Reprinted from JOURNAL OF HEAT TRANSFER, *Vol. 102, No. 3, August 1980*

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On the Prediction of the Minimum Pool Boiling Heat Flux

A criterion is offered for the collapse of film boiling in a saturated liquid at the minimum heat flux. The criterion says the vapor film collapse occurs when insufficient vapor is generated to sustain the growing wave after it reaches a constant rate of increase of amplitude. This criterion yields an accurate prediction for horizontal flat plates and cylinders. The prediction requires the use of empirical generalizations about the configuration of film boiling, which are also developed here.

Objective

The literature on the prediction of $q_{\min}$, the minimum saturated pool boiling heat flux, includes several basic papers, each of which was written before certain relevant information became available. We wish to refine the theory on the basis of more recent information. First let us review what has been done.

History

Chang [1] appears to be the first investigator who recognized that, to understand film boiling behavior, one had to understand the Taylor unstable wave action in the liquid-vapor interface above the heater. Zuber and Tribus (see, e.g., [2]) showed how $q_{\min}$ could be predicted on the basis of this action. Their development was restricted to the infinite horizontal flat heater. It can be paraphrased in the following way. They began by noting that

$$\frac{q_{\min}}{\rho_g h_{fg}} = u_g = \left(\frac{\text{volume of bubbles}}{\text{heater area-cycle}}\right)\left(f\ \frac{\text{cycle}}{\text{s}}\right) \tag{1}$$

Here we designate volume of vapor created per unit area of heater as the velocity, u_g, and the lowest frequency at which bubbles can leave each node, as f. Figure 1 is a typical photograph of film boiling from a horizontal wire. It illustrates the collapsing Taylor wave motion. (It is much harder to photograph film boiling on a large flat plate, although Dhir, Castle, and Catton [3] have photographed analogical behavior during the sublimation of a slab of dry ice under warm water.) Above a horizontal plate, the Taylor wave action is as shown in Fig. 2.

Zuber completed his derivation subject to several assumptions. They are

1 The bubble radius is $\lambda_{d_1}/4$ where λ_{d_1} is the length of a one-dimensional wave given by [4]

$$\lambda_{d_1} = \frac{2\pi\sqrt{3}}{\sqrt{\dfrac{g(\rho_f - \rho_g)}{\sigma}}} \tag{2}$$

2 The minimum frequency was taken as the inverse of the time needed for a wave to grow to an amplitude of λ_{d_1}, based on its amplitude-averaged growth rate up to an amplitude of $0.4\lambda_{d_1}$ (assuming that during this period, the growth can be predicted by the simple linear theory.)

3 Two bubbles are released per cycle per area equal to $\lambda_{d_1}^2$

The first assumption has stood up to several people's subsequent measurements (see, e.g., [5]). Berenson [6] noted in 1961 that Zuber's amplitude average should have been a time average and that nonlinear growth, beyond an amplitude of $0.4\lambda_d$, could not be obtained by an extrapolation in any event. His $q_{\min}$ formula therefore took the form

$$u_g = A\sqrt[4]{\frac{\sigma g(\rho_f - \rho_g)}{(\rho_f + \rho_g)^2}} \tag{3}$$

where A was set at 0.09, based on two carefully measured data points.

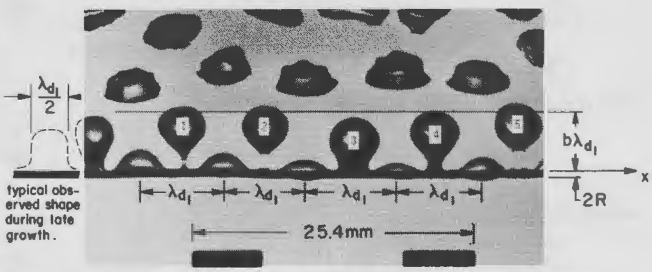

Fig. 1 Typical bubble departure pattern during film boiling (Radius of wire, R = 0.323mm, $q = 1.3 \times 10^5$ W/m², fluid is acetone at 1 atm, R' = 0.0203, b_4 = 0.65)

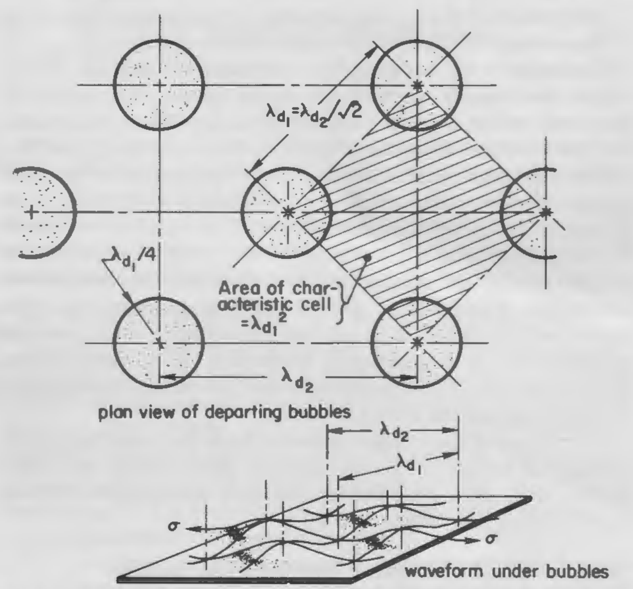

Fig. 2 The array of vapor jets as seen on an infinite horizontal heater surface

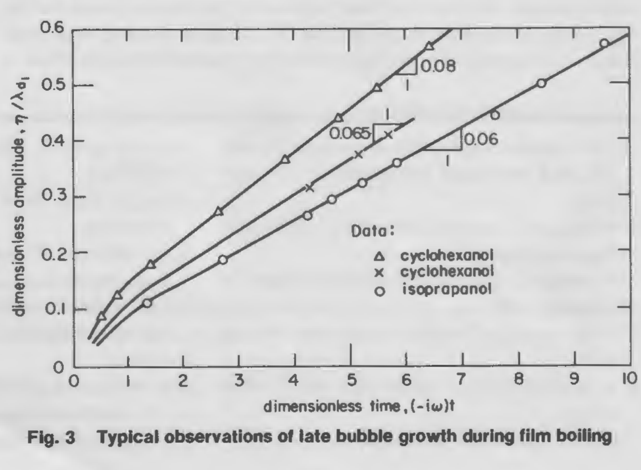

Fig. 3 Typical observations of late bubble growth during film boiling

[1] This work was done when the author was with the Department of Mechanical Engineering, University of Kentucky, Lexington, KY.

Contributed by the Heat Transfer Division for publication in the JOURNAL OF HEAT TRANSFER. Manuscript received by the Heat Transfer Division January 11, 1980.

Zuber's prediction gave $A = 0.177$.

In 1969, Sernas [7] showed that the wavelength in a horizontal plane interface—what we shall call the two-dimensional wavelength, λ_{d_2},—is

$$\lambda_{d_2} = \sqrt{2}\lambda_{d_1} \tag{4}$$

(see Fig. 2). He attempted to correct Zuber's formula but he assumed that the bubble diameter was still $\lambda_{d_1}/2$ instead of $\lambda_{d_2}/2$, and following Zuber he counted two bubbles in each area of λ_{d2}^2, instead of the correct number, four. (See Fig. 2. He corrected the latter point in 1973 [8].) He also retained Zuber's estimate of f.

Meanwhile Lienhard and Wong [5] had redone Zuber's flat plate analysis for a horizontal cylinder, after they had first derived λ_{d_1} for such a configuration:

$$(\lambda_{d_1})_{cyl.} = \frac{2\pi\sqrt{3}}{\sqrt{\dfrac{g(\rho_f - \rho_g)}{\sigma} + \dfrac{1}{2R^2}}} \tag{5}$$

The resulting $q_{\min}$ prediction can be written as

$$u_g = B\sqrt[4]{\frac{g\sigma(\rho_f - \rho_g)}{(\rho_f - \rho_g)^2}} \Big/ [R'^2(2R'^2 + 1)]^{1/4}, \tag{6}$$

where

$$R' \equiv R\sqrt{\frac{g(\rho_f - \rho_g)}{\sigma}} \tag{7}$$

The Criterion for the Collapse of Film Boiling

The prediction of $q_{\min}$ using equation (1) only requires knowledge of the average velocity of vapor needed to maintain the cyclic growth and collapse of the interface. To understand the physical circumstances that specify the minimum vapor volume flow that can be sustained, we first consider the observed vapor removal configuration on a flat plate and above a horizontal cylinder (see Figs. 1 and 2).

The horizontal cylinder and horizontal flat plate generate vapor at a nearly constant rate and deliver it to an escapement process in the collapsing Taylor wave above the heater. Thus, $q_{\min}$ will occur when, as q is reduced, there is one point in the cycle where the constant rate of vapor production becomes insufficient to supply the instantaneous natural rate of wave motion. Then the wave can no longer stay off the heater, and film boiling must collapse. This criterion differs from that used by Zuber [2] in obtaining u_g in equation (1). As we have noted, Zuber attempted to determine the slowest overall natural frequency of oscillation that the wave can sustain.

To make use of the criterion we have to know the instantaneous rate of change of volume of a bubble during its growth period, and to find this we must set down some experimental observations of bubble growth during film boiling near $q_{\min}$.

Observed Bubble Action During Film Boiling

Consider the numbered bubbles in Fig. 1. Bubbles No. 2 and 5 have severed from the vapor film removing themselves as an avenue of vapor escape. Bubble No. 1 has just barely broken away and bubble No. 3 has a little way to go. Bubble No. 4 is just ceasing to provide means for vapor to escape, and its top has reached a height which is a fraction, $b = 0.65$, of the wavelength. We also observe that, although bubble height at departure is about $0.65\lambda_{d_1}$, the bubble radius of curvature is never less than $\lambda_{d_1}/4$ (see Fig. 1)

In the early stages, the growth process is exponential with an amplitude, η_m, given by

$$\eta_m = \eta_{m0}e^{-i\omega t}\cos\frac{2\pi x}{\lambda_{d_1}} \tag{8}$$

where x is an axial coordinate (see Fig. 1), η_m, evaluated at time, $t = 0$; and $(-i\omega)$ is the wave growth rate—a positive real number. Several observations of wave amplitude, η_m, as a function of time in isopropanol [9, 10] and cyclohexanol [10], show that exponential growth blends into a linear growth after the amplitude reaches a value of $0.12 \leq \eta_m/\lambda_{d_1} \leq 0.25$. A sampling of these data is shown in Fig. 3. It is also very clear from the data plotted in Fig. 3 that a linear growth rate is maintained until the bubble detaches from the interface. The growth rate in the linear region can be written as

$$\frac{d\eta}{dt} = c\lambda_{d_1}(-i\omega) \tag{9}$$

The value of the constant c is listed in Table 1 along with the values of b, the bubble height at detachment divided by λ_{d_1}.

Model for Instantaneous Bubble Volume Growth Rate

Figure 4(a) shows a geometrical model for bubble shape in the exponential and early linear growth period. If the growing interface is a part of a sphere, the bubble volume at any instant is

$$V = \pi R\eta^2\left(1 - \frac{\eta}{3R}\right) \tag{10}$$

The base radius, r, of the bubble is generally less than or equal to its curvature. To distinguish between r and R we observe that

$$r^2 = R^2 - (R - \eta)^2$$

The instantaneous rate of change of bubble volume is obtained by differentiating equation (10).

$$\frac{dV}{dt} = \pi(R\eta - \eta^2)\frac{d\eta}{dt} + \pi\eta^2\frac{dR}{dt} \tag{12}$$

However, we noted earlier that the radius of curvature of the bubble stays constant after the bubble becomes hemispherical and its height reaches $\lambda_{d_1}/4$. Thus for bubble heights equal to or slightly greater than $\lambda_{d_1}/4$, equation (12) reduces to

$$\frac{dV}{dt} = \pi(2R\eta - \eta^2)\frac{d\eta}{dt} \tag{13}$$

In all cases we have observed, the bubble reaches a linear growth

Table 1 Observed values of c and b during film boiling in organic liquids [5,9,10], for various wire sizes and (in some cases) at elevated gravity

$\bar{c}$	Standard Deviation of c	$\bar{b} \equiv \dfrac{\text{height}}{\lambda_{d_1}}$	Standard Deviation of b
0.069	0.011	0.645	0.097

Nomenclature

A,B = constants defined in equations (3) and (6), and evaluated in equations (19) and (23)

b = (height of top of a detaching bubble) ÷ (appropriate λ_d)

c = constant of proportionality as used in equation (9)

f = frequency of bubble departures from a node

g = acceleration of gravity or other body force

h_{fg} = latent heat of vaporization

$-i\omega$ = growth rate of a wave (a real number)

q,q_{min} = heat flux; minimum pool boiling heat flux

u_g = velocity of vapor leaving a surface at $q_{\min}$, equal to $q_{\min}/\rho_g h_{fg}$

r,R,R' = radius of bubble at base, radius of top of bubble, $R\sqrt{\sigma/g(\rho_f - \rho_g)}$

t = time

V = volume of growing bubble

x,y = coordinates parallel with, and normal to, liquid-vapor interface

η_m,η_{m0} = amplitude of wave; amplitude at time $t = 0$

$\lambda_{d_1},\lambda_{d_2}$ = the most susceptible wavelength in a one-dimensional interface, in a two-dimensional interface

ρ_f,ρ_g = saturated liquid and vapor densities

σ = surface tension

τ = period of bubble cycle, equal to $1/f$

ω = circular frequency of a traveling wave (a pure imaginary number) rate by the time the bubble is a hemisphere. Therefore we can substitute $d\eta/dt$ from equation (9) in equation (13) and get

$$\frac{dV}{dt} = \pi c(-i\omega)\lambda_{d_1}(2R\eta - R^2) \tag{14}$$

We are interested in a maximum of dV/dt during a growth cycle. According to equation (14), dV/dt will be maximum when the interface height, η, equals the radius of curvature, R. This is true when the bubble is hemispherical and $\eta = R = \lambda_{d_1}/4$. Thus the equation for the maximum value of dV/dt is

$$\frac{dV}{dt} = \frac{\pi c}{16}(-i\omega)\lambda_{d_1}^3 \tag{15}$$

Following the criterion developed earlier, film boiling will collapse when the vapor generation rate at the heater falls below the value given by equation (15).

The Minimum Heat Flux on Flat Plates

Figure 2 shows that, on the average, one bubble at a time is supported in a heater area, $\lambda_{d_1}^2$. Thus we obtain the minimum vapor velocity, u_g, from equation (15), as

$$u_g = \frac{dV}{dt}\frac{1}{\lambda_{d_1}^2} = \frac{\pi c}{16}(-i\omega)\lambda_{d_1} \tag{16}$$

But the growth rate is given [4] as

$$(-i\omega) = \sqrt{\frac{\pi}{\lambda_{d_1}}\left[\frac{g(\rho_f - \rho_g)}{\rho_f + \rho_g} - \frac{\pi^2\sigma}{\lambda_{d_1}^2(\rho_f + \rho_g)}\right]} \tag{17}$$

Substituting $(-i\omega)$ from equation (17), and λ_{d_1} from equation (2), into equation (16) we get

$$u_g = 1.32c\sqrt[4]{\frac{g\sigma(\rho_f - \rho_g)}{(\rho_f + \rho_g)^2}} \tag{18}$$

Finally, making use of the value of c from Table 1, we obtain minimum heat flux on a flat plate as

$$(q_{\min})_{\text{flat plate}} = \underbrace{0.091\,{}^{0.078}_{0.104}}_{A}\,\rho_g h_{fg}\sqrt[4]{\frac{g\sigma(\rho_f - \rho_g)}{(\rho_f + \rho_g)^2}} \tag{19}$$

where the range of A corresponds with the standard deviation of c.

Berenson's measurements indicated that the best experimental values for n-pentance and CCl_4, respectively, gave the lead constant in equation (19) equal to 0.089 and 0.091. Berenson chose a mean value of 0.09 for the constant. The constant obtained using the present criterion is nearly the same as Berenson's. (Other flat heater data can be found, but all are inadequate for one or more of three reasons: They lack vertical sidewalls to eliminate induced flows, they are not enough larger than λ_{d_2} in size, or they have not been obtained with Berenson's attention to cleanliness.)

A few minimum heat flux data obtained during the sublimination of a horizontal slab of dry ice placed beneath a pool of water or benzene have been reported in reference [3]. During sublimation, the gas-liquid interface was found to grow, collapse and regrow at the same location rather than to alternate between nodes and antinodes as observed in film boiling on cylinders (Fig. 1) and as proposed in Fig. 2 for film boiling on flat plates. This peculiar behavior during "pseudo film boiling" is caused by uneven sublimation of the surface underneath the gas releasing nodes and under the gas film. The antinodes of the wave that coincided with the valleys on the surface remained inactive because of the concave nature of the valley. Visual observations showed that, on the average, the bubble release nodes fall on a square grid with a spacing of λ_{d_1} (See Fig. 2). For the cyclic bubble release process to continue on dry ice with a nearly uniform gas evolution rate, the neighboring bubbles must have a staggered growth pattern. This suggests that pseudo film boiling on dry ice will be sustained as long as the gas volume flux can support a maximum volume growth rate between one-half and one bubble in an area of $\lambda_{d_1}^2$.

This means that during pseudo film boiling the lead constant in equation (19) should be cut in half. The minimum heat flux data reported in [3] for water over dry ice are indeed correlated to within 3 percent with equation (19), when the numerical constant is reduced from 0.091 to 0.0455. Likewise, pseudo film boiling data for benzene are within 5 percent when the numerical constant in equation (19) is reduced by 75 percent. This is because the shift from one bubble to one half bubble is incomplete in the case of benzene.

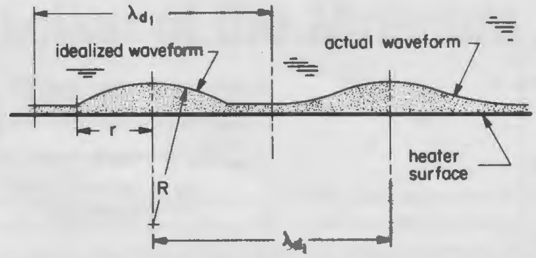

Fig. 4(*a*) Idealization of waveform during early growth for a flat heater

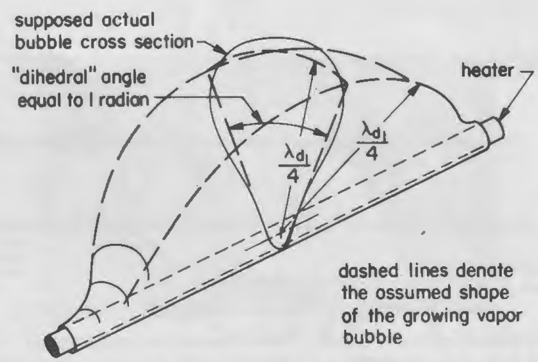

Fig. 4(*b*) Idealization of bubble growing on a horizontal wire

Fig. 4 Bubble shapes during film boiling

The Minimum Heat Flux on a Horizontal Cylinder

An average of one bubble is supported on a surface area, $2\pi R\lambda_{d_1}$, of a cylindrical heater. However, because the diameter of the cylinder is much smaller than the width of the bubble at the top, the bubble will be a spherical wedge, as shown in Fig. 4(*b*), rather than a hemisphere during its early period of growth. The top of the bubble is taken to be a spherical surface with both radius of curvature and arc length equal to $\lambda_{d_1}/4$. The contribution of the volume of the heater to the volume of the spherical wedge is ignored. After correcting equation (15) to give the volume of a spherical wedge whose diahedral angle is unity instead of π, we get the following expression for the vapor volume flux.

$$u_g = \frac{\frac{\pi c}{16\pi}(-i\omega)\lambda_{d_1}^3}{2\pi R\lambda_{d_1}} = \frac{c}{32\pi}(-i\omega)\frac{\lambda_{d_1}^3}{R} \tag{20}$$

The growth rate for waves above a cylinder has been given in [5] as

$$(-i\omega) = \sqrt{\frac{2\pi}{\lambda_{d_1}}}\left[\frac{g(\rho_f - \rho_g)}{\rho_f + \rho_g} - \frac{4\pi^2\sigma}{\lambda_{d_1}^2(\rho_f + \rho_g)} + \frac{\sigma}{2(\rho_f + \rho_g)(R + \delta)^2}\right]^{1/2} \tag{21}$$

Substituting the wavelength from equation (7), and this growth rate, into equation (20), we get

$$u_g = 0.866c\sqrt[4]{\frac{g\sigma(\rho_f - \rho_g)}{(\rho_f + \rho_g)^2}}\,[R'^2(2R'^2 + 1)]^{-1/4} \tag{22}$$

where $R' \equiv R\sqrt{\sigma/g(\rho_f - \rho_g)}$. Using a mean value of c from Table 1,

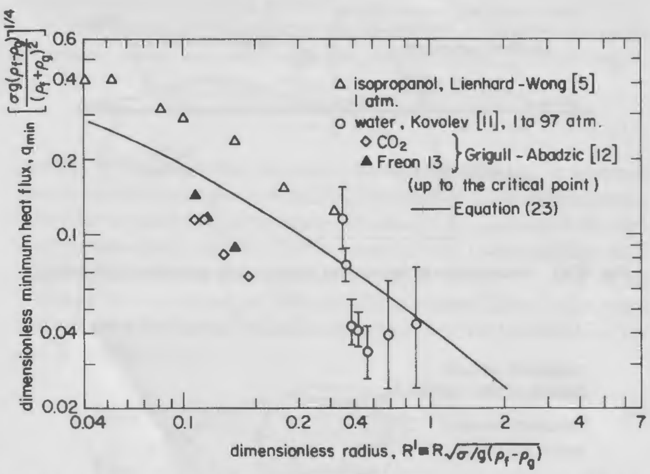

Fig. 5 **Comparison of the q_{min} prediction with the rather scattered available data**

we obtain for the minimum heat flux on a cylindrical heater

$$q_{min} = \underbrace{0.06^{0.051}_{0.068}}_{B} \rho_g h_{fg} \sqrt[4]{\frac{g\sigma(\rho_f - \rho_g)}{(\rho_f + \rho_g)^2}} [R'^2(2R'^2 + 1)]^{-1/4} \quad (23)$$

Very few of the available q_{min} data for cylinders can be used to test the validity of equation (23). All but the data of Kovalev [11] involve end mounting effects[1] that have the effect of raising q_{min} above its true value. The data in [5] probably suffer from end effects although precautions were taken to minimize them. The data of Grigull and Abadzic [12] were obtained with submerged ends in CO_2 near the critical point. Their wires doubtless had thick vapor blankets around them. This would have the influence of making q_{min} lower than it is when the blanket is thin, in conformity with the theory.

Consequently, the data exhibit about ±50 percent scatter on appropriately nondimensionlized coordinates in Fig. 5. But the important fact is that equation (23) falls in the middle of these data as the discussion above would lead us to expect. It is also interesting to note that the present equation falls within 3 percent of the expression established experimentally by Lienhard and Sun [9].

[1] Kovalev bent the ends of his heater wire upward, in a U-shape, so they exited from the liquid without ending in it.

Conclusions

A criterion for minimum pool boiling heat flux on horizontal flat plates and cylindrical heaters has been proposed. According to this criterion, the minimum heat flux occurs when the vapor volume flux is just sufficient to maintain the maximum volume growth rate of a bubble in a given cycle. This maximum volume flux occurs when interface grows linearly with time, the radius of curvature of the bubble is $\lambda_{d_1}/4$ and the bubble is either a hemisphere (for a flat plate) or a spherical wedge (for a cylindrical heater). The numerical constants obtained in this work are nearly the same as those obtained experimentally, and previously reported in the literature.

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