectra, and included for the reader's interest. These points compare well with Delany and Sorensen's data. With the exception of eight of Relf and Simmons' data points, all data in Fig. 4 are recent.

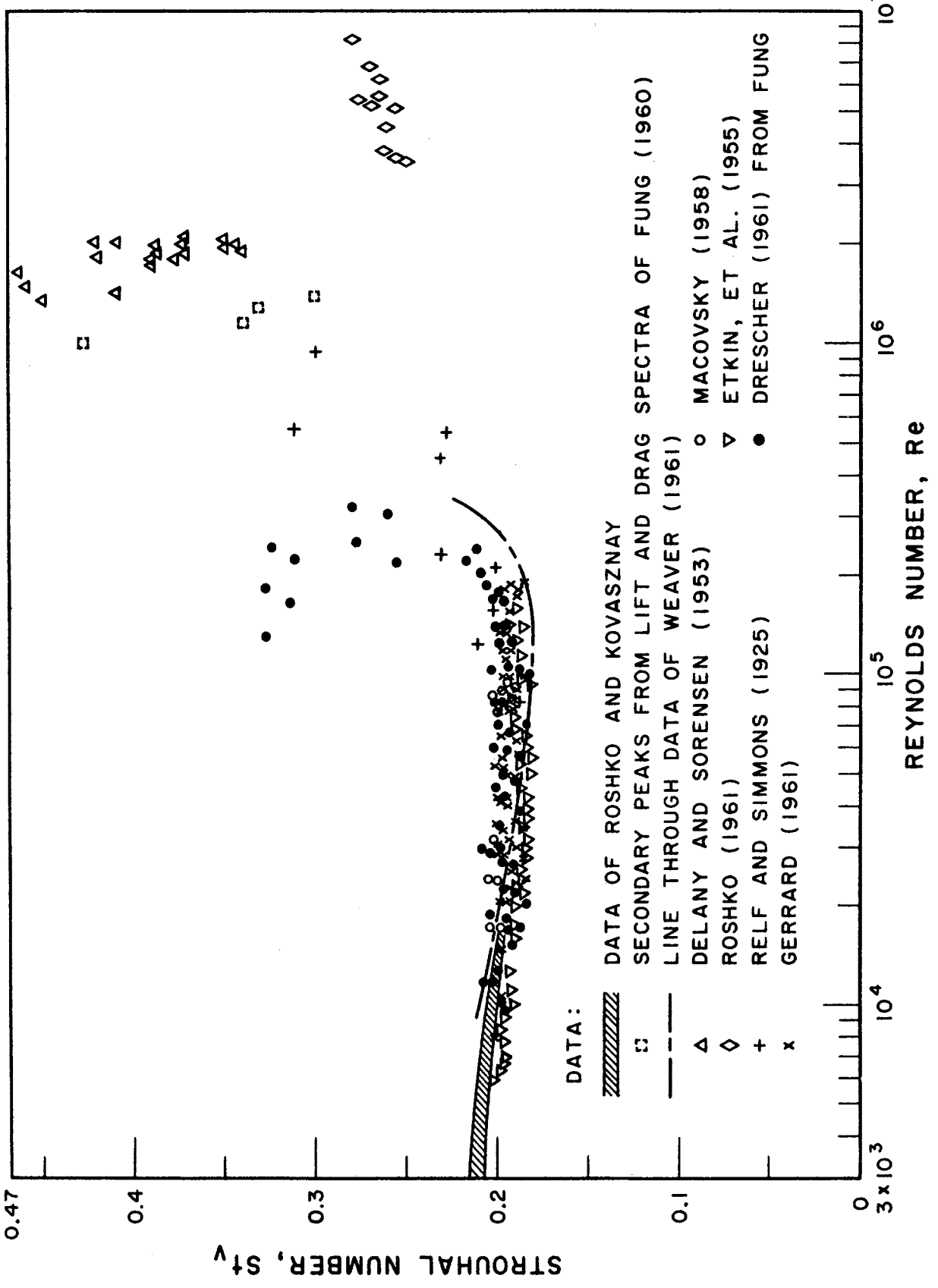


Fig. 4 Strouhal-Reynolds Number Data for Circular Cylinders; $3 \times 10^3 \leq Re < 10^7$.

An envelope which contains virtually all of the data in Fig. 4 has been fitted to these data. This envelope and the one shown in Fig. 3 have been combined in Fig. 5. The writer advances Fig. 5 as the best estimate of the Strouhal-Reynolds relationship for eddies on rigid cylinders that can be made at this time. Our knowledge of St_v is accurate within $\pm 5\%$ over most Reynolds numbers. Little can be said about the variable region of boundary layer transition, and we can only conjecture about the region of $Re > 8 \times 10^6$. Eddy-shedding behavior in the range $Re_{\text{transition}} \leq Re < 3.5 \times 10^6$ is poorly understood and should be studied further.

The dimensionless frequency, fD^2/ν , suggested in equation (1a) has not been used here since it passes through about seven orders of magnitude in the Reynolds number ranges of interest. Accordingly the 5 or 10% variations in f_v that interest us are lost when equation (1a) is plotted on logarithmic coordinates. Equation (1a) is more useful in limited Reynolds number ranges. Equation (1b) will be of some interest in the next section.

The results presented in this section have included nothing related to the classical hydrodynamic studies of cylinder wakes by VonKarman and subsequent workers. These works are discussed very fully by Birkhoff and Zarantonello [25].

LIFT AND DRAG FORCES

The hydrodynamic driving forces exerted on a cylinder will be resolved into two components. The first of these is the drag force, F_D , which acts in the direction of motion of the fluid. Drag is the only force component that acts on a cylinder below $Re = 40$ because all flows in that regime are symmetrical with respect to the direction of flow. Vortex instability and shedding

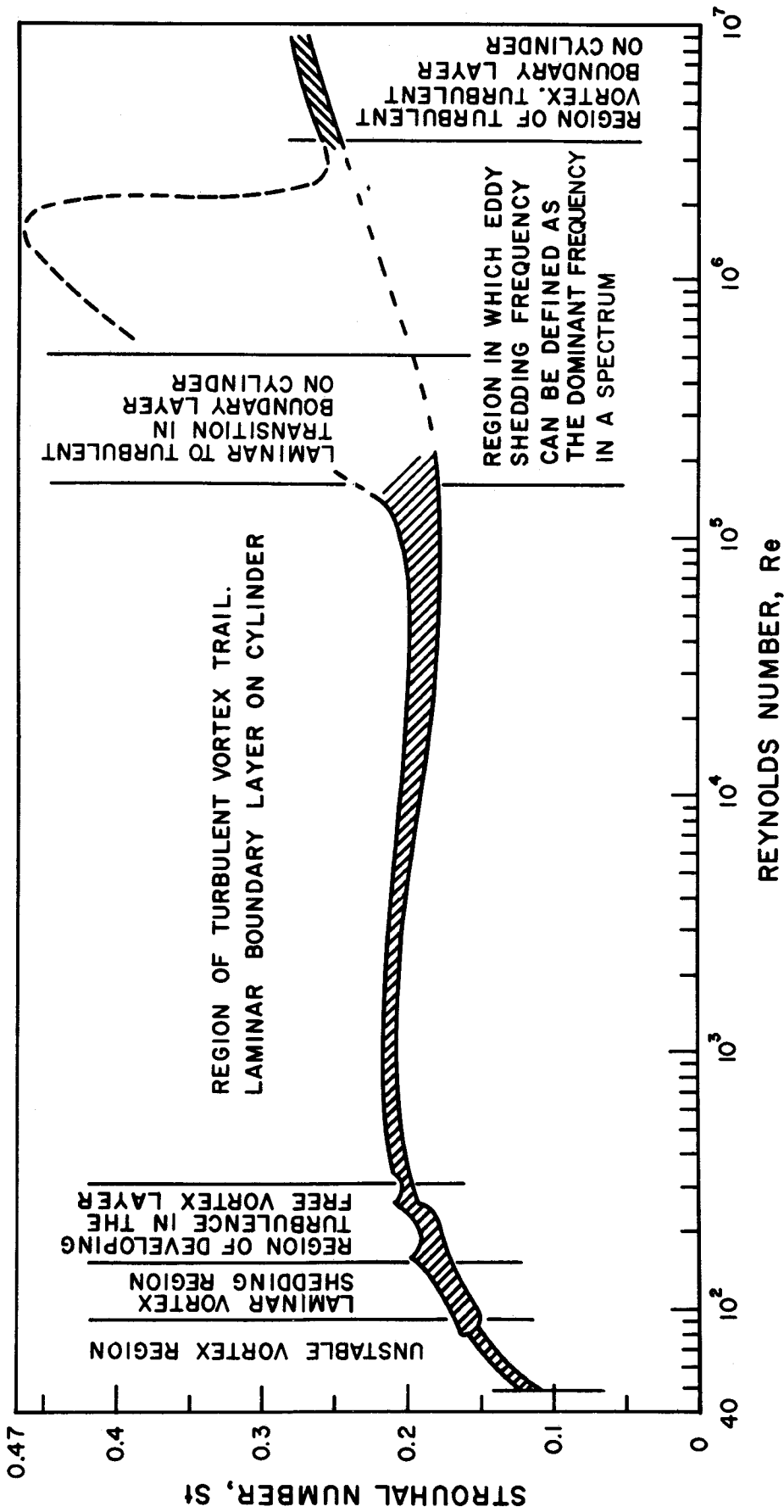


Fig. 5. The Strouhal-Reynolds Number Relationship for Circular Cylinders as Defined by Existing Data.

in the regime: $Re > 40$ gives rise to an additional force component called lift, F_L , which is normal to both the flow direction and to the axis of the cylinder. The drag force is steady until vortex instability begins. Then it oscillates either periodically or randomly about a mean value. The lift force oscillates either periodically or randomly about zero.

The application of dimensional analysis to the relevant variables again shows that the lift and drag forces should be uniquely related by relationships of the form:

$$C_D \equiv \frac{\bar{F}_D}{\frac{1}{2} \rho A U^2} = f_n(Re) \quad , \quad (2)$$

where A is the projected area that the cylinder presents to the flow, ρ is the density of the fluid and \bar{F}_D is the time-average of the drag force, $F_D(t)$. Several complications arise in the identification of the various force components, however, and we are forced to discriminate among a variety of lift and drag coefficients.

The drag force, F_D , is equal to the sum of two very different kinds of force (see, e.g., [26]). The first is pressure drag:

$$F_{D_p} = \int_A p \sin \alpha \, dA \quad , \quad (3)$$

where F_{D_p} is the pressure at any point on the surface of the body, and α is the angle between the forward stagnation streamline and a vector normal to the surface. The second component is frictional drag:

$$F_{D_f} = \int_A \tau_o \cos \alpha \, dA \quad , \quad (4)$$

where τ_o is the viscous shear stress at the surface. At very low Reynolds numbers ($Re < 5$), drag will consist almost entirely of F_{D_f} . When vortices form, the pressure distribution will become more severe from the forward to the aft

stagnation point and eventually it will become asymmetrical with respect to the direction of flow. As the oscillating lift force appears, F_{Df} gradually becomes negligible in comparison with F_{Dp} .

The presence of oscillating force components complicates the use of equation (2). For a simply harmonic lift force a modification of equation (2) called "Karman coefficient" is generally used:

$$C_K \equiv \frac{F_L}{\frac{1}{2}\rho AU^2 \cos(2\pi f_v t)} \quad (5)$$

Equation (5) is easily adapted to describe the oscillating drag component ($F_D(t) - \bar{F}_D$):

$$C_{KD} \equiv \frac{F_D(t) - \bar{F}_D}{\frac{1}{2}\rho AU^2 \cos(2\pi f_v t)} \quad (5a)$$

Except for very low Reynolds numbers, the lift and drag forces oscillate in an increasingly random fashion and equations (5) and (5a) become inapplicable. A variety of coefficients is in use in these higher Reynolds number regimes. They are all modifications of equation (2). They include: $C_{D_{rms}}$ and $C_{L_{rms}}$ which are based upon the root-mean-square variations of F_D and F_L ; $C_{D_{max}}$ and $C_{L_{max}}$ which are based upon maximum absolute values of F_D and F_L and C_{Dp} , C_{Df} , C_{D0} which are based upon F_{Dp} , F_{Df} , and the amplitude of the drag force oscillation, respectively.

For many years we have only had information relating to C_D , the simple average drag coefficient, at our disposal. Only during the last eight years has serious experimentation been directed at obtaining lift coefficients. The first important evaluations of C_D were made by Reif [27] (1914) at N.P.L. and by Wieselsberger [27] (1921) at the Göttingen Aerodynamic Laboratories, over the large Reynolds number range, $4 < Re < 8 \times 10^5$. Reif's and Wieselsberger's results are represented by faired lines in Fig. 6 (low Re Range) and Fig. 7

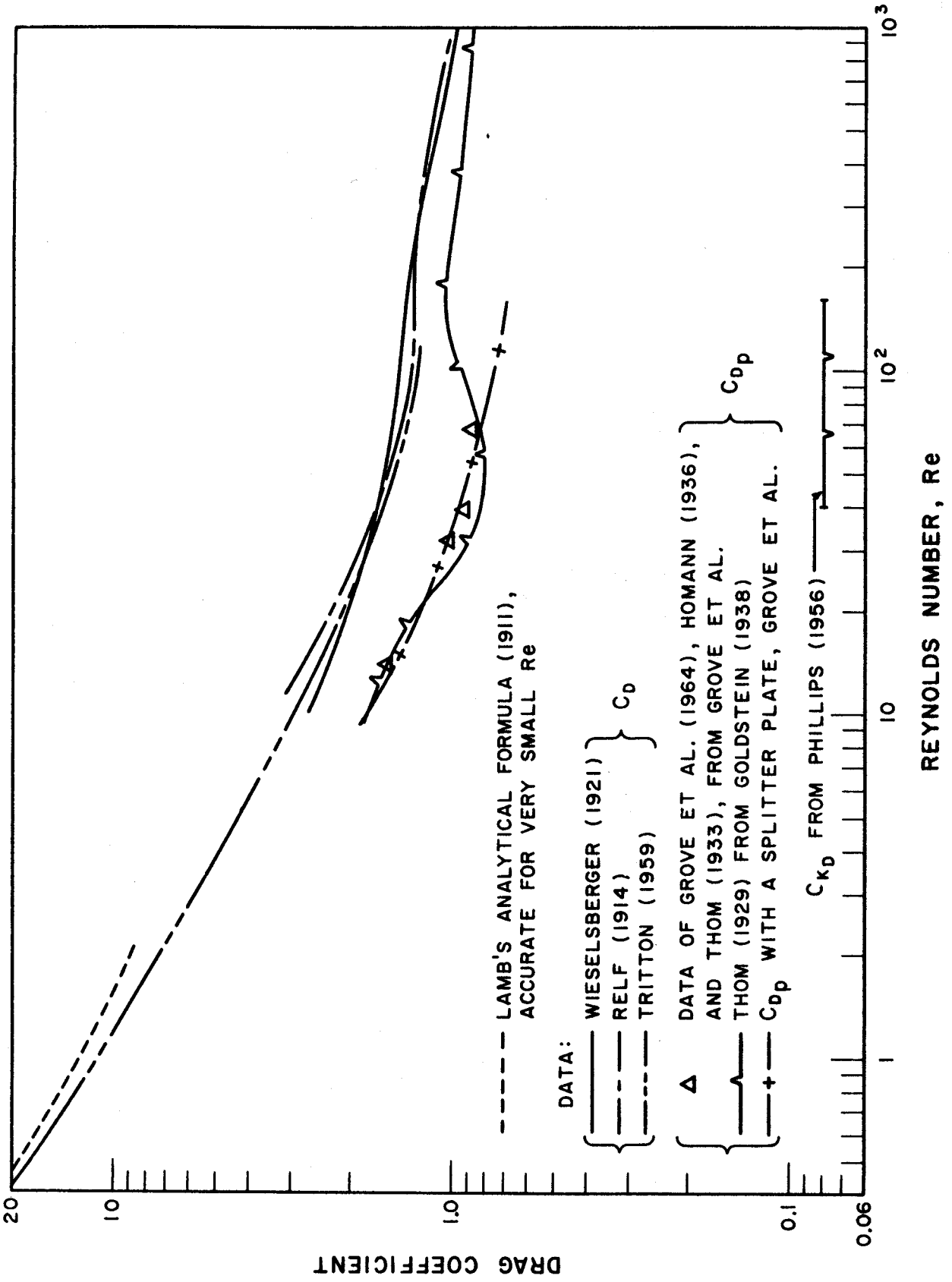


Fig. 6. Drag Coefficient Results for Circular Cylinders; $0.4 \leq Re < 10^3$.

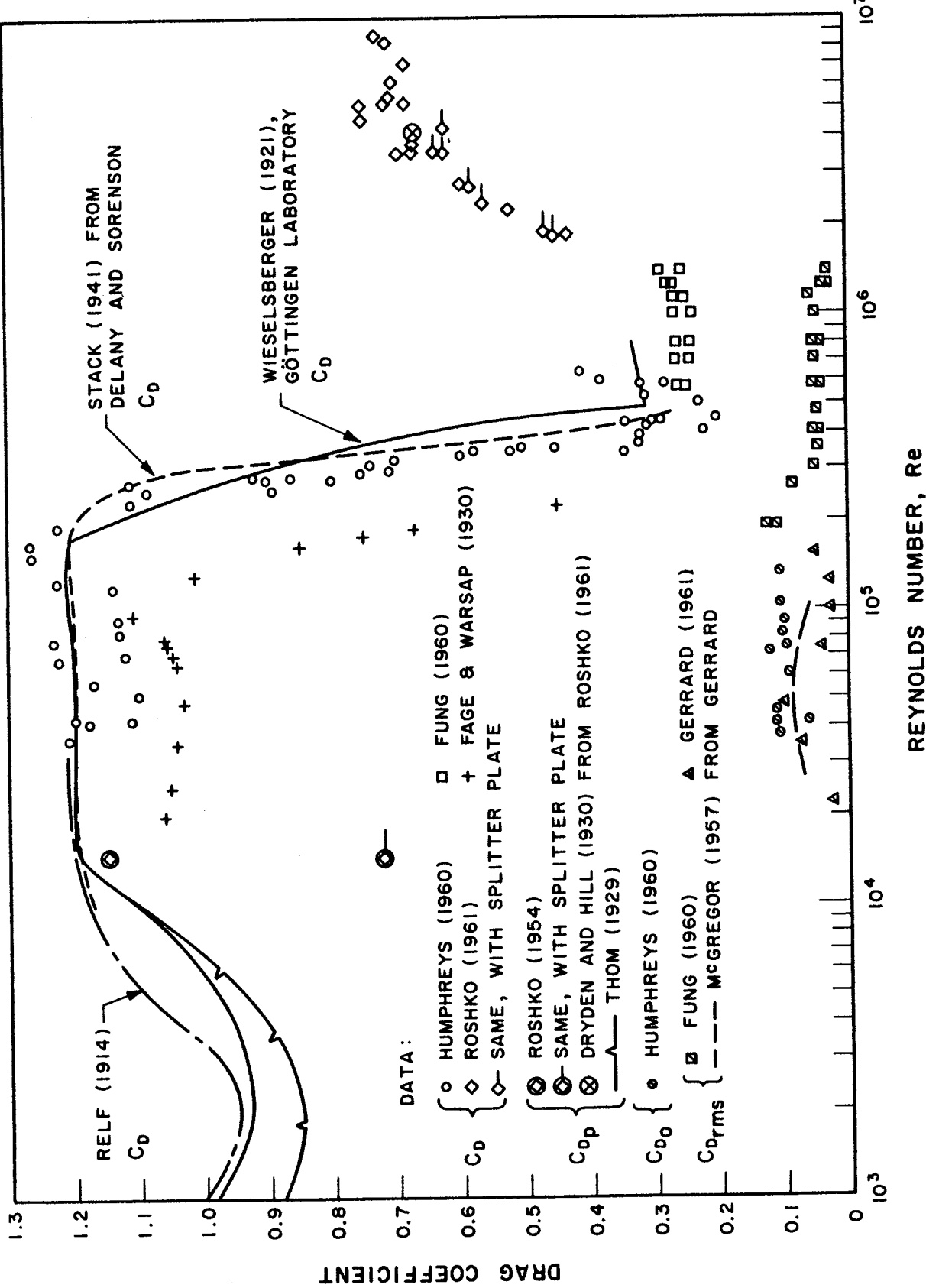


Fig. 7. Drag Coefficient Results for Circular Cylinders; $10^3 \leq Re < 10^7$.

(high Re range). Figure 6 also displays the best line through Tritton's careful observations of C_D in the very low Re regimes, and Lamb's [29] analytical result which is accurate only at extremely low Reynolds numbers. Together these results define our knowledge of C_D within $\pm 10\%$ in the entire range: $Re < 10^3$.

Pressure drag coefficients can be determined by applying equation (3) to measured pressure distribution data. Thom [30,31] (1929 and 1933) provided such C_{D_p} evaluations in an experimental and analytical study of pressure and frictional drag. His work is summarized in an excellent chapter on "Flow Round Symmetrical Cylinders" by Goldstein [32]. Figures 6 and 7 display the line through his observations along with additional values from the data of Dryden and Hill [33] given by Roshko (1961), and from the data of Grove et al. [34], Hohmann [35], and Roshko (1954), given in [34]. Thom found both by an approximate analytical formulation, and by the difference between observed values of C_D and C_{D_p} , that:

$$C_{D_f} \simeq 4/\sqrt{Re} \quad , \quad 30 < Re < 10^4 \quad . \quad (6)$$

This corroborates our earlier assertion that friction drag becomes relatively unimportant at higher Reynolds numbers.

Additional observations of C_D at higher Reynolds numbers by Humphreys, Fung, Roshko (1961), Stack [36] (1941) given by Delany and Sorensen, Fage and Warsap [37] (1930) given by Goldstein are included in Fig. 7. Goldstein reports that the latter results are "uncorrected for wall or gap interference effects." Since they fall significantly lower than the other data they probably should be distrusted. In this connection Goldstein also presents data from a Göttingen report [38] (1923) which show a marked decrease of C_D on short

cylinders, owing to end effects.

The following additional information has been included in Figs. 6 and 7:

Both figures display data for cylinders with "splitter plates" given by Grove et al. and Roshko (1954, 1961). The splitter plate is a vortex inhibiting device, which will be discussed in the last section.

Figure 6 displays a semi-analytical prediction of C_{KD} , the oscillating drag coefficient, given by Phillips, who claims 40% accuracy for it. His value of C_{KD} is 0.08 in the range: $40 \leq Re \leq 160$. This is about 4% of C_D in this range.

Figure 7 presents McGregor's [39] (1957), and Gerrard's (1961) $C_{D_{rms}}$ data, both from Gerrard, along with the C_{D_0} data of Fung and Humphreys. These experimenters generally agree that their data are not very accurate in this range. The best that we can infer from these measurements of the oscillating drag force is that F_D is relatively steady and that its maximum deviation (about 20%) occurs in the boundary layer transition regime.

Missing from Fig. 7 are Drescher's data which were not locally available, and the relatively scattered data of Macovsky, who observed both C_D and $C_{D_{max}}$ in the range $10^4 < Re < 10^5$. Macovsky's data bracketed the data in Fig. 7, while his $C_{D_{max}}$ data fell about 10% higher. Since $C_{D_{max}} = C_D + C_{D_0}$, this is consistent with the data of Humphreys in Fig. 7.

We have already noted that the laminar to turbulent boundary layer transition point varies with the level of free stream turbulence, cylinder roughness, and other disturbance levels. Fage and Warsap's experiments (reported by Goldstein) show the effect of such variables upon C_D in the Reynolds number range $10^4 < Re < 2.5 \times 10^5$. They increased surface roughness both by covering their cylinders with different grades of glass paper and by installing tripping wires in the boundary layer; they changed the free stream turbulence level by placing a coarse rope mesh at different distances upstream from the cylinder.

The results of Fage and Warsap are presented in Fig. 8. Although we noted that there was some doubt as to the absolute accuracy of these data in the discussion of Fig. 7, their qualitative value is clear. In all cases the transitions are shifted strongly to the left with increasing disturbance. While a variation in upstream turbulence does not appear to alter the severity of the dip in C_D , increases in cylinder roughness very definitely do. Indeed the dip in C_D almost vanishes on an extremely rough cylinder.

The available lift force data are far less plentiful and far less consistent than the available drag force data. Furthermore, most observations appear to have been made in the transitional range. Figure 9 displays the available $C_{L_{max}}$ and $C_{L_{rms}}$ results in the range: $10^4 < Re < 1.5 \times 10^6$. These include data from the previously cited works of Macovsky, Fung, Humphreys, McGregor, and Weaver, as well as additional results of Goldman [40] (1957), Bingham [41] (1952), and Fujino [42] (1957), all given by Weaver.

The existing $C_{L_{max}}$ data are surprisingly similar to the drag data in the same transitional Reynolds number range. They differ, however, in that they exhibit much broader scatter--scatter which might be even worse if all individual data points were available. Humphreys made an important point related to scatter by presenting two sets of $C_{L_{max}}$, and two sets of $C_{L_{rms}}$, data. In general his uppermost points in each plot were obtained with the cylinder ends sealed at the wall. His lower points at each Reynolds number, and the relatively low data of Weaver, were obtained with a gap between the cylinder and the wall where the cylinder passed through the wall. Macovsky obtained generally low, but widely scattered, values that were observed on a loose central cylindrical segment mounted between two dummy segments.

Figure 10 displays the collected C_D and $C_{L_{max}}$ information over the entire range of Reynolds numbers. The C_D curve is a best line through the data in

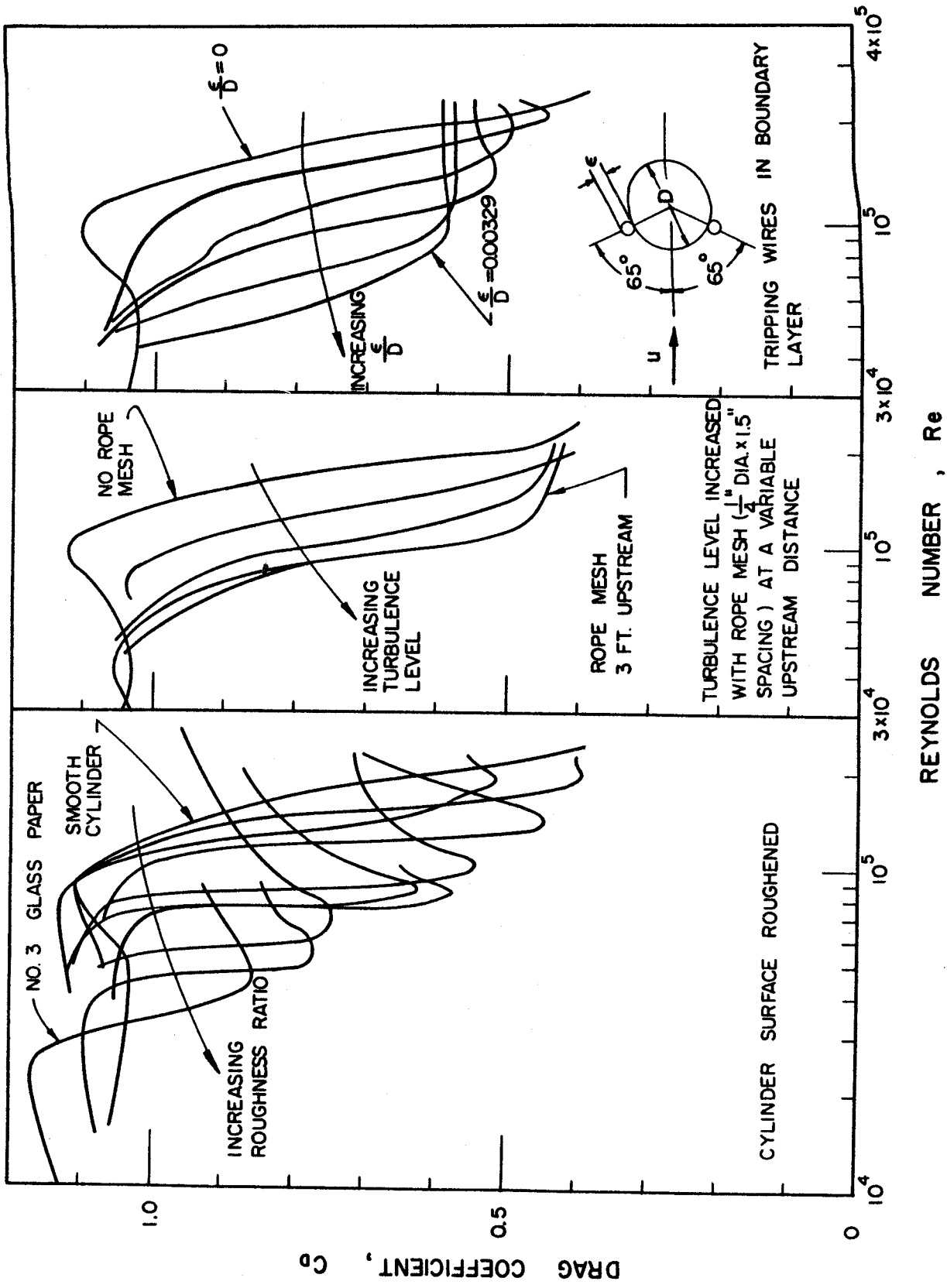


Fig. 8. Effects of Disturbances upon Drag in the Boundary Layer Transition Regime (after Fage & Warsap).

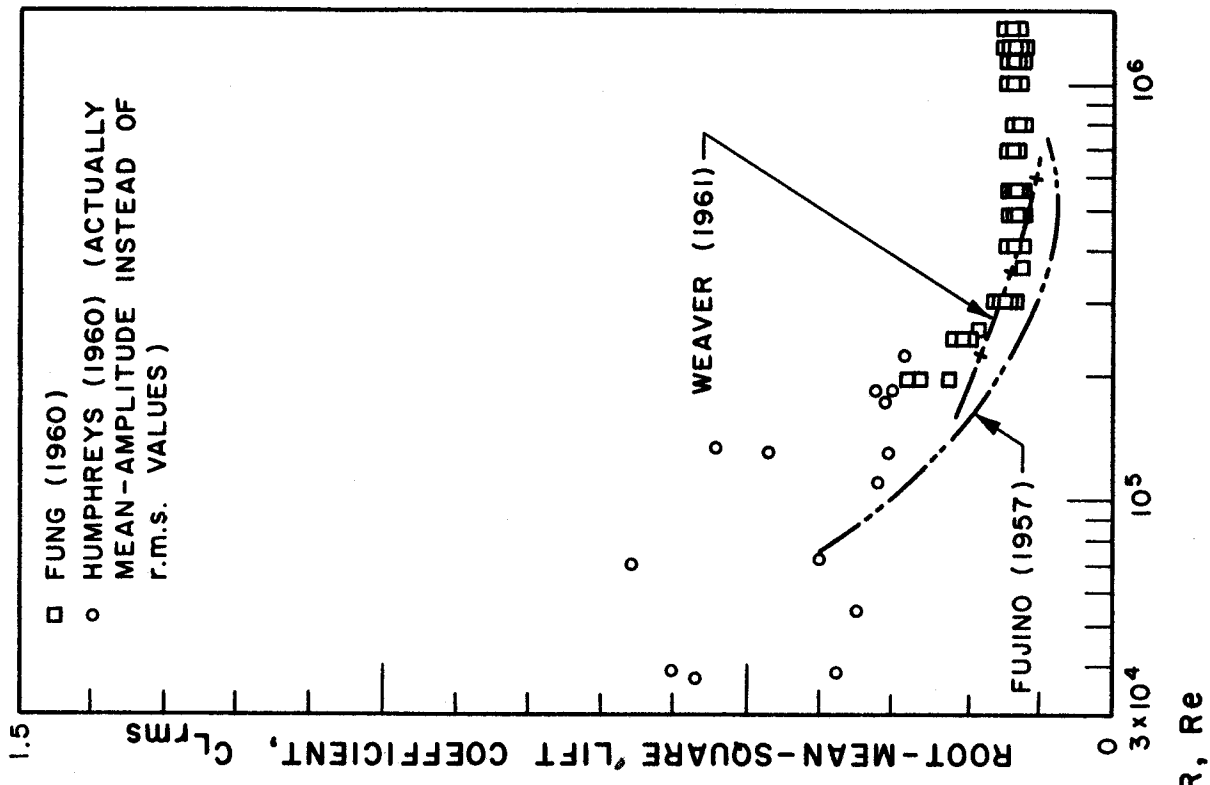
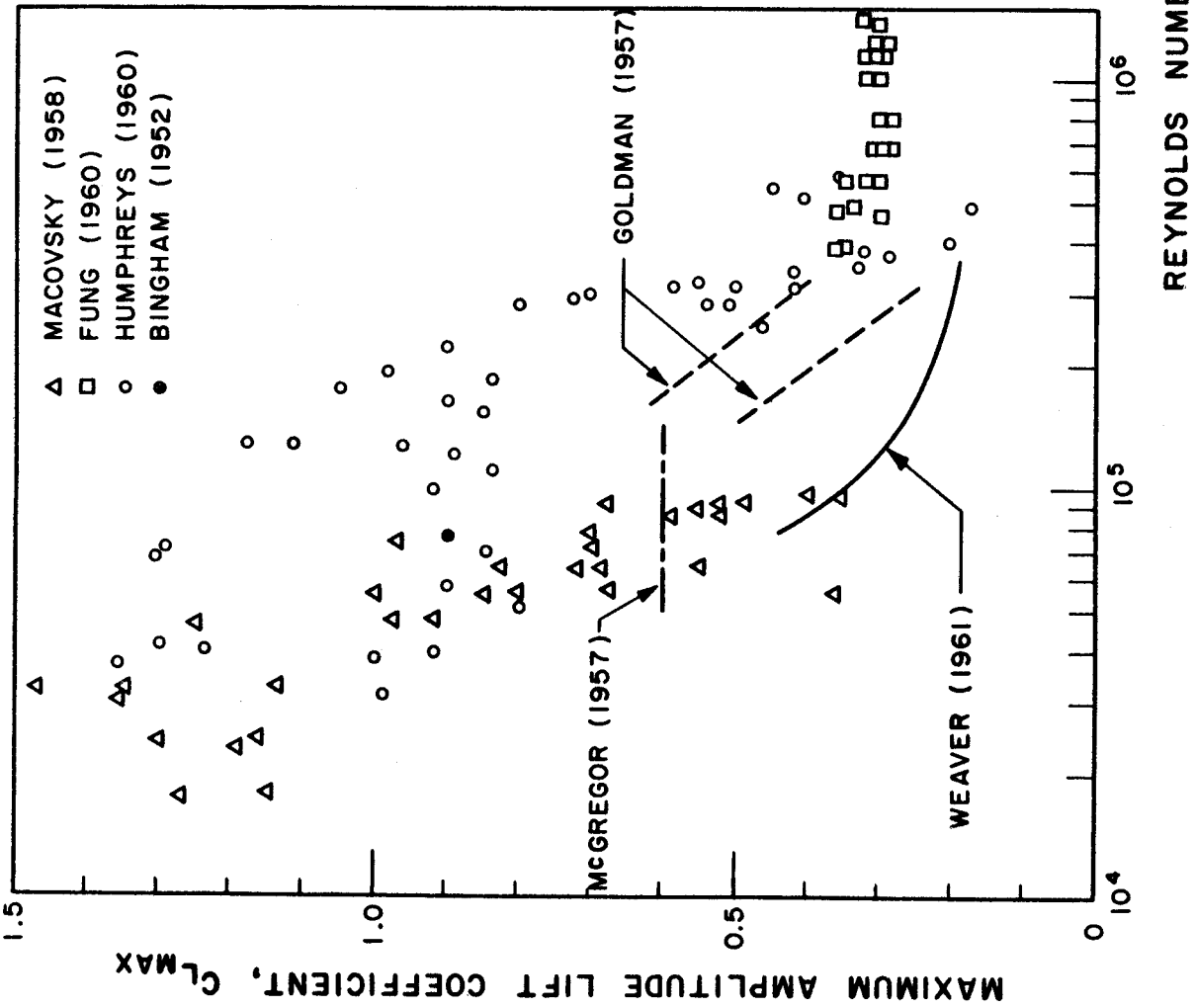


Fig. 9. Lift Coefficient Data for Circular Cylinders in the Boundary Layer Transition Regime.

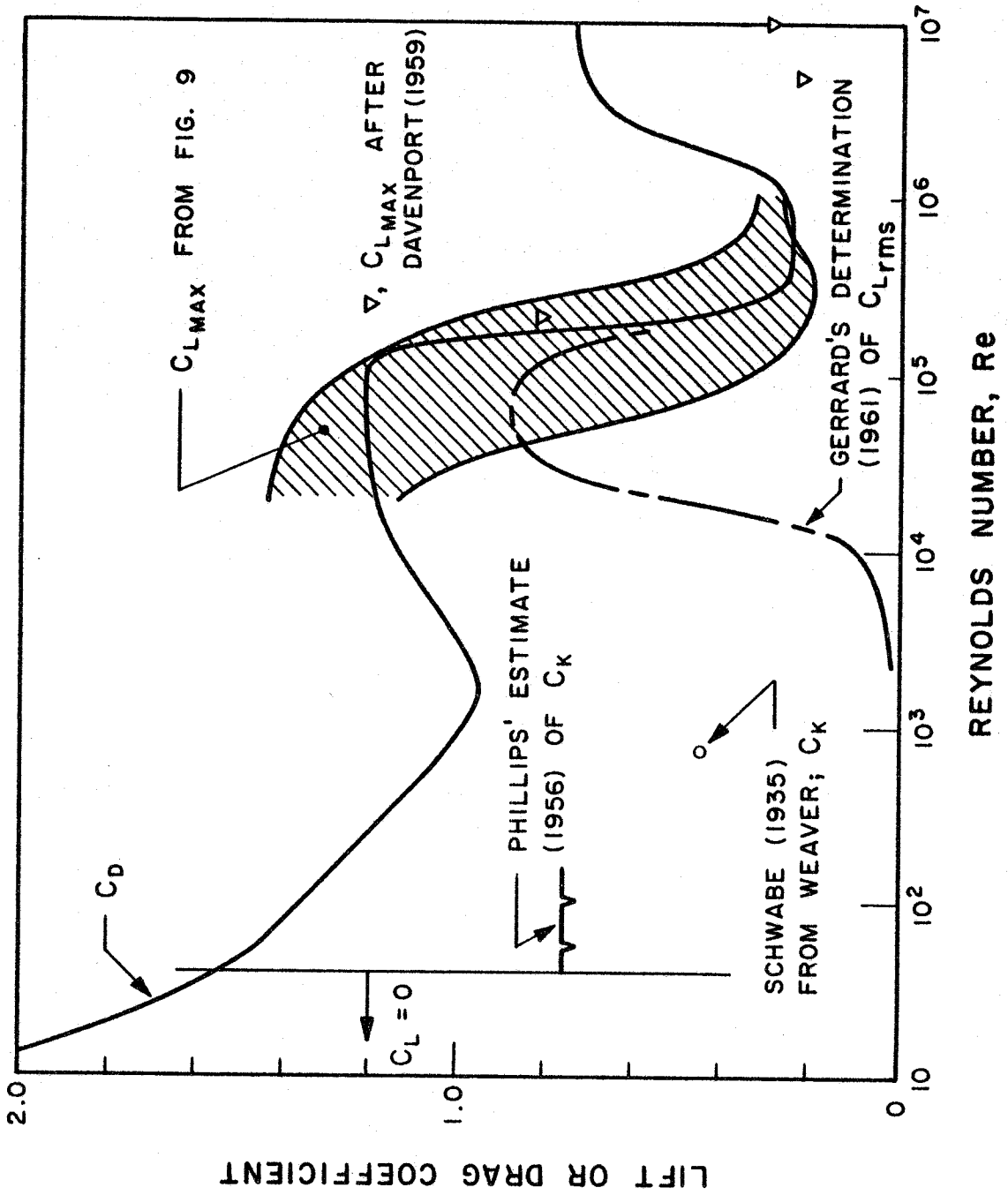


Fig. 10. Combined Lift and Drag Information for Circular Cylinders.

Figs. 6 and 7. The cross-hatched area represents almost all of the $C_{L_{max}}$ data from Fig. 9. Three $C_{L_{max}}$ observations made by Davenport [43] (1959) on large smokestacks indicate that the cross-hatched area can be extrapolated to higher Reynolds numbers.

The most important force on a transmission line insofar as vibration is concerned is, unfortunately, lift in the range: $10^3 < Re < 10^5$. However, it is clear that we are badly informed on lift in the range: $40 < Re < 2 \times 10^4$. Weaver reports a measurement by Schwabe (1935) of C_K (?) at $Re = 735$. Phillips presents a semi-empirical determination of C_K in the range $40 \leq Re \leq 160$ based upon the data of Kovasznay. He claims "40% accuracy" for his result. Finally, Gerrard measured pressure distributions about cylinders in 1961 and used them to compute $C_{L_{rms}}$ in the range $4 \times 10^3 < Re < 1.8 \times 10^5$. His results are strange in that $C_{L_{rms}}$ virtually disappears at low Reynolds numbers. However, F_L should approach a simple harmonic function for which $C_{L_{max}} = C_K = 2 C_{L_{rms}}$, at low Reynolds numbers. Gerrard's results thus imply that $C_{L_{max}}$ should also disappear at low Reynolds numbers! The writer would therefore strongly point to the need for further study of the lift force on rigid cylinders in the range $40 \leq Re < 10^5$.

ROSHKO'S "SPLITTER PLATE" AND OTHER VORTEX INHIBITORS

Roshko (1961) presented a plot of both C_D and $1/St_v$ against Re which makes an important commentary on the relation between eddy shedding frequency and drag. We have employed the information from Figs. 5, 6, and 7 to form such a plot in Fig. 11. The quantity $0.22/St_v$ has been used in place of $1/St_v$ to get a curve that lies almost on top of the C_D curve. The similarity of these curves below the boundary layer transition region makes it possible for us to write within $\pm 10\%$ accuracy:

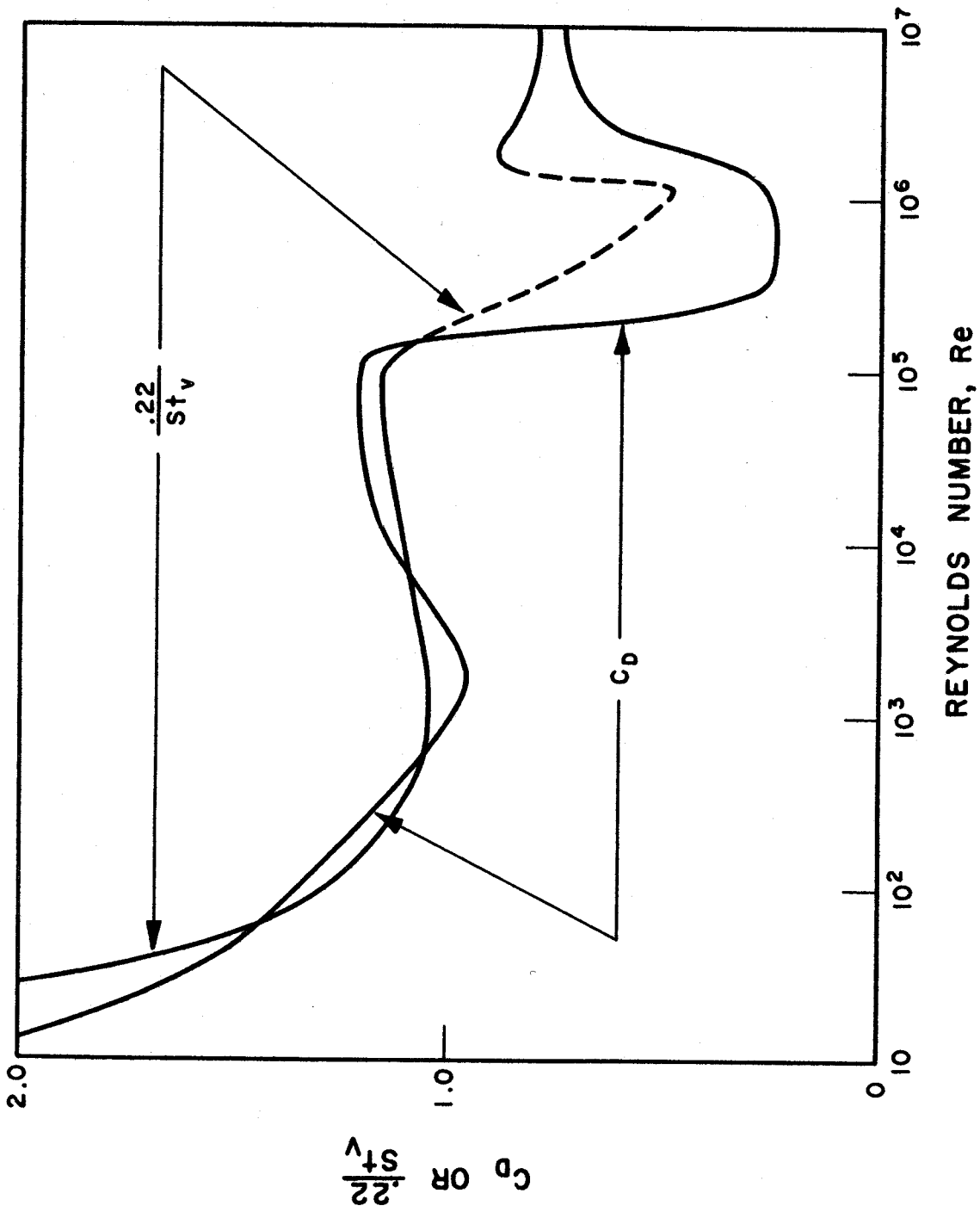


Fig. 11. The Similarity of Drag Coefficient and Inverse Strouhal Number.

$$C_D St_V \approx 0.22 \quad , \quad 40 \leq Re < 2 \times 10^5 \quad (7)$$

Roshko wrote two NACA reports in 1954 ([44] and [7]) which show in a more detailed way how C_D and St_V are related. He first showed that the classical free-streamline theory of Kirchhoff (see, e.g., Lamb, op. cit., Sect. 76) could be applied to the free vortex layers leaving the cylinder. To do so he first had to modify the theory by setting the velocity of these layers at the separation point, equal to a velocity which corresponds with the actual aft pressure. The classical theory assumes that the free vortex layer departs at the free stream velocity, U . Figure 12a shows the free vortex layers that would leave a cylinder if the pressure after separation were that of the free stream. Figure 12b shows the free vortex layers that actually leave as a result of the reduced wake pressure.

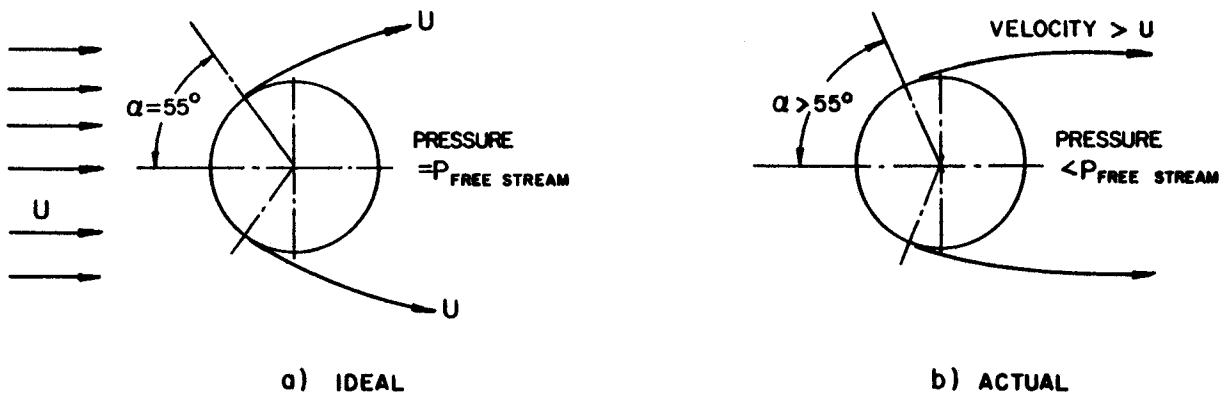


Fig. 12. Effect of Aft Pressure upon the Free Vortex Layers.

After determining the wake width in terms of the aft pressure--a knotty problem in conformal mapping, in its own right--there remained the problem of determining the aft pressure. Roshko achieved this by the clever correlation

of experimental results. He found that a unique correlation could be formed between the Reynolds and Strouhal numbers, if they were both based upon the wake width and velocity. This single correlation would apply for any bluff cross-sectional form. Then, given the shedding frequency, the wake pressure could be obtained by trial and error solution of the graphical relation:

$$\frac{(f_v) \text{ (wake width)}}{\text{velocity of vortex layer}} =$$

$$f_n \left(\frac{\text{(velocity of vortex layer)} \text{ (wake width)}}{2s} \right)$$

and the analytical relation:

$$\text{wake width} = f_n(\text{wake pressure})$$

Finally, C_D could be determined when the free vortex layer configuration and the aft pressure were known.

Roshko's analysis makes the important point that drag is related to a vortex-induced low-pressure region in the wake. Thus, not only lift, but much of drag as well, might be eliminated if vortex action could be inhibited. To this end both Roshko (1954) and Grove et al. made experiments related to the use of a splitter plate or splitter vane to suppress vortex formation.

The splitter plate is nothing more than a horizontal plate, parallel with the cylinder axis and the free-stream, placed in the wake of the cylinder. The results of two of Roshko's splitter plate experiments are shown in Fig. 13. The first of these shows the effect of a splitter plate almost five diameters in length upon the local pressure coefficient, C_p , where:

$$C_p \equiv \frac{P_{\text{interest}} - P_{\text{free stream}}}{\frac{1}{2}\rho U^2} \quad (8)$$

In this case P_{interest} is the mean pressure at points immediately downstream from the cylinder. Clearly, the splitter plate obstructs vortex formation and

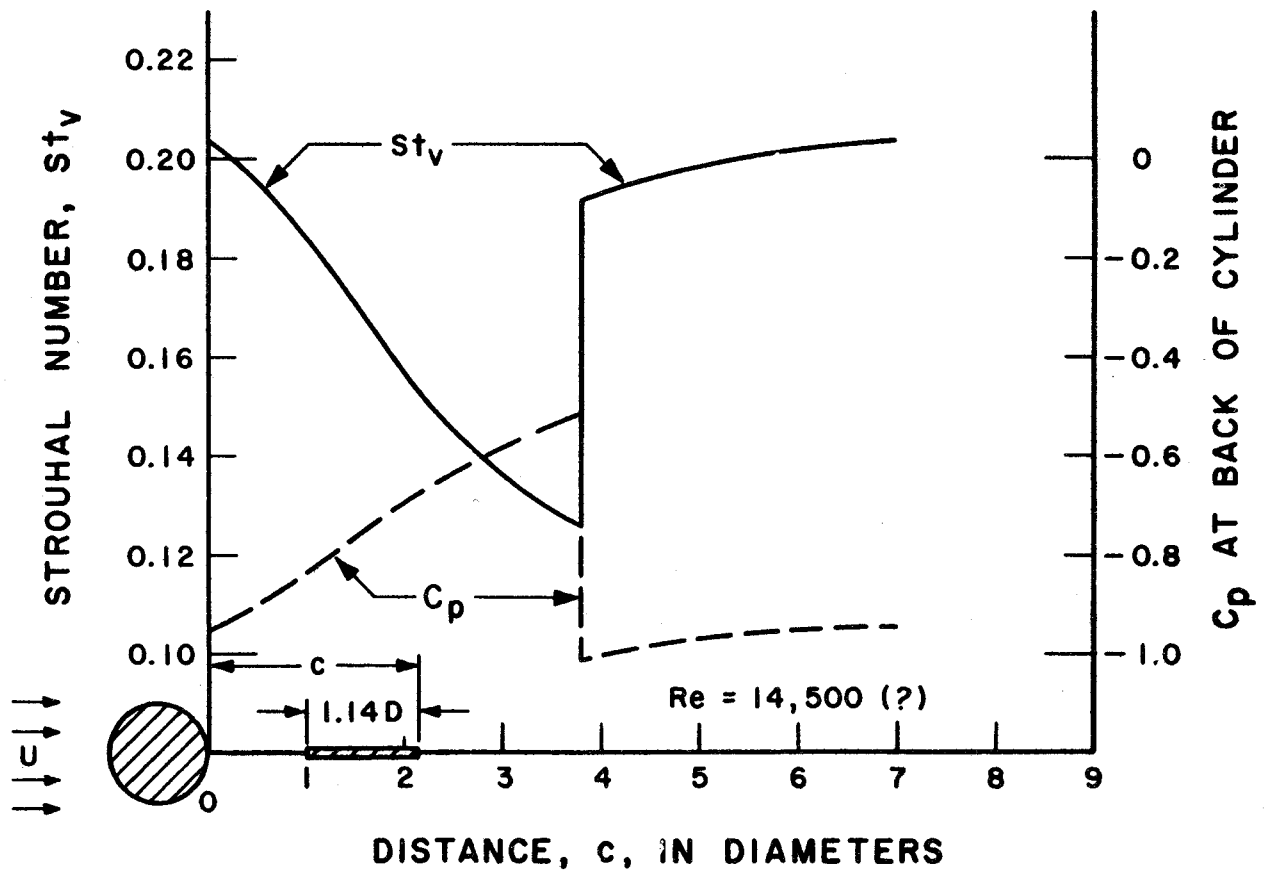
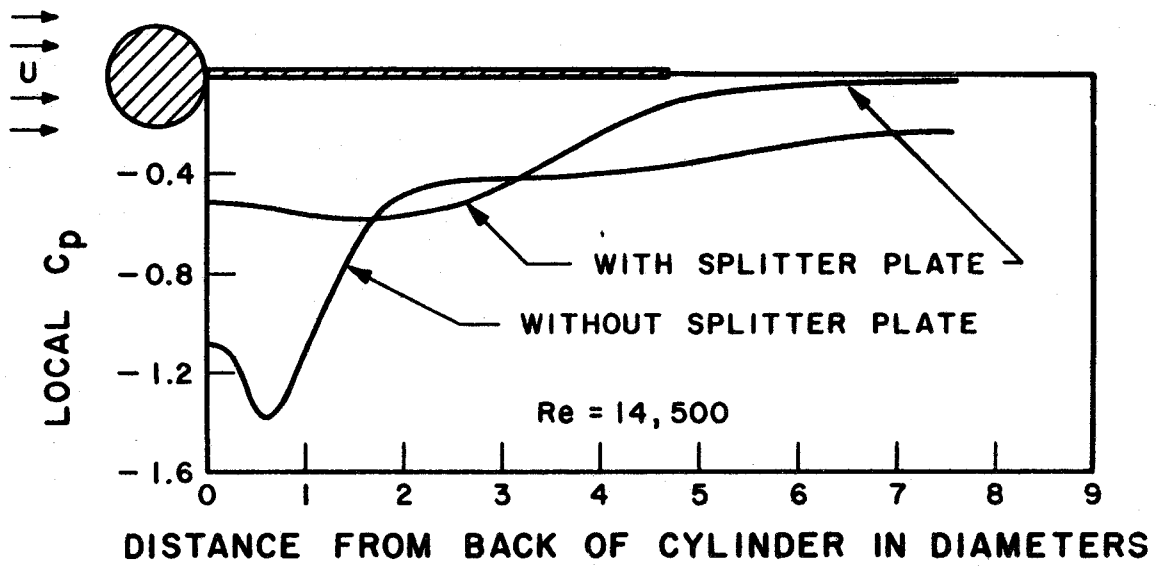


Fig. 13. Effects of Splitter Plates upon St_v and Localized Pressure Coefficients.

eliminates the extreme reduction in pressure in the wake. The effect of the splitter plate in this case has already been shown (see Fig. 7) to reduce C_{Dp} by more than one third.

The second experiment shows the effect of spacing a short splitter plate at various distances from the cylinder. The plate shown has its greatest effect when the distance, c , is 3.85 cylinder diameters. Then it abruptly loses its entire effect. It is for this spacing that alternate vortex shedding can re-establish itself between the aft end of the cylinder and the splitter plate.

Very few drag data, and no lift data, are available for cylinders with splitter plates. In addition to his result for $Re = 14,500$, Roshko (1961) also observed C_D in the range $Re > 3.5 \times 10^6$ with a splitter plate 2-1/3 diameters in length located immediately aft of the cylinder. These results, shown with the regular C_D data in Fig. 7, suggest that for supercritical flow the splitter plate has no effect whatsoever. Roshko offers an heuristic explanation in terms of the complexities of the separation process; however, the explanation must still be regarded as not totally resolved.

The very recent study of the Föppl vortex regime by Grove et al. employed a splitter plate to stabilize the wake up to $Re = 150$. They obtained C_{Dp} data for a splitter plate 4 diameters in length with a c dimension of about 7 diameters. Their results, shown in Fig. 6, reveal as much as a one third reduction in pressure drag.

The lack of lift data for cylinders with splitter plates should not prevent us from guessing that lift would be even more dramatically reduced than drag. Since the observed reduction of drag is almost certainly the result of inhibiting the alternating vortex-shedding action, the lift force--and with it aeolian vibration--should be strongly attenuated or even eliminated with a splitter plate.

Another scheme for inhibiting vortex shedding has already been investigated by Weaver in the work that we cited earlier. He wound smooth cylinders with a single helix of small diameter tubing. In much the same way that transmission line conductors are wound with an outer lay of wire. He obtained data for stationary cylinders only in the boundary layer transition regime. As one might anticipate, he observed that the decrease in C_K with Reynolds number is reduced, just as Fage and Warsap's data (see Fig. 8) showed that the dip in C_D is attenuated by cylinder roughness. The real importance of Weaver's work lies in its relevance to the aero-elastic problem of the wind excitation of vibrating cylinders--a matter that we shall not consider here.

LITERATURE CITED

- 1.) A.W. Marris, "A Review on Vortex Streets, Periodic Waxes, and Induced Vibration Phenomena," Jour. Basic Engr., vol. 86, no. 2, 1964, p. 185.
- 2.) L. Föppl, Sitzungsberichte d. k. bayer. Akad. d. Wissensch. Zu Nunchen (1913) 1 - 17. The Föppl Analysis is given in essence as an example in Milne-Thomson, Theoret. Hydrodynamics, fourth edition, Macmillan, 1960, p. 366.
- 3.) L. S. G. Kovasznay, "Hot Wire Investigations of the Wake Behind Cylinders at Low Reynolds Numbers," Proc. Roy. Soc. A, vol. 198, 1949, p. 174.
- 4.) D. J. Tritton, "Experiments on the Flow Past a Circular Cylinder at Low Reynolds Numbers," Jour. Fluid Mech., vol. 6, part 4, November, 1959, p. 547.
- 5.) J. W. Schaeffer and Salamon Eskinazi, "An Analysis of the Vortex Street Generated in a Viscous Fluid," Jour. Fluid Mech., vol. 6, part 2, August 1959, p. 241.
- 6.) A. Roshko, "On the Development of Turbulent Wakes from Vortex Streets," NACA TN 2913, 1953.
- 7.) A. Roshko, "On the Development of Turbulent Wakes from Vortex Streets," NACA Report 1191, 1954, p. 801; or A. Roshko, "On the Wake and Drag of Bluff Bodies," Jour. Aero. Sci., vol. 22, no. 2, 1955, p. 801.
- 8.) Morris S. Macovsky, "Vortex Induced Vibration Studies," David Taylor Model Basin Report 1190, 1958.
- 9.) John S. Humphreys, "On a Circular Cylinder in a Steady Wind at Transition Reynolds Numbers," Jour. Fluid Mech., vol. 9, part 4, December, 1960, p. 603.
- 10.) O.M. Phillips, "The Intensity of Aeolian Tones," Jour. Fluid Mech., vol. 1, 1956, p. 607.
- 11.) M. S. Bloor, "Transition to Turbulence in the Wake of a Circular Cylinder," Jour. Fluid Mech., vol. 19, part 2, June, 1964, p. 290.
- 12.) N. K. Delany and N. E. Sorensen, "Low Speed Drag of Cylinders of Various Shapes," NACA TN 3038, 1953.
- 13.) Y. C. Fung, "Fluctuating Lift and Drag Acting on a Cylinder in a Flow at Supercritical Reynolds Numbers," Jour. Aero. Sci., vol. 27, no. 11, 1960, p. 801.
- 14.) A. Roshko, "Experiments on the Flow Past a Circular Cylinder at Very High Reynolds Number," Jour. Fluid Mech., vol. 10, part 3, 1961, p. 345.
- 15.) E. Tyler, Phil. Mag., vol. 11, 1931, p. 849.

- 16.) E. F. Relf and L. F. G. Simmons, "On the Frequency of Eddies Generated by the Motion of Circular Cylinders through a Fluid," Phil. Mag., 6th series, vol. 49, 1925, p. 509.
- 17.) E. F. Relf, "On the Sound Emitted by Wires of Circular Section When Exposed to an Air Current," Phil. Mag., series 6, vol. 42, 1921, p. 173.
- 18.) V. R. Lehnert, "Akustische Messungen an Wirbelstrassen hinten Kreiszyylinder und Ebener Platte," Phys. Zeit., vol. 38, 1937, p. 476.
- 19.) J. H. Gerrard, "Measurements of the Sound from Circular Cylinders in an Air Stream," Proc. Phys. Soc., vol. 68, series B, 1955, p. 453.
- 20.) B. Etkin, G. K. Korbacher and R. T. Keefe, "Acoustic Radiation from a Stationary Cylinder in a Fluid Stream (Aeolian Tones)," Jour. Acoustical Soc. Am., vol. 29, no. 1, 1957, p. 30.
- 21.) G. K. Korbacher and L. R. Fowell, UTIA Rev., vol. 8, 1955.
- 22.) H. Drescher, "Messung der auf Querangeströmte Zylinder Ausgeübten Zeitlich Veränderten Drücke," Z. f. Flugwiss., vol. 4, heft 1/2, 1956, p. 17.
- 23.) W. Weaver, "Wind-Induced Vibrations in Antenna Members," Jour. Engr. Mech. Div., ASCE, vol. 87, no. EMI, 1961, p. 141.
- 24.) J. H. Gerrard, "An Experimental Investigation of the Oscillating Lift and Drag of a Circular Cylinder Shedding Turbulent Vortices," Jour. Fluid Mech., vol. 11, part 2, 1961, p. 244.
- 25.) G. Birkhoff and E. H. Zarantonello, Jets, Wakes and Cavities, Academic Press, Inc., New York, 1957, Chap. XIII.
- 26.) J. K. Vennard, Elementary Fluid Mechanics, John Wiley and Sons, Inc., New York, third edition, 1954.
- 27.) E. F. Relf, A.R.C., R. and M., no. 102, 1914. (See, e.g., Goldstein [32].)
- 28.) Von C. Wieselsberger, "Neuere Feststellungen über die Gesetze des Flüssigkeits- und Luftwiderstandes," Phys. Zeit., vol. 11, June, 1921, p. 321.
- 29.) Sir Horace Lamb, Hydrodynamics, Dover Publications, New York, sixth edition, 1945, sect. 343.
- 30.) A. Thom, A.R.C., R. and M., no. 1194, 1929. (See, e.g., Goldstein [32].)
- 31.) A. Thom, "The Flow Past Circular Cylinders at Low Speeds," Proc. Roy. Soc. A, vol. 227, 1933, p. 367.
- 32.) S. Goldstein, Modern Developments in Fluid Dynamics, vol. 11, Oxford Press, London, 1938, chap. IX.
- 33.) H. L. Dryden and G. C. Hill, "Wind Pressure on Circular Cylinders and Chimneys," Bur. Std. J. Res., Wash., vol. 5, 1930, p. 653.

- 34.) A. S. Grove, F. H. Shair, E. E. Petersen, and A. Acrivos, "An Experimental Investigation of the Steady Separated Flow Past a Circular Cylinder," Jour. Fluid Mech., vol. 19, part 1, 1964, p. 60.
- 35.) F. Homann, "Einfluss Größer Zähigkeit Bei Strömung um Zylinder," Forsch. Ing. Wis., vol. 7, no. 1, 1936.
- 36.) John Stack, "Compressibility Effects in Aeronautical Engineering," NACA ACR, 1941.
- 37.) Fage and Warsap, A.R.C., R. and M., no. 1283, 1930.
- 38.) "Ergibnisse der Aerodynamischen Versuchsanstalt Zu Göttingen," 2, no. 28 and 34, 1923.
- 39.) D. M. McGregor, "An Experimental Investigation of the Oscillating Pressure on a Circular Cylinder in a Fluid Stream," Univ. Toronto Inst. Aerophys., T.N. 14, 1957.
- 40.) R. Goldman, "The Generation and Suppression of VonKarman Vortex Forces," Engr. Rept. No. 8984, Martin Co., Baltimore, 1957.
- 41.) B. H. Bingham, D. K. Weimer, and W. Griffith, "The Cylinder and Semi-Cylinder in Subsonic Flow," Tech. Rept. No. 11-13, Princeton Univ. Dept. of Physics, July, 1952.
- 42.) Fujino et al., Proceedings, Seventh Japan National Congress for Applied Mechanics, 1957.
- 43.) A. Davenport, Jr., "The Wind-Induced Vibration of Guyed and Self-Supporting Cylindrical Columns," Trans. Engr. Inst. Canada, vol. 3, no. 4, 1959.
- 44.) A. Roshko (1954), "A New Hodograph for Free-Streamline Theory," NACA TN 3168, 1954.