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Locked-in Vortex Shedding Behind Oscillating Circular Cylinders, with Application to Transmission Lines[1]

J. H. LIENHARD

Associate Professor, Mechanical Engineering, Washington State University, Pullman, Wash.
Mem. ASME.

L. W. LIU[2]

Engineer, The Boeing Company, Seattle, Wash.

The vortex shedding frequency has been measured for a water flow over cylinders undergoing forced vibration normal to the flow. A Reynolds number range of $970 \leq Re \leq 24{,}800$ and an amplitude-to-diameter ratio of 1/8 were employed in the tests. The results show that the vortex frequency locks in on the cylinder frequency over a frequency band whose width varies with Reynolds number. Locking-in also appears to occur over narrower bands at certain multiples and submultiples of the cylinder frequency. Steidel's transmission-line data are rationalized on the basis of these data.

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[2]Formerly, Research Assistant, Mechanical Engineering Department, Washington State University.

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J. H. LIENHARD   L. W. LIU

NOMENCLATURE

A = amplitude of vibration of cylinder
D = diameter of cylinder
f = frequency
f_V = frequency of vortex shedding
f_{V_O} = frequency of vortex shedding behind a stationary cylinder
f_I = frequency of input cylinder vibration
Δf_V = difference between frequency at end of locked-in region and f_{V_O}
fn = denoting an arbitrary function
Re = Reynolds number, $U_\infty D/\nu$
S = Strouhal number, fU_∞/D
S_V, S_{V_O}, S_I, ΔS_V = Strouhal numbers based upon f_V, f_{V_O}, f_I, and Δf_V
U = free stream velocity of liquid crossflow over cylinder
ν = kinematic viscosity

INTRODUCTION

The prediction of the frequency of aeolian vibration of freely suspended cables is a problem of great practical importance. Such lines will vibrate at one of many closely spaced natural frequencies. The difficulty is that of finding which frequency the line will "choose" under the influence of the wind.

We have sought to answer this question by inverting it as follows: What will be the frequency of vortex shedding, f_V, behind a circular cylinder vibrating at frequency, f_I; and how will this f_V relate to the frequency, f_{V_O}, for a stationary cylinder? The vortex shedding frequency in such an experiment will depend upon: the input frequency, f_I; the kinematic viscosity, ν, of the fluid; the amplitude, A, of imposed vibration; the normal velocity, U_∞, of the fluid far from the cylinder; and the diameter, D, of the cylinder. Dimensional analysis shows that:

$$S_V = S_V\,(S_I,\ \mathrm{Re},\ A/D) \qquad (1)$$

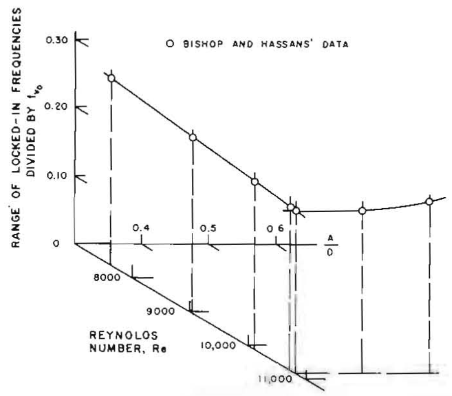

Fig. 1 Bishop and Hassans' locked-in regime

where S is the Strouhal number fU_∞/D, and Re is the Reynolds number $U_\infty D/\nu$.

Bishop and Hassan (1)[3] studied lift and drag forces in an experiment of this kind. They found that the frequencies, f_V, are usually very nearly equal to f_{V_O}. However, when f_I falls in a band surrounding f_{V_O} or certain of its multiples, then f_V will "lock-in" on f_I. It is thus possible to eliminate a variable from equation (1) by considering only the locked-in range, ΔS_V or Δf_V, above and/or below f_{V_O}. Thus

$$\Delta S_V/S_{V_O} = \mathrm{fn}(\mathrm{Re},\ A/D) \qquad (2)$$

Bishop and Hassan illustrated this relation over narrow ranges of Re and A/D with 7 data points shown in Fig.1. These data imply that the influence of A/D is weak, especially when its magnitude is small; and they indicate that Δf_V drops with increasing Re. We wish to extend the mapping of this locked-in regime and to show how it relates to aeolian vibration. In doing this, we (like Bishop and Hassan) have used simple-harmonic motion to approximate what is more of a

[3] Underlined numbers in parentheses designate References at the end of the paper.

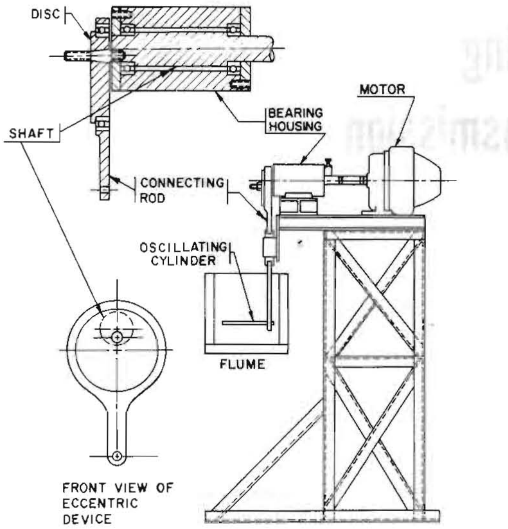

Fig. 2 Vibrating cylinder and flume apparatus

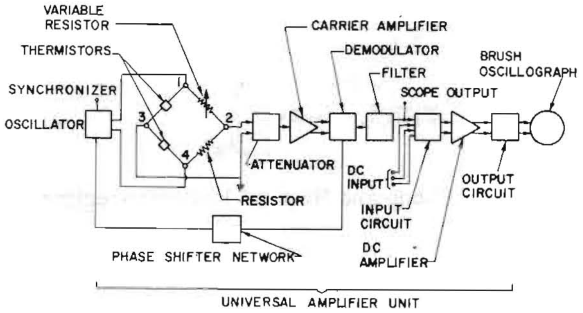

Fig. 3 Thermistor velocity-recording device

figure-eight motion in the case of aeolian vibration.

EXPERIMENT

Details of the experiment -- given fully in reference (2) -- will be sketched briefly here. An oscillating cylinder, driven by a variable-speed motor, moves up and down in a water flume as shown in Fig.2. The adjustable eccentric mechanism (shown in Fig.2) is a slider-crank device with a crank-to-connecting-rod ratio that is always less than 0.20. Accordingly, the simple harmonic motion is almost perfect. The water flume is 17 in. high, 12 in. wide, and 28 ft long, with the cylinder located about 8 ft from the downstream end.

Two schemes were used to determine the vortex frequency behind the cylinder. At low flow velocities hydrogen bubbles were used to mark the flow and a visual count of vortices was timed with a stop watch. At higher velocities a homemade

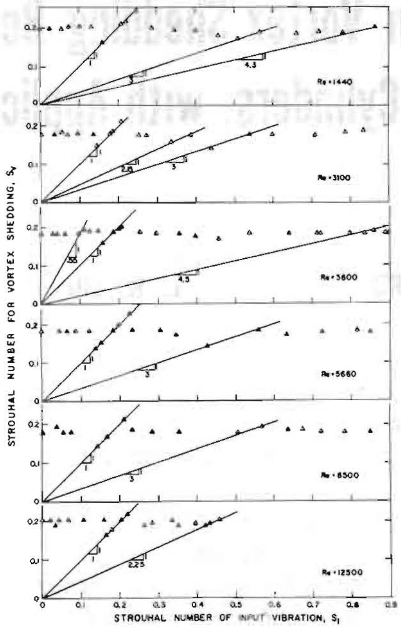

Fig. 4 Influence of cylinder frequency upon vortex frequency, 6 typical cases, A/D = 0.125

hot-wire anemometer was used for this purpose. This device employed two thermistors in place of conventional hot wires. These were placed one diameter apart (in the upper and lower vortex trails), and between 4 and 6 diameters downstream. The circuitry for this device is shown in Fig.3

The thermistors compensated one another for temperature and, to some extent, for effects of gross cylinder motion. The vortex shedding frequency could thus be counted directly from an oscillograph without any calibration. These observations were consistent with the visual observations made in the low-velocity range. The probable error of both schemes was found to be about 3 percent.

Sets of data were obtained by varying f_I, with U_∞ and D held constant. The ratio, A/D, was adjusted to 0.125 in all of the work reported here. This is a reasonable value for the aeolian vibration of large transmission-line conductors. It also provides a way of checking the relatively minor dependence of $\Delta S_V/S_{V_0}$ upon A/D, beyond Bishop and Hassan's range.

Typical plots of S_V versus S_I are given for six Reynolds numbers in Fig.4. In each case, S_V is generally equal to approximately S_{V_0}, except in regions in which it is a multiple or a submultiple of S_I. These locked-in regions are comparatively indistinct, with the exception of the fundamental one, $f_V = f_I$. The lesser ones that appeared repeatedly were: $f_I \simeq 0.55f_V$, $f_I \simeq 2.2f_V$, $f_I \simeq 3.0f_V$, and $f_I \simeq 4.4f_V$. The reader will note

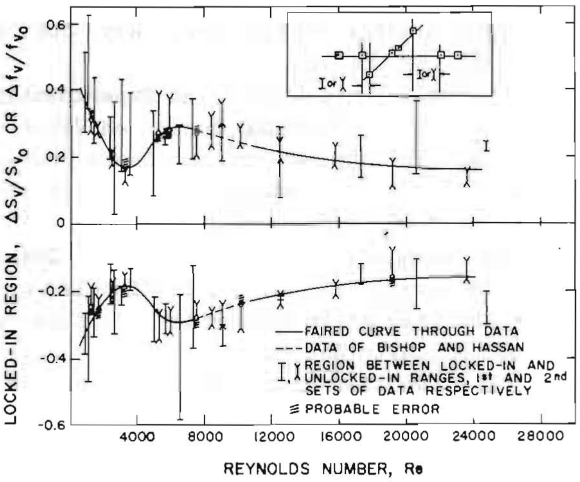

Fig. 5 Dependence of fundamental locked-in region upon Reynolds number, A/D = 0.125

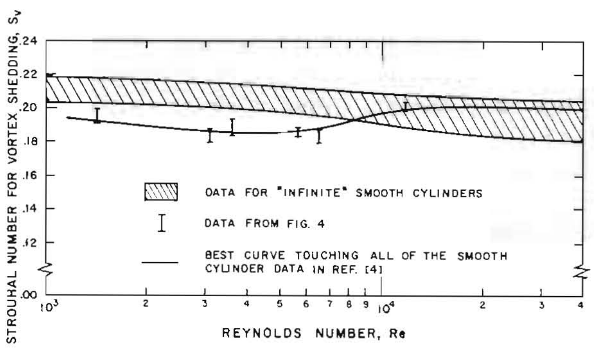

Fig. 6 Dependence of S_{v_0} upon Re for "infinite" cylinders and for present tests

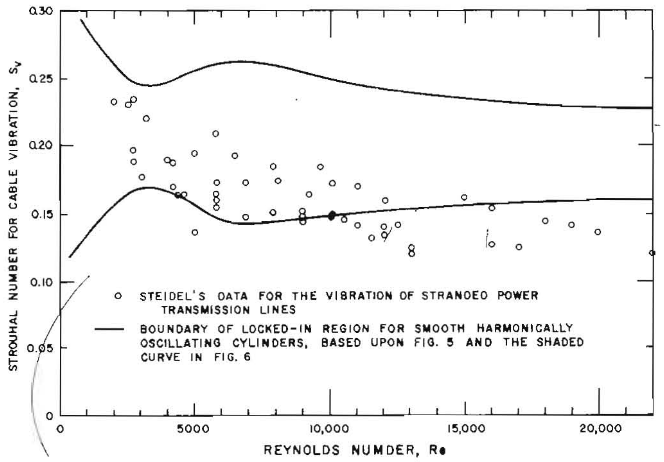

Fig. 7 Comparison present locked-in region with Steidel's transmission line data

that other minor locked-in regions are suggested by Fig. 4. Indeed, what we have called regions of constant frequency might well have turned out to be a fine sawtooth pattern of successive locked-in regions if the data had been continuous.

Two sets of runs were made, each of which provided a series of plots similar to those shown in Fig. 4. The first set revealed the fundamental locked-in region and indicated the need for more data points near the ends of the locked-in region. The second set of data provided these data specifically. Fig. 5 shows the collected results of both sets of runs shown in the form suggested by equation (2). The indeterminate range between the locked-in and unlocked-in regions has been presented in this figure, with a "best curve" through these data. This curve touches all of the data, although in a very few cases the probable error had to be included. Very little alteration of this curve would be permitted within the data.

Fig. 5 is interesting in several respects. The fundamental locked-in range appears to be symmetrical about S_{v_0}. Its form is complex; indeed, one data point at Re = 24,800 implies that more local maxima and minima might appear if the range of Re were extended even further. Finally, the data of Bishop and Hassan for A/D = 0.3 align perfectly with our data for A/D = 0.125.

Fig. 6 shows a "best estimate" of the Strouhal number for infinite, rigid smooth cylinders (3) based upon the data of many prior investigators. Data for stationary cylinders in the present apparatus (4) define the solid line shown in this figure. This line shows that the end and side effects present in our flume (see the cylinder mounting shown in Fig. 2) have slightly altered the S_{v_0} versus Re relation. Fig. 5 is normalized on the basis of these values of S_{v_0}.

APPLICATION TO TRANSMISSION-LINE VIBRATION

In 1956 Steidel (5) obtained vibration frequencies for a 900-ft span of 1-in-dia stranded cable. These results are plotted on a Strouhal-Reynolds number plot in Fig. 7. With the data is shown the locked-in region, obtained from Fig. 5, and based upon S_{v_0} for "infinite" cylinders. In the range Re $\leq 10^4$, the cable locks-in within the range indicated by our experiments. At higher Re's, the cable vibrates more slowly than it would have to, to fall within the locked-in region.

To see more clearly why this should be, we should note that transmission lines cannot vibrate at high frequencies because the energy dissipation rate increases rapidly with increasing frequency. Since, at high Re's, frequencies must be higher to get the same S_v, it is likely that the line begins to lock-in on submultiples of f_{v_0}

at these Re's. Conversely, at low Re's, cables tend not to vibrate at very low frequencies (i.e., in very long loops). Thus vibration frequencies seek the upper end of the fundamental locked-in region when Re is low.

ACKNOWLEDGMENTS

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