A SET OF NOTES ON TURBO-MACHINERY

by Harold Iversen

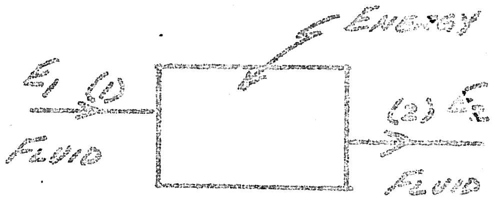
Harold Walter Iversen was a member of the Mechanical Engineering faculty at the University of California at Berkeley from 1941 until his death in 1973. He was an expert on turbo-machinery, with considerable industrial and consulting experience – including work on wave action during the atomic bomb tests at Bikini Atoll. He was also a dedicated teacher.

These are Iversen's notes for his exacting graduate course on turbo-machinery. They provide a rare and remarkably complete perspective. It is a pity and a great loss that he never got around to polishing them in book form. Iversen, a heavy smoker, died of cancer at the age of only 60.

Scope: Pumps, turbocompressors, hydraulic and gas turbines. Analysis of fluid machinery performance with emphasis on the application to fluid systems.

1.0 Classes of Fluid Machinery

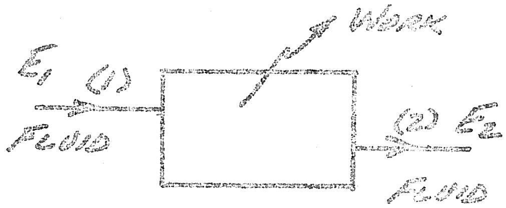
1.1 Pumps, compressors, blowers, fans.
Devices used to transport a fluid by adding energy to the fluid usually by a force acting on the fluid.



E – ENERGY

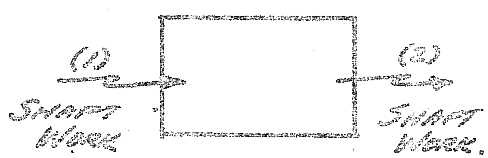
$E_2 > E_1$

1.2 Turbines, motors.
Devices used to transform the energy of a fluid stream into shaft work output.



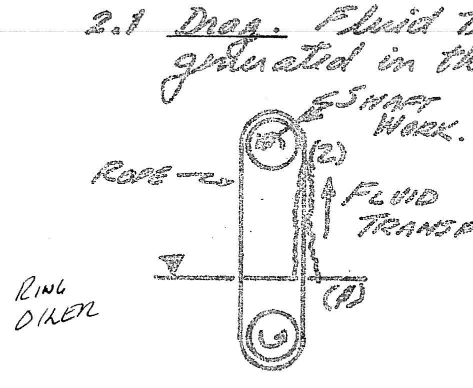
$E_1 > E_2$

1.3 "Hydraulic" couplings and torque converters.
Devices used to transfer work between two shafts primarily through fluid interaction.

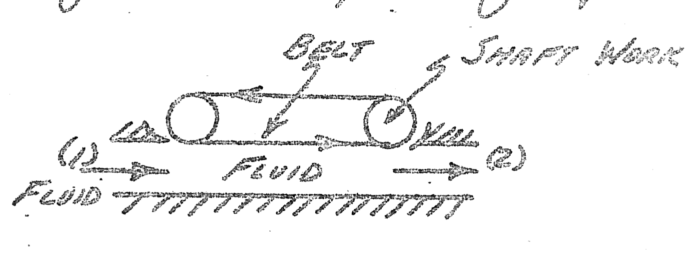


2.0 Classes of Pumps, Compressors, Blowers, Fans

2.1 Drag.
Fluid transport produced by shear forces generated in the fluid by the action of moving surfaces.

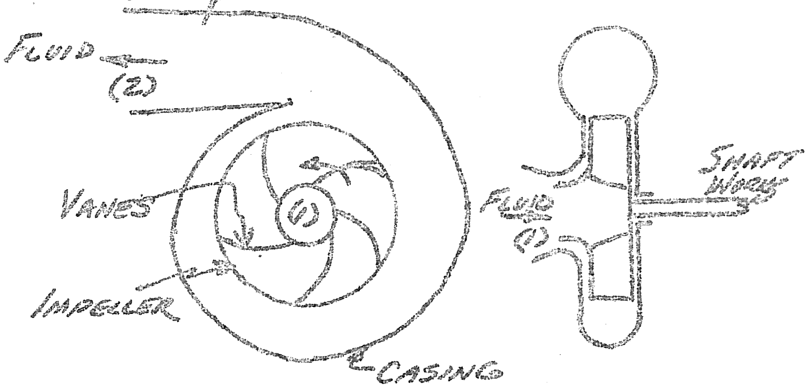


ROPE PUMP



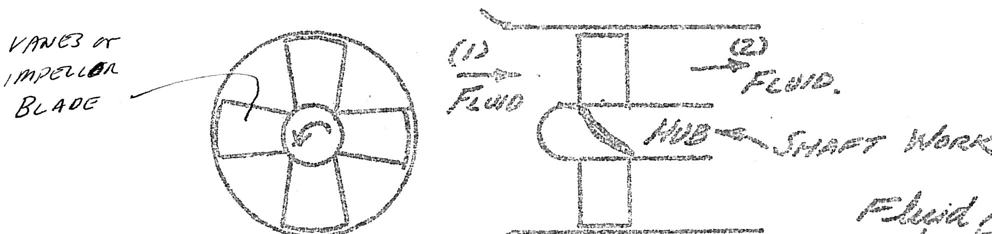
BELT PUMP

2.2 <u>Centrifugal.</u> Fluid transport produced by forces causing a change in the moment of momentum of the fluid.



RADIAL FLOW

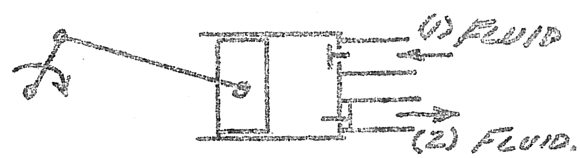
"Momentum and energy addition to the fluid by forces acting on the fluid from the vanes of the impeller as the fluid moves in radial planes.



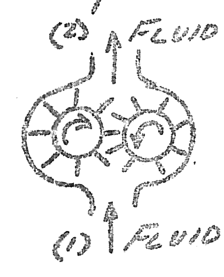
Fluid moves at constant radius.

AXIAL FLOW (Propeller)

2.3 <u>Positive Displacement.</u> Fluid transport produced by a decreasing in size solid-bounded volume containing the fluid.



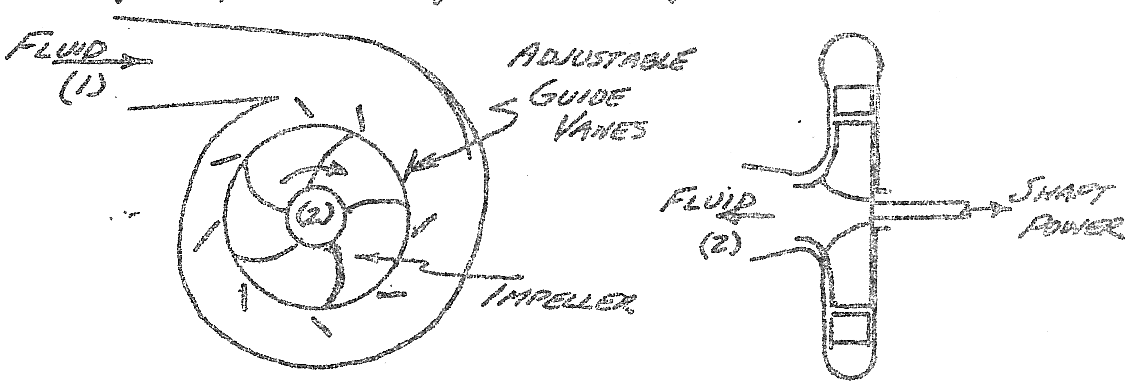
PISTON PUMP



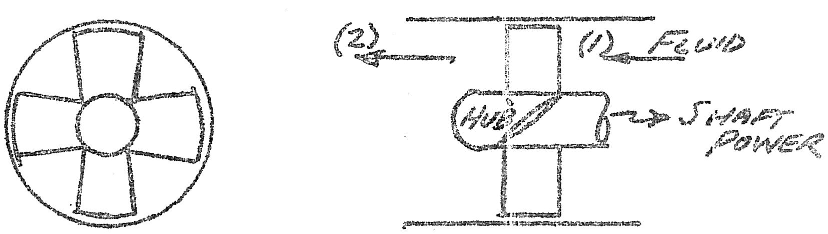
GEAR PUMP

2.4 <u>Jet.</u> Fluid transport produced by another fluid stream with mixing of the two fluids.

3.2 <u>Centripetal.</u> Power output produced by forces caused by a change in the moment of momentum of a fluid acting on vanes of an impeller.

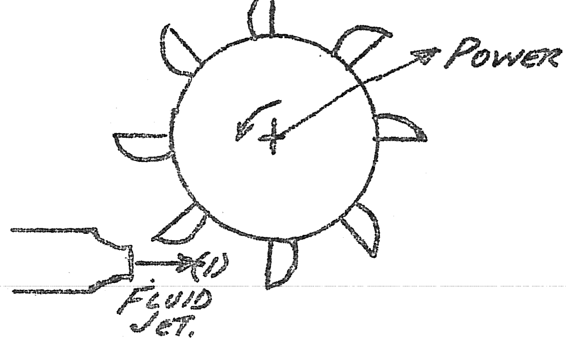


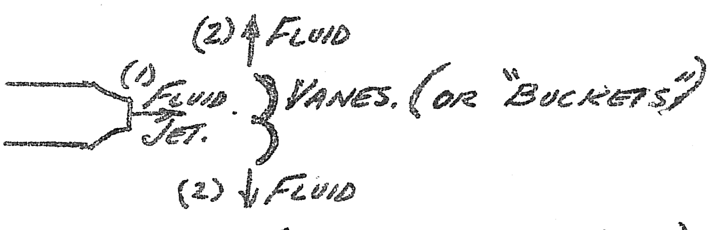
RADIAL FLOW (FRANCIS)



AXIAL FLOW (PROPELLER) (WITH ADJUSTABLE IMPELLER VANES—KAPLAN)

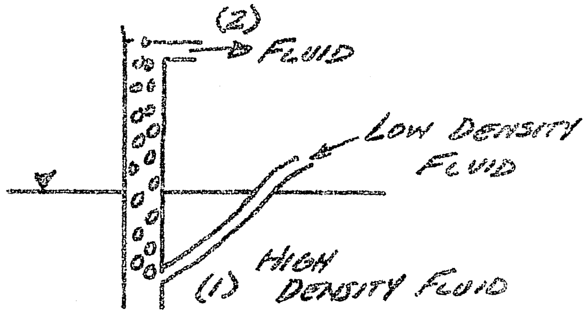
3.3 <u>Impulse.</u> Power output produced by forces caused by a change in the momentum of a fluid acting on vanes of an impeller with no pressure drops across the impeller.





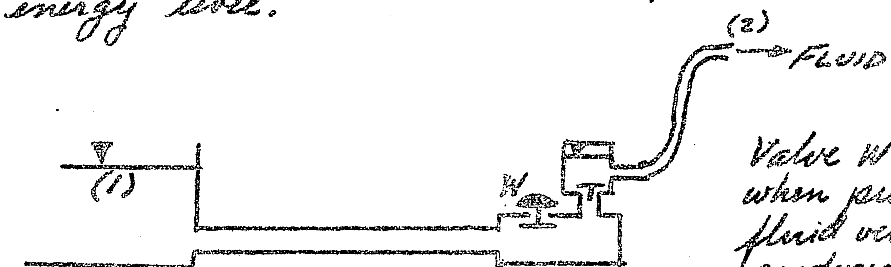
IMPULSE (PELTON WATER WHEEL).

2.5 Density. Fluid transported vertically by the introduction of a lower density fluid.



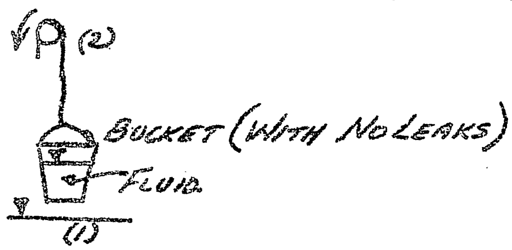
COFFEE PERCOLATOR

2.6 Intermittent Momentum. Rapid decelleration of a large mass of low energy fluids produces a transport of a small mass of fluid to a higher energy level.



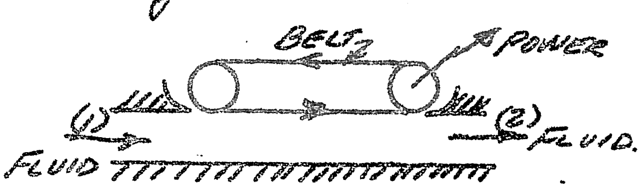
Valve W closes when pumping fluid velocity produces sufficient force to overcome valve weight.

2.7 Mechanical. Fluid transport purely by mechanical action and independent of the fluid dynamics.



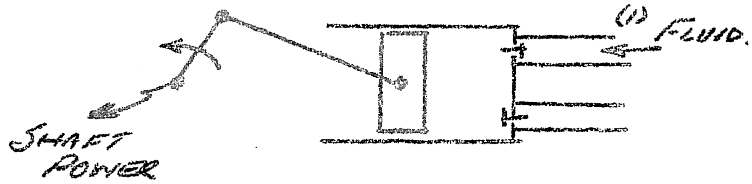
3.0 Classes of Turbines, Fluid Motors.

3.1 Drag. Power output from a moving surface dragged by the shear force of a fluid flowing past the surface.



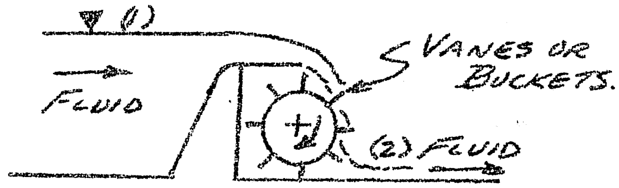
BELT TURBINE

3.4 Positive Displacement. Power output produced by the pressure of a fluid acting in a volume against a moving boundary of the volume.

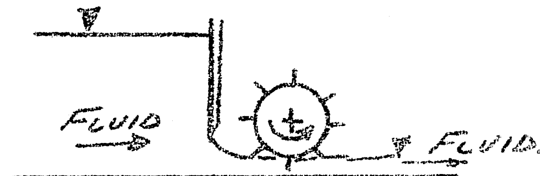


PISTON MOTOR

3.5 Mechanical (Water Wheels)



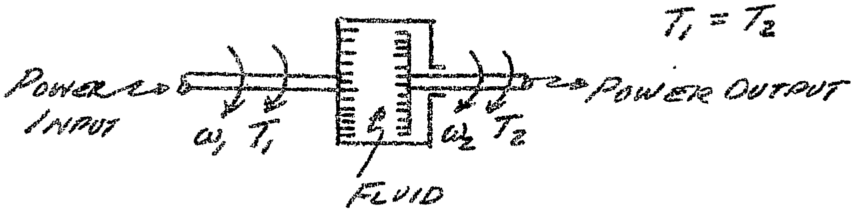
OVERSHOT



UNDERSHOT

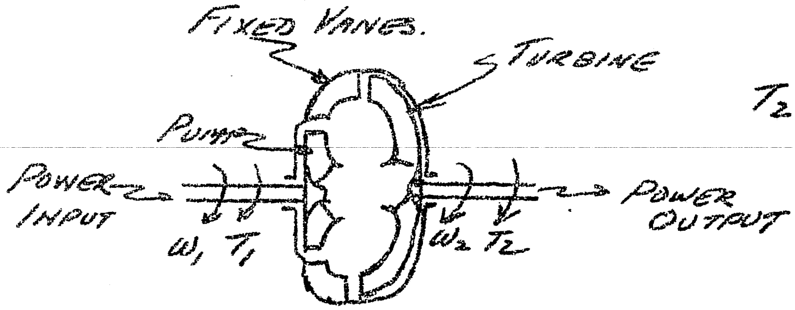
4.0 Classes of Hydraulic Transmissions.

4.1 Couplings. Shaft power transmission with equal input and output shaft torques.



$T_1 = T_2$

4.2 Torque Converters. Shaft power transmission with output torque usually greater than input torque.



$T_2 > T_1$

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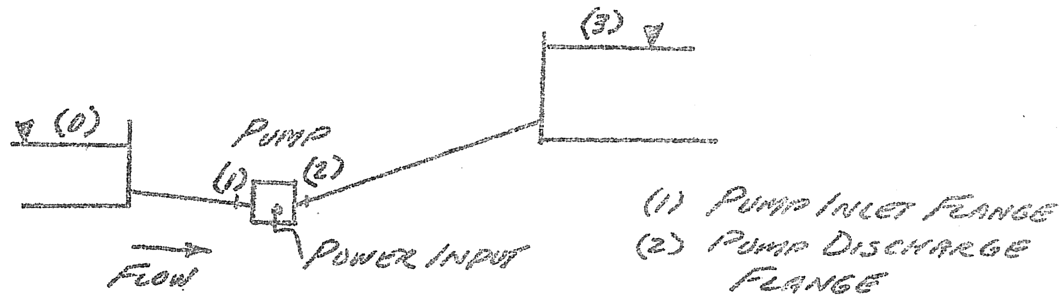
5.0 Performance Representation.

5.1 Pumps - Incompressible flow.



PUMP AND SYSTEM

The independent variable is the flow rate Q, GPM or CFS

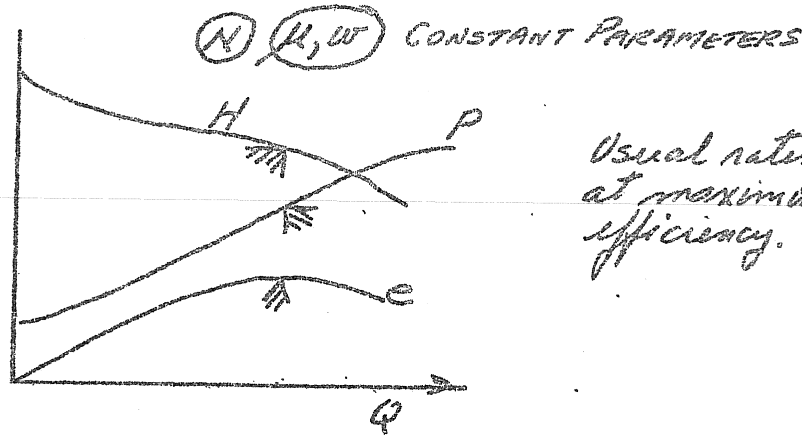
Head $H = \left(\frac{P_2}{w} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{P_1}{w} + z_1 + \frac{V_1^2}{2g}\right)$, ft.

Power input to pump shaft P, horsepower

Efficiency $e = \frac{QwH}{P}$ (dimensionless)

The interrelationships of Q, H, P and e are the pump performance and are determined by test of the pump. The performance is predicted by analysis of the pumping actions. Representation is usually in graphical form with

$$\left.\begin{aligned} H &= \Phi_1(Q) \\ P &= \Phi_2(Q) \\ e &= \Phi_3(Q) \end{aligned}\right\}$$ With other parameters such as pump speed and fluid properties.



Usual rating at maximum efficiency.

"Shut-off" head and power are the values at $Q=0$.

5.2 Pump and System. The system is from (0) to (1) and from (2) to (3), exclusive of the pump. For the system

$$\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g} = \frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} + h_{\ell_{0-1}}$$

$$\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{w} + z_3 + \frac{V_3^2}{2g} + h_{\ell_{2-3}}$$

where, h_ℓ = head loss.

Combining,

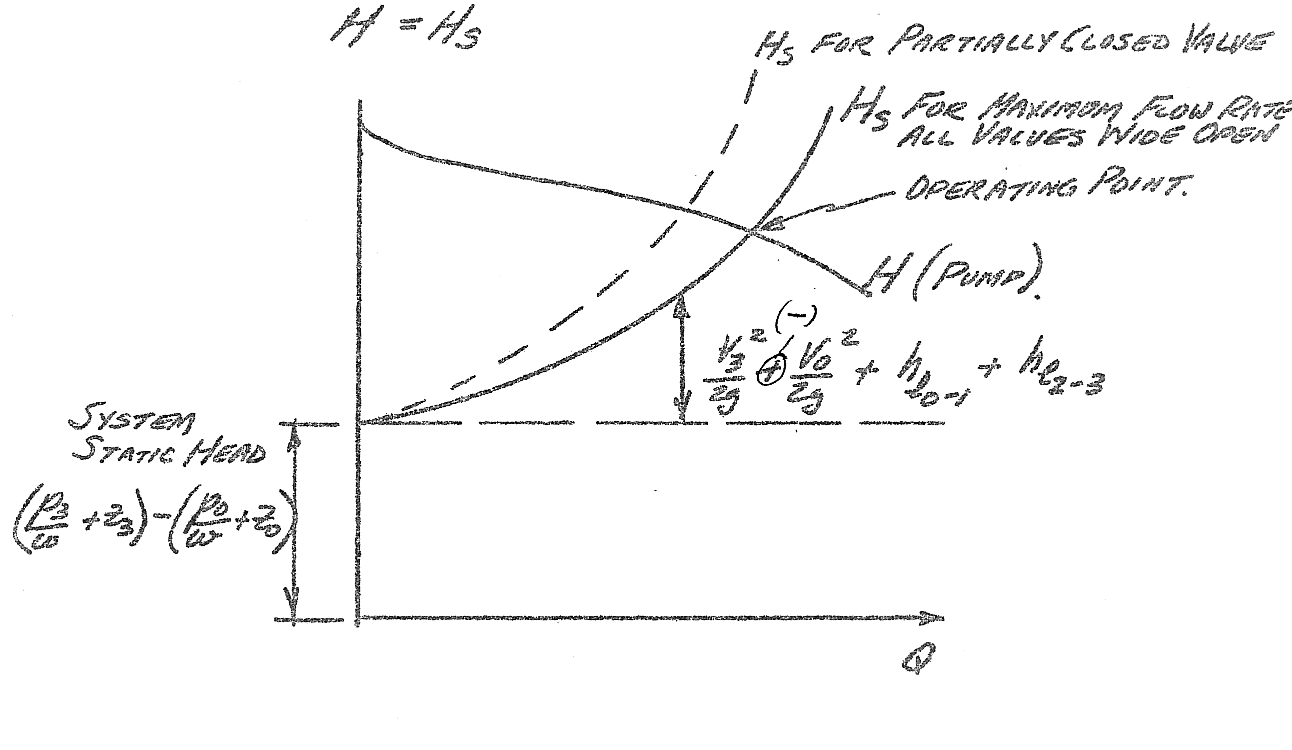
$$\left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right)$$
$$= \left(\frac{p_3}{w} + z_3 + \frac{V_3^2}{2g}\right) - \left(\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g}\right) + h_{\ell_{0-1}} + h_{\ell_{2-3}}$$

Define the system Head,

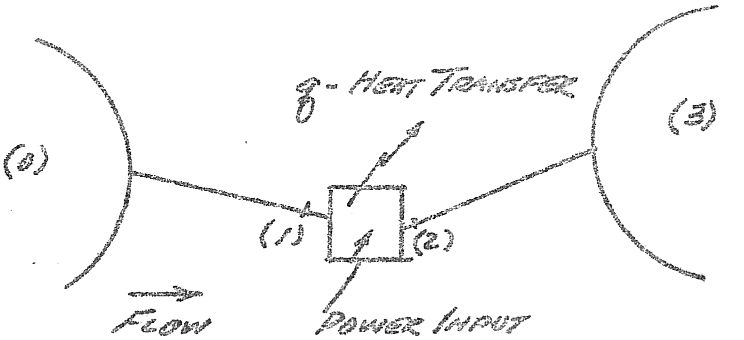
$$H_S = \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right)$$
$$H_S = \left(\frac{p_3}{w} + z_3 + \frac{V_3^2}{2g}\right) - \left(\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g}\right) + h_{\ell_{0-1}} + h_{\ell_{2-3}}$$

The system head can be represented on a head-capacity (or flow rate) basis on the same representation as the pump performance. The operating condition of the pump and the system is

$$H = H_S$$



5.3 Blowers and Compressors — Compressible Flow



The independent variable is the flow rate Q, CFM at a reference pressure and temperature, or G, lbs per sec.

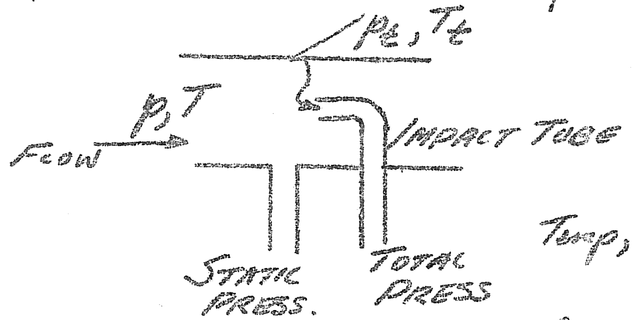
Pressure rise $(p_2 - p_1)_{static}$ or $(p_2 - p_1)_{total}$, psi.

Power input to compressor shaft P, horsepower

Efficiency $$e = \frac{G\left[\left(h_2 + \frac{V_2^2}{2g} + z_2\right) - \left(h_1 + \frac{V_1^2}{2g} + z_1\right)\right]_{isentropic}}{P}$$

(Note: Sometimes in water-jacketed compressors, the isothermal flow work is used as the ideal work)

Ratings are given for both static and total pressure addition to the fluid by the compressor.



p, T - static

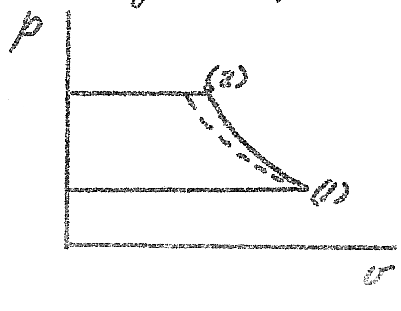
p_t, T_t - total (or stagnation)

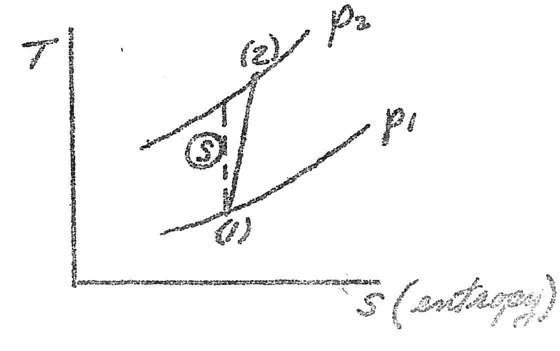
Temp, $T_t = T + \frac{V^2}{2gc_p}$ } ideal gas

Press, $p_t = p\left(\frac{T_t}{T}\right)^{\frac{k}{k-1}}$

Enthalpy, $h_t = h + \frac{V^2}{2g}$

Thermodynamic process



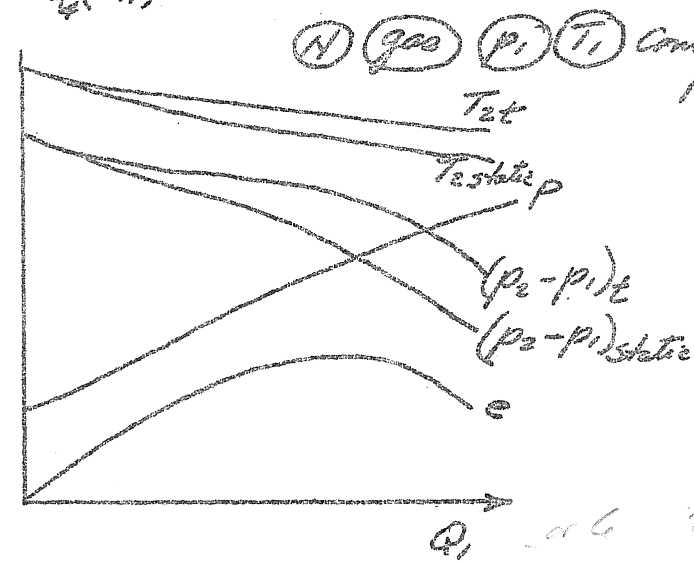


The interrelationships of Q (or G), $p_2 - p_1$, P, and e are the compressor performance and are determined by test of the compressor. The performance is predicted by analysis of the compressing action. Representation is usually in graphical form with

$$\left.\begin{aligned} (p_2 - p_1) &= \phi_1(Q_1) \\ P &= \phi_2(Q_1) \\ e &= \phi_3(Q_1) \end{aligned}\right\} \text{ with other parameters such as speed and fluid properties}$$

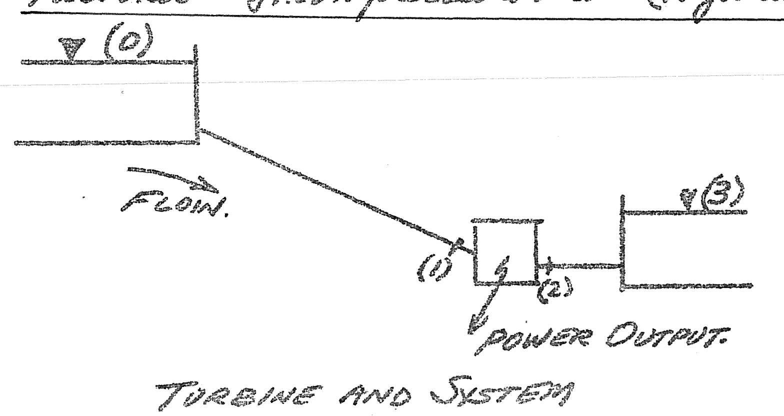
In addition, the discharge temperature is given

$$T_2 = \phi_4(Q_1)$$



5.4 <u>Compressor and System.</u> The system performance together with the compressor performance may be represented on a combined diagram in a similar fashion to that shown in Sect 5.2 for the pump and system. The variables are $(p_2 - p_1)$ and Q_1. However, there is no simple formulation for the system pressure difference due to the fluid specific weight variation with state of the fluid.

5.5 <u>Turbines – Incompressible Flow (Hydraulic).</u>



(1) Turbine inlet flange
(2) Turbine outlet flange

TURBINE AND SYSTEM

The independent variable is the power output P, horsepower

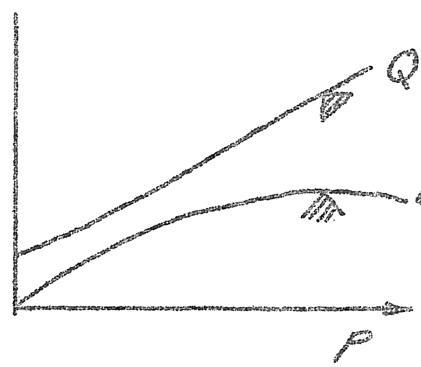
Head. $H = \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right)$, ft.

Flow rate Q, CFS

Efficiency $e = \frac{P}{QwH}$ (dimensionless)

The interrelations of P, Q, and e are the turbine performance with constant parameters of H and speed, N. The performance is determined by test of the turbine. Performance predictions are made by analysis of the turbine action. Representation is usually in graphical form with

$$\left.\begin{array}{l} Q = \phi_1(P) \\ e = \phi_2(P) \end{array}\right\}$$ With parameters of constant head and speed and also fluid properties.



H N FLUID
CONSTANT PARAMETERS

Usual rating at maximum efficiency

Important diff, Between Turb & Pump Head -- a f(load) static head -- is taken as constant.

5.6 Turbine and System. The system is from (0) to (1) and from (2) to (3). Power control is obtained by throttling or by adjustable area flow rate control (guide vanes) with these devices considered to be part of the turbine. For the system

$$\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g} = \frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} + h_{e_{0-1}}$$

$$\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g} = \frac{p_3}{w} + z_3 + \frac{V_3^2}{2g} + h_{e_{2-3}}$$

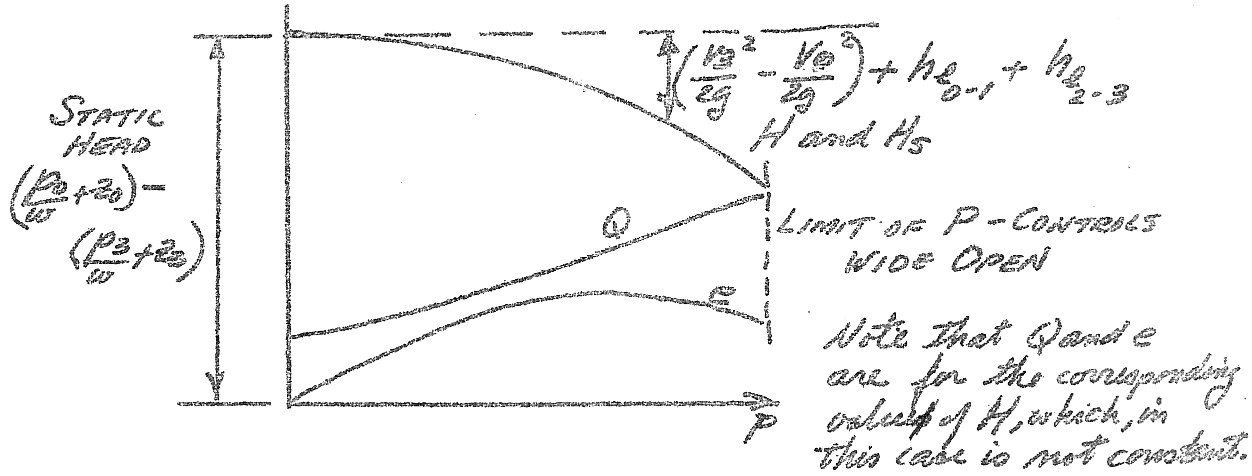
Combining, and defining the system head

MODIFY for DRAFT tube TURBINES or for total head

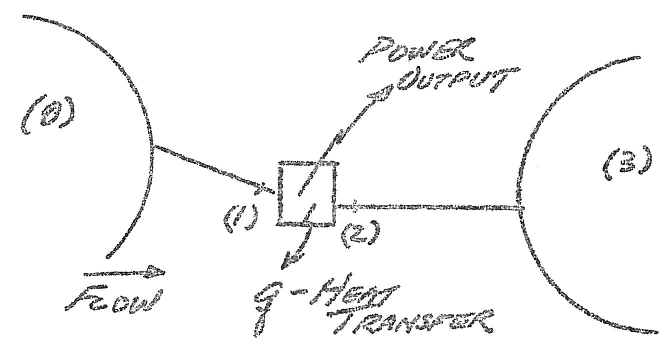
$$H_S = \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right)$$

$$= \left(\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g}\right) - \left(\frac{p_3}{w} + z_3 + \frac{V_3^2}{2g}\right) - h_{e_{0-1}} - h_{e_{2-3}}$$

The system head can be represented on the same head-power basis as the turbine. In this case the system head and the turbine head are the same since the control device is part of the turbine. Thus the turbine operates over a head range furnished by the system.



5.7 Turbines - Compressible Flow.



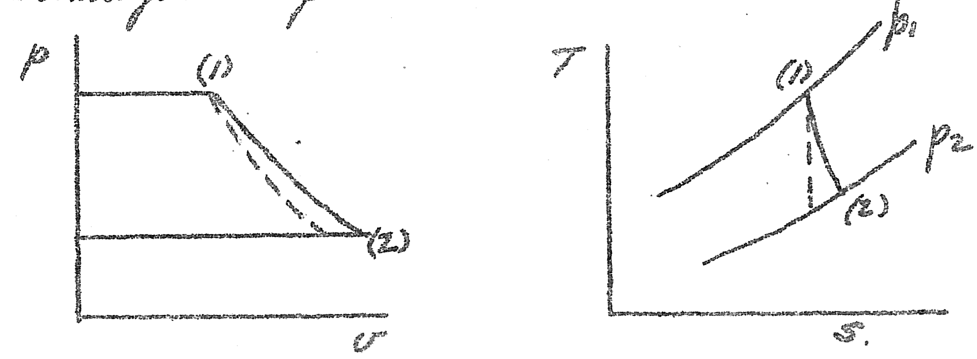
The independent variable is the power output - P, horsepower

total Enthalpy difference $(h_1 - h_2)_t$ Btu per lb.

Flow rate G lbs. per sec (or minute)

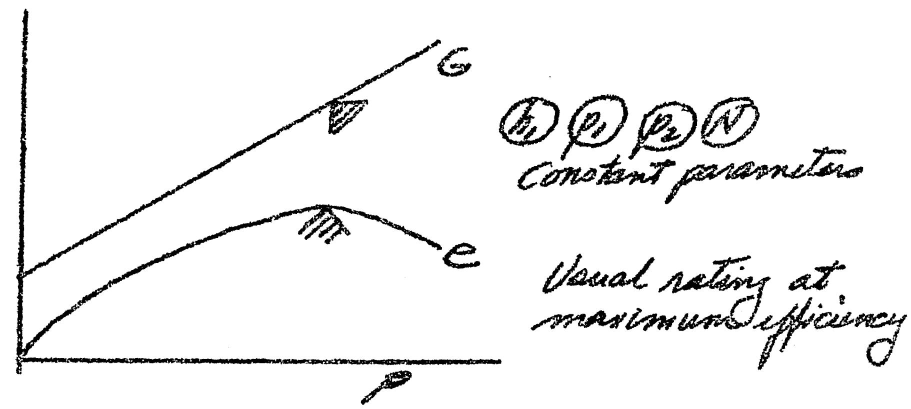
Efficiency $e = \dfrac{P}{G(h_1-h_2)_{\text{isentropic}}}$ total ?

Thermodynamic process



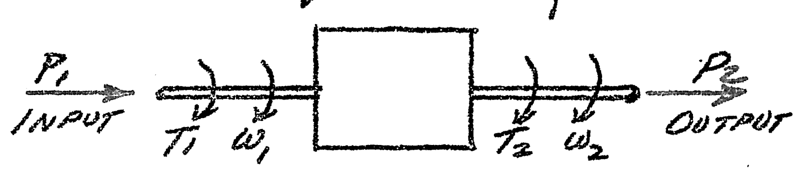
The interrelations of P, G, and e are the turbine performance with constant parameters of h_1, p_1, p_2 and N. The performance is determined by test of the turbine. Performance predictions are made by analysis of the turbine action. Representation is usually in graphical form with

$$\left.\begin{array}{l} G = \phi_1(P) \\ e = \phi_2(P) \end{array}\right\} \text{ With parameters of } h_1, p_1, \text{ and } N.$$



5.8 <u>Compressible Flow Turbine and System.</u> The representation of the system performance together with the turbine performance may be made in a similar fashion to that shown in Section 5.6 for the incompressible flow turbine and system. However, there is no simple formulation for the system pressure change due to the change in fluid specific weight with pressure changes. System heat transfer effects also complicate the formulation of the system performance.

5.9 <u>Fluid Couplings and Torque Converters.</u>



The independent variable is the power output, P_2. However, representation of the converter performance is best made using the output speed as the independent parameter with the input speed as a constant parameter.

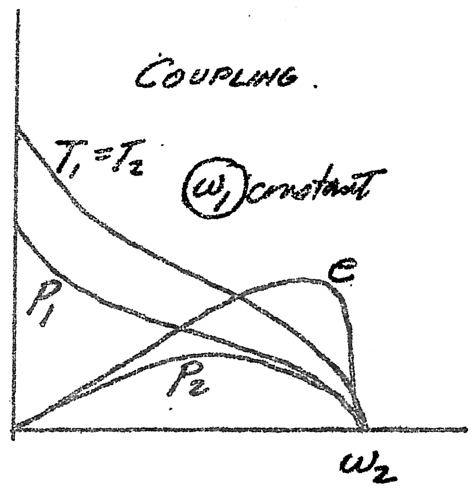
Output power P_2, horsepower
Output torque T_2 ft. lbs.
Efficiency $e = P_2/P_1$

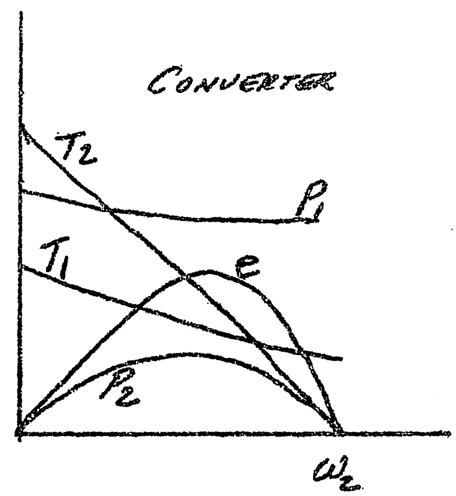
For the coupling $T_2 = T_1$

For the torque converter $T_2 > T_1$ over part of the speed ratio.

Test performance is usually represented in graphical form with

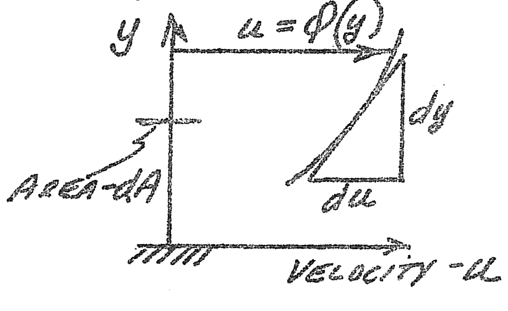
$$\left.\begin{aligned} T_1 &= \phi(\omega_2) \\ T_2 &= \phi(\omega_2) \\ P_1 &= \phi(\omega_2) \\ P_2 &= \phi(\omega_2) \\ e &= \phi(\omega_2) \end{aligned}\right\} \text{ with constant parameter of } \omega_1$$





6.0 Fluids and Fluid Systems

6.1 Fluid. A material which deforms continuously under the action of a shear (tangential) stress. The resistance to shear is the viscous property of a fluid.



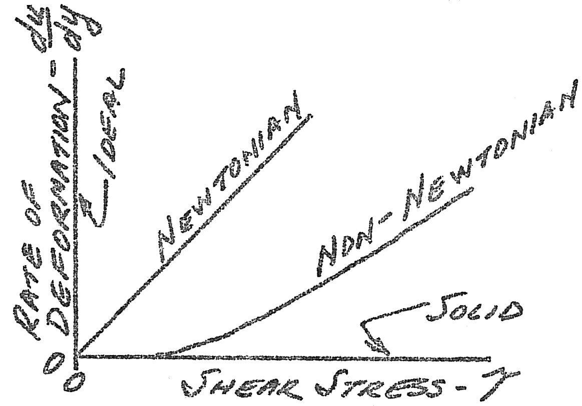
y - distance from boundary

Shear stress on the area, dA, = τ

$$\tau = \mu \frac{du}{dy}$$

μ = absolute viscosity

6.2 Classification of fluids. For a fixed state of a fluid the equilibrium relationship between the shear stress and the rate of deformation (velocity gradient) determines the fluid type.



Fluid Types

Ideal $\mu = 0$

Newtonian μ = constant

Non-Newtonian $\mu = \phi\left(\frac{du}{dy}\right)$

[Solid $\mu = \infty$]

6.3 Fluid System. A completely bounded aggregate of material. Two general classes of systems are used:

(a) Finite boundaries consisting of solid surfaces or walls with no fluid flow across the surfaces, and of other selected boundaries in the flow streams entering and leaving the walled portion. The combination of walls and selected surfaces completely encloses an aggregate of fluid material.

(b) Infinitesmal fluid particle array within finite boundaries.

7.0 Prediction of the Behavior of Fluid Systems.

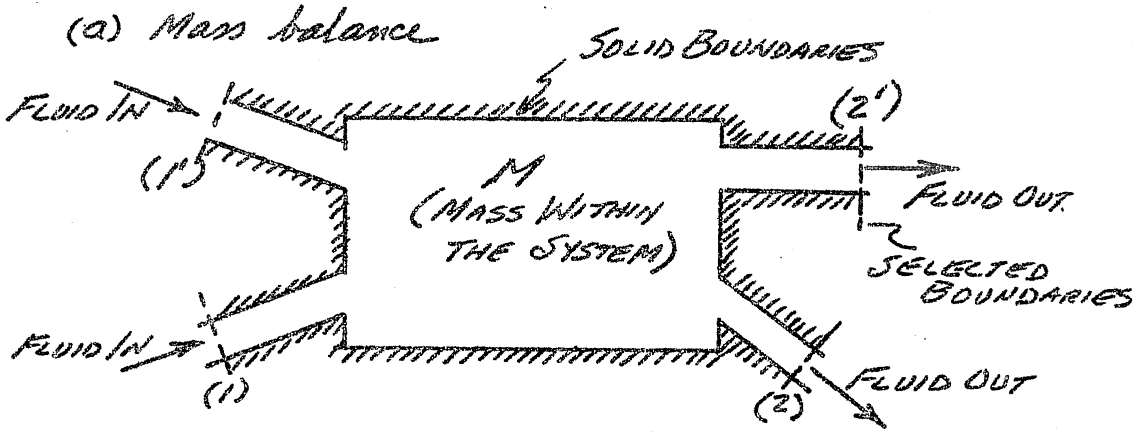
7.1 Basic concepts. Laws of behavior.

(a). Mass balance
(b) Force balance
(c) Energy balance
(d) Properties (equation of state)
(e) Limitations.

The basic concepts are applied to systems depending upon the known information and upon the desired result. When applied to a finite system information is known and desired at the boundaries. When applied to an infinitesmal system details of the internal behavior are determined as well as information at the boundaries.

7.2 Finite system analysis.

(a) Mass balance



Mass rate in = mass rate out + mass rate of storage

$$\sum^{n}\int^{A_1}\rho_1 u_1 dA_1 = \sum^{n}\int^{A_2}\rho_2 u_2 dA_2 + \frac{dM}{dt}$$

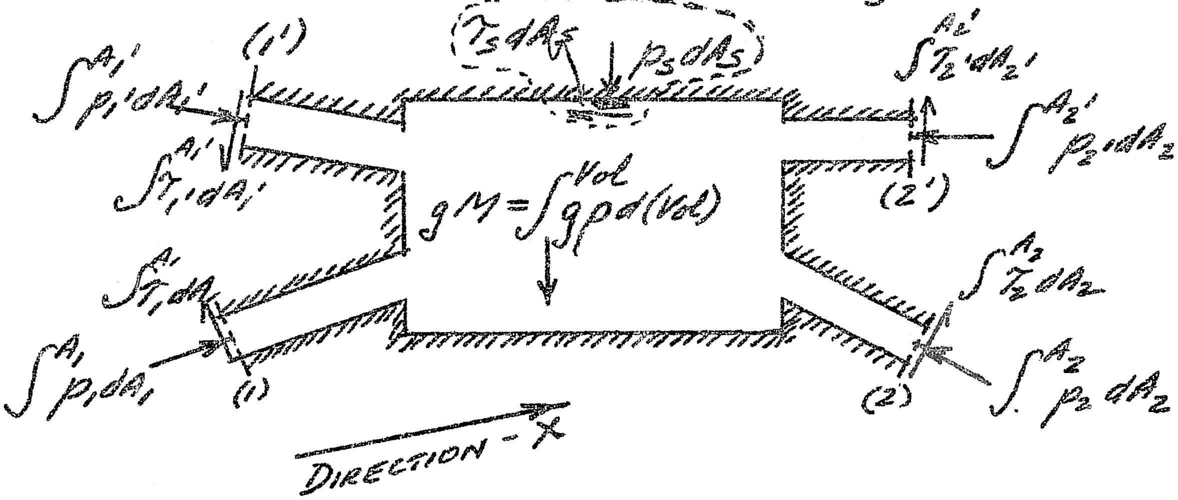
The summations indicate terms for each of n streams. The integrals are used for the general case of non-uniform velocity and density distributions at each flow boundary.

Simplifications: Steady flow, steady state, $\frac{dM}{dt}=0$

Uniform flow $\int^{A}\rho V dA = \rho VA$

Definition of average velocity, $V_{avg} = \frac{\int^{A} u\,dA}{A} = \frac{Q}{A}$

(b) Force balance ON THE AREA dA_S



Pressures are normal to areas. Shears are tangent to areas. X is any selected axis. Force components in the direction of the selected axis only enter the force balance as indicated by the subscript x.

[Summation of forces acting on the fluid at all boundaries]$_x$

= [change in the momentum rate of the fluid]$_x$

= [Final momentum rate]$_x$ − [Initial momentum rate]$_x$

$$\sum^{n}\left[\int^{A_1} p_1 dA_1\right]_x + \sum^{n}\left[\int^{A_1} \tau_1 dA_1\right]_x + \sum^{n}\left[\int p_2 dA_2\right]_x + \sum^{n}\left[\int^{A_2} \tau_2 dA_2\right]_x$$

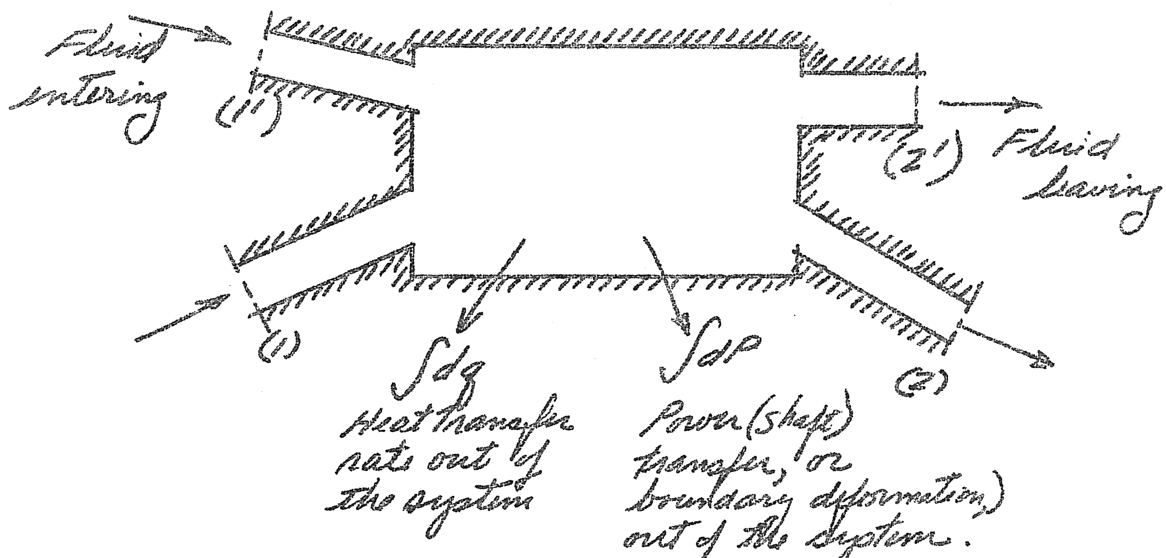
$$+ \left[\int^{A_S} \tau_S dA_S\right]_x + \left[\int^{A_S} p_S dA_S\right]_x + \left[\int^{vol} g\rho\, d(Vol)\right]_x$$

$$= \sum^{n}\left[\int^{A_2} \rho_2 V_2^2 dA_2\right]_x - \sum^{n}\left[\int^{A_1} \rho_1 V_1^2 dA_1\right]_x$$

The distributed pressure and shear on the solid boundaries may be difficult or impossible to evaluate. Very often the net force from these effects is required. Then,

$$F_x = \left[\int^{A_S} p_S dA_S\right]_x + \left[\int^{A_S} \tau_S dA_S\right]_x$$

(c) Energy balance



Energy rate in = energy rate out + energy rate of storage.

$$\sum^{n}\int^{A_1} g\rho_1 u_1\left(\frac{p_1}{w_1}+z_1+\frac{u_1^2}{2g}+I_1\right)dA_1$$

$$=\sum^{n}\int^{A_2} g\rho_2 u_2\left(\frac{p_2}{w_2}+z_2+\frac{u_2^2}{2g}+I_2\right)dA_2$$

$$+\int dq+\int dP+\frac{d}{dt}\int^{vol}\left[\rho g\left(z+\frac{u^2}{2g}+I\right)\right]_{vol} d(vol)$$

where I – internal thermal energy per unit weight
z – elevation

The sign convention is established by the diagram. The storage term includes changes of elevation, kinetic energy, and internal thermal energy <u>within</u> the boundaries.

7.0 Model Laws.

7.1 The first approach to the prediction of system behavior is by analytical techniques. However, many systems are not amenable to complete precise analysis. Either an inadequate approximate prediction is accepted or a more adequate experimental solution is attempted.

7.2 Experimental solutions. There are three general classes, each with a different objective:

(a) Measurements on a system to check the analytical prediction of behavior.

(b) Measurements on a few samples of a group of systems having common parameters to correlate the measured information into generally useful information applicable to all other systems of the same group.

(c) Measurements on a single system to predict the performance of the system under different operating conditions or of another similar system of a different size.

All three classes may be considered as applications of model techniques. The first is standard laboratory or field testing. The second and third require knowledge of applicable groupings of variables or of relative magnitudes of variables in relation to the different operating conditions or size. Model laws are needed.

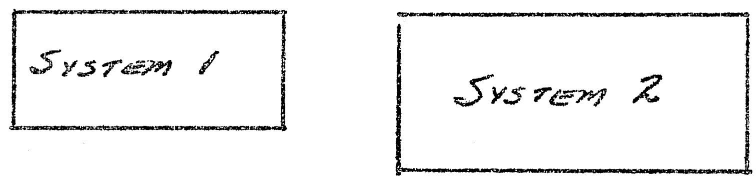
7.3 General techniques for determining model laws. The model laws must be determined, or derived, from conditions which describe the system behavior. This does not mean a complete knowledge of behavior, but it does mean a knowledge of those factors which result in the behavior. All causes and effects of any system can be expressed in a mathematical function, or more than one function. Perhaps these functions for a particular system are differential equations for which solutions can not be found by known techniques. Perhaps the behavior of the system is known in manageable algebraic form, either partially or completely.

The general technique for obtaining the model laws for a given system may be shown by considering the behavior equation for an arbitrary system.

$$x + y = z \qquad 7.1$$

where x, y, and z are variables of the system.

For two systems of the same type where information from one is to be used to predict performance of the other, or where information from one or both is to be correlated to be made more generally useful, then the behavior equation 7.1 applies to both systems and may be written for both systems.



The subscript, 1, is used to designate variables of system 1, and the subscript, 2, is used to designate variables of system 2.

Then, for system 1, $\quad x_1 + y_1 = z_1 \qquad 7.2$

and, for system 2, $\quad x_2 + y_2 = z_2 \qquad 7.3$

Any variable in one system is related to the corresponding variable in the other system. This relationship is expressed in a ratio, That is,

$$\left.\begin{array}{lll} b_x = \dfrac{x_1}{x_2} & \text{or} & x_1 = b_x x_2 \\ b_y = \dfrac{y_1}{y_2} & \text{or} & y_1 = b_y y_2 \\ b_z = \dfrac{z_1}{z_2} & \text{or} & z_1 = b_z z_2 \end{array}\right\} 7.4$$

Substitution of eqs. 7.4 in eq. 7.2 gives

$$b_x x_2 + b_y y_2 = b_z z_2 \qquad 7.5$$

Eq. 7.5, which was obtained from the behavior equation of system 1, is now given in terms of the variables of system 2 and hence must be a behavior equation of System 2. Equation 7.3 is also a behavior equation of system 2. Therefore, since system 2 can have only one behavior for a given set of input conditions, equations 7.3 and 7.5 must be identical. A sufficient condition for this identity is equality of the coefficients.

$$b_x = b_y = b_z \tag{7.6}$$

Equations 7.6 give the relationship of the variables that must be satisfied for modeling. Thus, these equations are the model laws for systems with the behavior expressed by eq. 7.1.

Note that the model laws may be written directly by inspection of eq. 7.1. In each term, place a ratio, b, with a subscript for the variable and equate the coefficients of the b's.

Further generalizations are possible. Divide eq. 7.6 by b_z.

$$\frac{b_x}{b_z} = \frac{b_y}{b_z} = 1 \tag{7.7}$$

Substituting eq. 7.4 in 7.7,

$$\frac{x_1/x_2}{z_1/z_2} = \frac{y_1/y_2}{z_1/z_2} = 1 \tag{7.8}$$

or,

$$\frac{x_1/z_1}{x_2/z_2} = \frac{y_1/z_1}{y_2/z_2} = 1 \tag{7.9}$$

Then,

$$\frac{x_1}{z_1} = \left(\frac{y_1/z_1}{y_2/z_2}\right)\frac{x_2}{z_2} = \frac{x_2}{z_2} \tag{7.10}$$

From eq. 7.10, $\frac{x_1}{z_1} = \frac{x_2}{z_2}$ only if $y_1/z_1 = \frac{y_2}{z_2}$ 7.11

The number of systems for which the behavior is described by eq. 1 may be increased to include more than the two described previously. Eqs 7.6 will apply for all. Eqs. 7.11 may be written to include all possible systems:

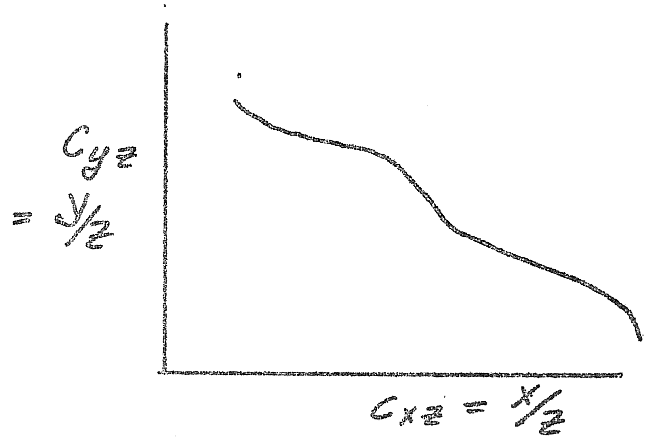
Then

$$\left.\begin{aligned} \frac{x_1}{z_1} = \frac{x_2}{z_2} = \frac{x_3}{z_3} = \cdots\cdots = C_{xz} \\ \frac{y_1}{z_1} = \frac{y_2}{z_2} = \frac{y_2}{z_3} = \cdots\cdots = C_{yz} \end{aligned}\right\} \quad 7.12$$

C_{xz} and C_{yz} are the correlating parameters of the systems described by eq. 7.1. A single function

$$C_{xz} = \Phi(C_{yz}) \qquad 7.13$$

describes all operating conditions of all systems to which eq. 7.1 applies.

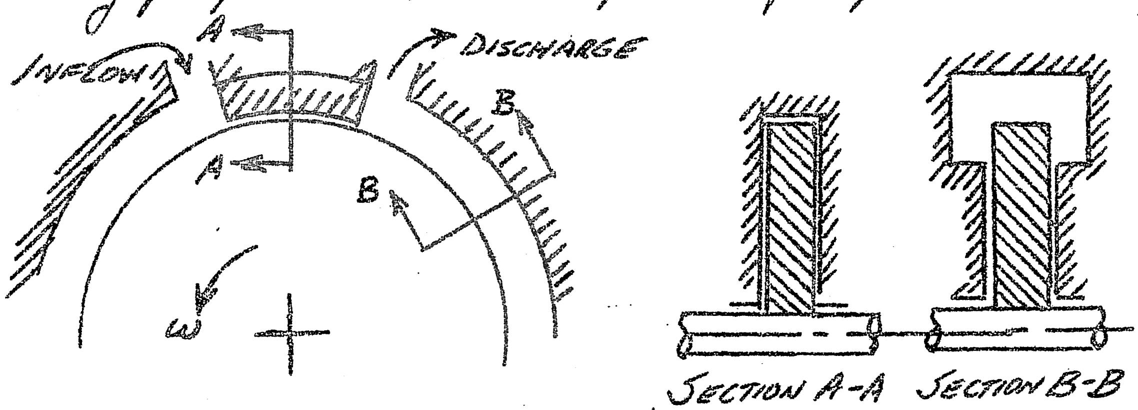


The function may be obtained from one system and the results applied to predict the behavior of all similar systems. Note that C_{yz} and C_{xz} are dimensionless. The function of eq 7.13 is dimensionless, or normalized.

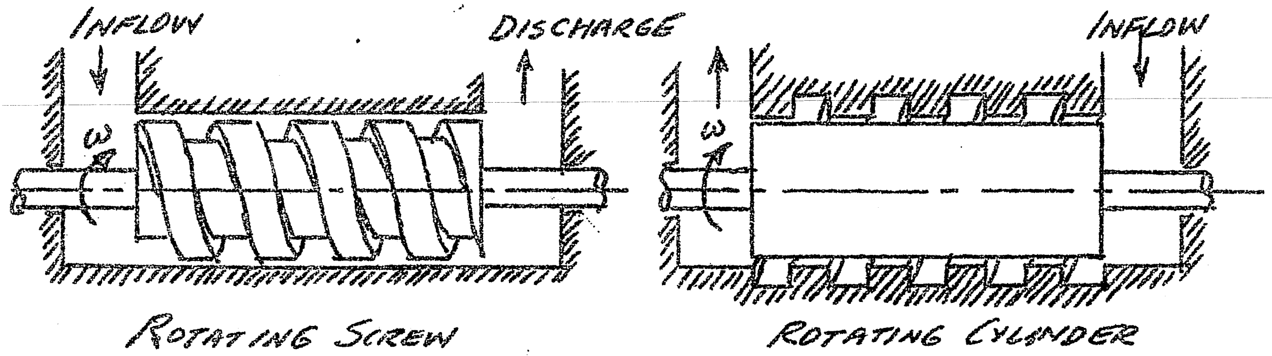
8.0 Drag Pumps. (See Sections 7.1 and 5.1)

8.1 Belt pumps. Fluid transport is produced by shear forces generated in the fluid by the action of moving surfaces. The belt pump is not a feasible mechanical device but serves to illustrate the possibilities of this type of pumps. A more practical shear pump is a disk rotating in a casing with a flow passage around the perimeter of the disk. Another possibility is a screw thread rotating in a cylindrical housing or a cylinder rotating inside a screw casing.

In all of these arrangements the flow may be either laminar or turbulent. If laminar, the shear forces are described by the fluid viscosity and the velocity gradients. If turbulent, the shear forces can only be described by shear coefficients which must be determined experimentally. Possible features of optimum performance may be determined analytically. The head, power, and efficiency are desired as functions of the flow rate. Interpretation of the simple belt pump results in a first approximation of the performance of drag pumps with more complicated flow patterns.



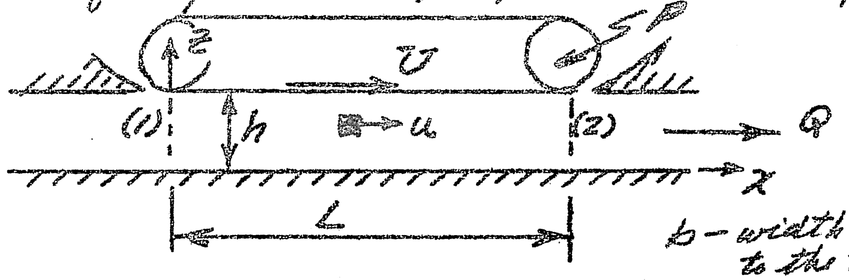
DISK PUMP



SCREW THREAD PUMPS

8.2 Analysis of the belt pump with laminar flow.

INCOMPRESSIBLE



b – width perpendicular to the diagram.

The moving surface drags the fluid adjacent to it in the direction of the movement. A velocity distribution results between the moving surface and the stationary surface with the possibility of a net flow from (1) to (2). The amount of flow will depend upon the relative pressures at (1) and (2), or upon the head difference between (1) and (2). The performance is desired as

$$H = \varphi_1(Q)$$
$$P = \varphi_2(Q)$$
$$e = \varphi_3(Q)$$

If the velocity distribution can be obtained, then integration of this distribution will give the flow rate. The head will appear in the final form of this integration since the velocity distribution depends upon the relative head at entrance and discharge.

To simplify the analysis the following conditions will be taken:

$h << L$ Neglect changes in boundary conditions of the flow at entrance and at exit.

$h << b$ Neglect side wall effects.

Neglect leakage at (1) and (2)

Neglect all power losses in bearings, leakage, etc.

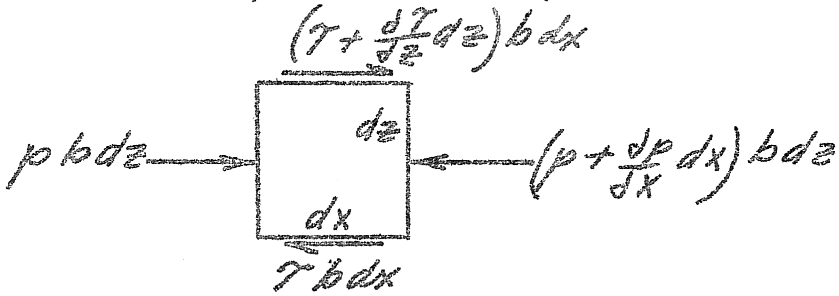
U = constant h = constant

μ = constant

w = constant (incompressible flow)

The system is then two-dimensional in x and z. The velocity distribution at any location x is the same for all locations within L, that is, u is a function of z only.

Consider a fluid element of sides dx, dz, and b



In laminar flow in this system with μ constant a particle remains at a constant location, z, from the x axis. There is also no acceleration of a fluid particle under the stated conditions.

The force balance is

$$p\,b\,dz - \left(p + \frac{\partial p}{\partial x} dx\right) b\,dz - \tau\, b\,dx + \left(\tau + \frac{\partial \tau}{\partial z} dz\right) b\,dx = 0$$

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial z} = 0 \qquad 8.1$$

For the two-dimensional system $\tau = \mu \frac{du}{dz}$ 8.2

Then, $-\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial z^2} = 0$ 8.3

An incompressible flow pump head is, by definition,

$$H = \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) \qquad 8.4$$

In this case, $z_2 - z_1 = 0$ with x horizontal, and $V_2 = V_1$. Since the velocity distributions are the same at every x, the shears are the same at every x and the pressure gradient is a constant.

Then, $\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{L} = \frac{Hw}{L}$ 8.5

Eq. 8.3 is $\frac{\partial^2 u}{\partial z^2} = \frac{Hw}{\mu L}$ 8.6

Integrating $\frac{\partial u}{\partial z} = \frac{Hw}{\mu L} z + C_1$ 8.7

$$u = \frac{Hwz^2}{2\mu L} + C_1 z + C_2 \qquad 8.8$$

The constants of integration are obtained from the boundary conditions.

$$\text{at } z=0,\ u=0 \text{ and } C_2=0$$

$$\text{at } z=h,\ u=U \text{ and } C_1=\frac{U}{h}-\frac{Hwh}{2\mu L}$$

SHOULD include diagram of VEL. DIST.

Then, $$u=\frac{Hw}{2\mu L}\left(z^2-hz\right)+\frac{Uz}{h} \qquad 8.9$$

$$Q=\int_0^h u\,b\,dz \qquad 8.10$$

$$=-\frac{Hwbh^3}{12\mu L}+\frac{Uhb}{2} \qquad 8.11$$

or, $$H=\frac{12\mu LU}{wh^2}\left(\frac{1}{2}-\frac{Q}{bhU}\right) \qquad 8.12$$

In normalized (dimensionless) form

$$\frac{H}{12\mu LU/wh^2}=\frac{1}{2}-\frac{Q}{bhU} \qquad 8.13$$

The power input to the belt, neglecting leakage, bearing losses and other extraneous losses, is

$$P=\tau_{z=h}\,bL\cdot U \qquad 8.14$$

$$\tau_{z=h}=\mu\left(\frac{du}{dz}\right)_{z=h} \qquad 8.15$$

From eq. 8.7, $$\left(\frac{du}{dz}\right)_{z=h}=\frac{U}{h}+\frac{Hwh}{2\mu L} \qquad 8.16$$

Then $$P=\frac{\mu UbLU}{h}+\frac{HwhbU}{2} \qquad 8.17$$

Substituting for H from eq. 8.12,

$$P=\frac{6\mu bLU^2}{h}\left(\frac{2}{3}-\frac{Q}{bhU}\right) \qquad 8.18$$

In normalized form,

$$\frac{P}{6\mu bLU^2/h}=\frac{2}{3}-\frac{Q}{bhU} \qquad 8.19$$

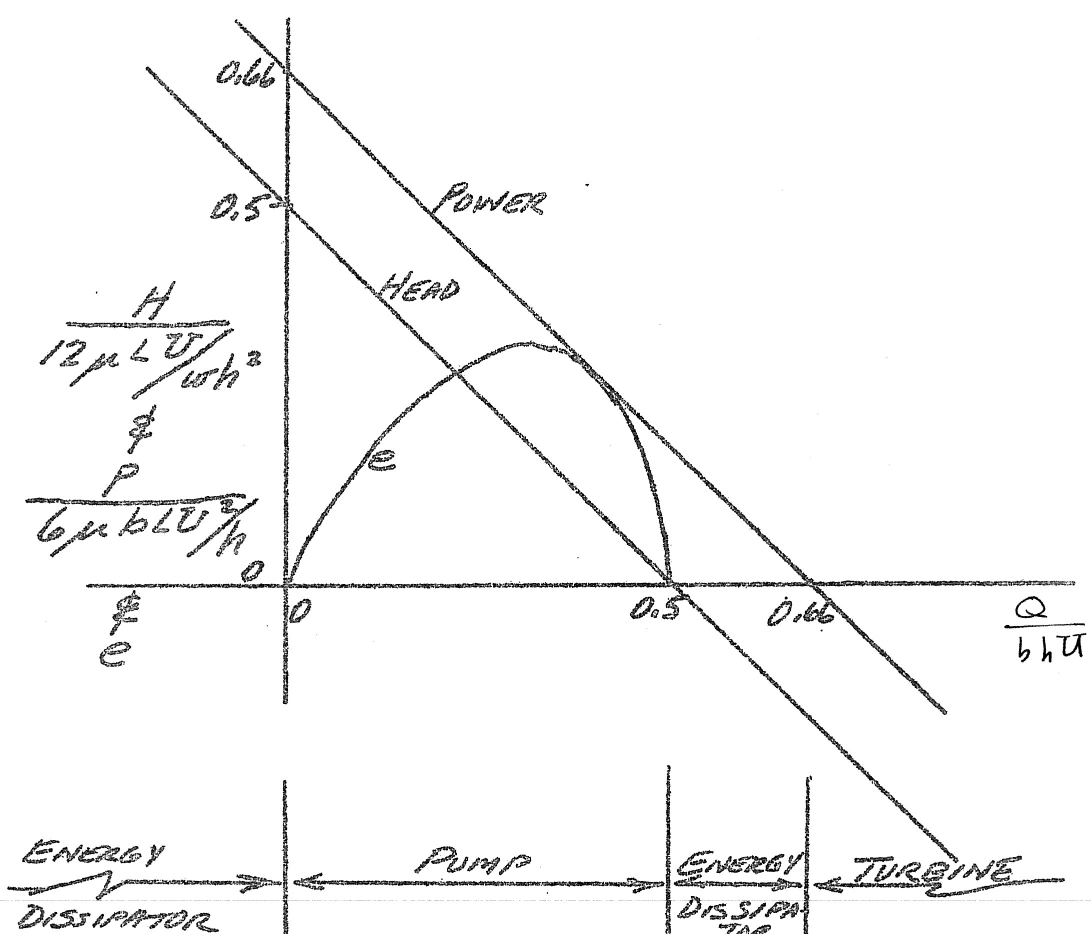
By definition, the efficiency is

$$e = \frac{QwH}{P} \qquad 8.20$$

Eliminating H and P using eqs. 8.12 and 8.18

$$e = \frac{2Q}{bhU}\left[\frac{\frac{1}{2} - \frac{Q}{bhU}}{\frac{2}{3} - \frac{Q}{bhU}}\right] \qquad 8.21$$

Maximum efficiency is $\frac{1}{3}$ (or $33\frac{1}{3}$%) at $\frac{Q}{bhU} = \frac{1}{3}$



Note that the analysis includes minus Q, P, and H.

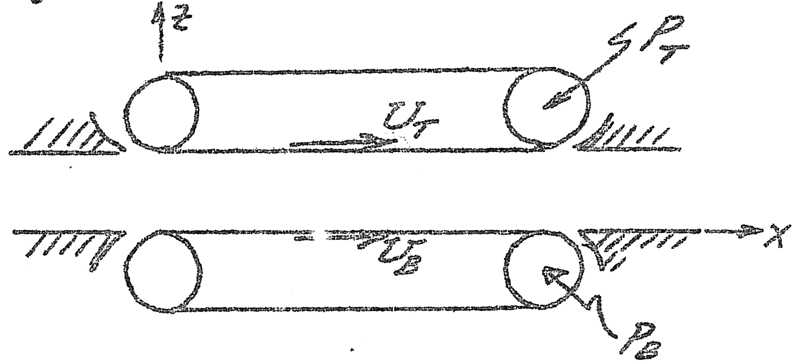
When,
H is + and Q is − Energy dissipator
H is + and Q is + and P is +, pump.
H is (+) and Q is + and P is +, energy dis.
(−) H is − and Q is + and P is −, turbine

8.3 Other possibilities of the laminar flow belt system.

The possibility of the system of Section 8.2 as a turbine has been shown in Section 8.2 except for the efficiency which must be evaluated separately from the definition of turbine efficiency which is

$$e = \frac{P}{Qw H}$$

If the bottom surface is also a belt with a power shaft which may be either an input or an output a number of different combinations may be investigated analytically.



(1) PUMP: $P_B = 0$. Bottom belt "free running". $\tau_B = 0$

(2) PUMP: $U_T = U_B$. Bottom and top belts geared together externally.

(3) TURBINE: $P_B = 0$ and P_T is an output

(4) TURBINE: $U_T = U_B$ and P_T is an output geared

(5) HYDRAULIC COUPLING: P_T is an input and P_B is an output.

If h varies linearly with x and the pressures at the two ends are the same, $p_2 - p_1 = 0$, then the single belt system of Section 8.2 can be shown to have a non-linear pressure distribution which will support a load in the direction z. This is a thrust bearing, or, with suitable modifications in interpretation a cylindrical journal bearing.

8.4 Test results.

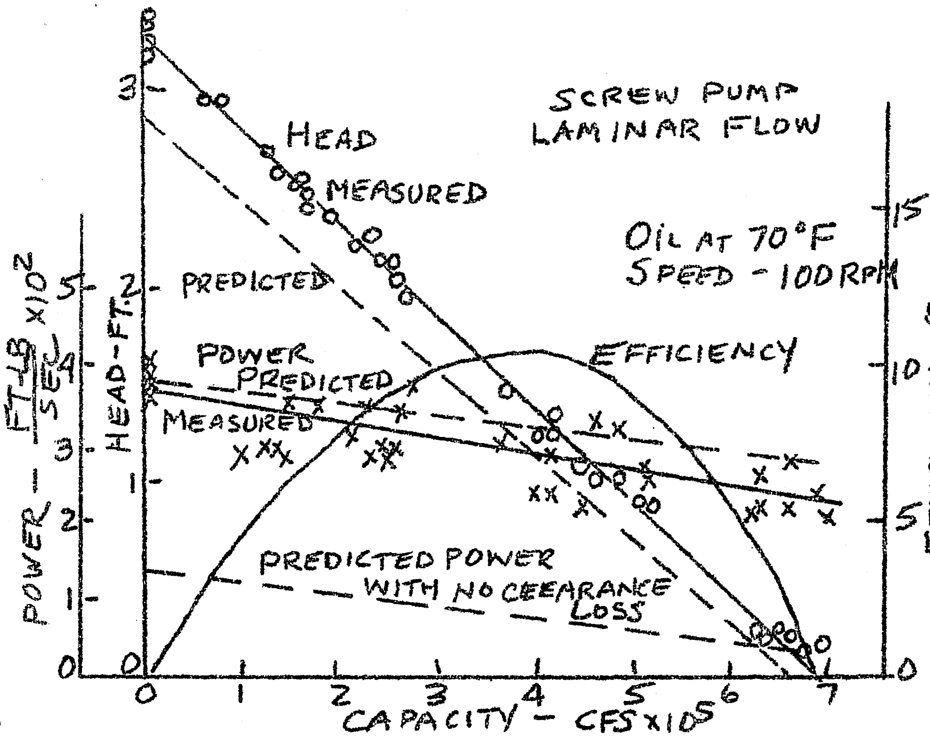


DIAGRAM of SCREW

LAMINAR FLOW SCREW PUMP PERFORMANCE
OIL: VISCOSITY - 2.12×10^{-3} LB.SEC/SQ.FT.
SPECIFIC GRAVITY - 0.88

This pump was tested at speeds from 90 to 150 RPM. all results were adjusted by the model laws to the presented speed of 100 RPM.

The model laws are obtained from either eq. 8.6 or eq. 8.12 for the head; eq. 8.10 or eq 8.12 for the flow rate; and eq. 8.14 (with eq. 8.15) or eq. 8.18 for power. These are, for geometrically similar pumps,

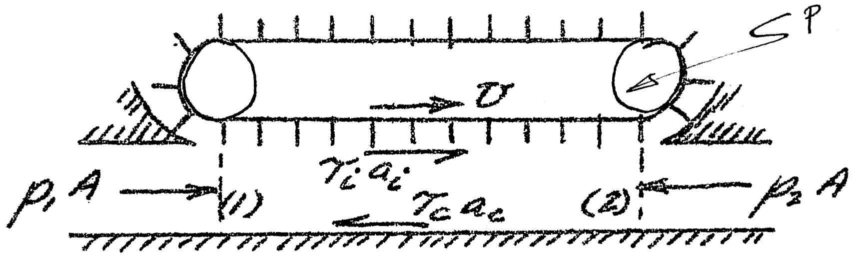
$$Q \propto N D^3$$

$$H \propto \frac{\mu N}{w}$$

$$P \propto \mu N^2 D^3$$

The predicted performance shown in the figure is from eq. 8.12 for the head. The actual performance includes effects from the side walls of the screw thread. The power prediction includes the power loss in the ~~clea~~ clearance between the thread tip and the casing.

8.5 Analysis of the belt pump with turbulent flow.



The shear stresses in the fluid in turbulent flow cannot be expressed in any explicit form in terms of the velocity gradients. Therefore the analysis is made on the fluid system bounded by the inlet, the discharge, the moving surface, and the stationary surface. Neglecting changes in velocity distributions between entrance of the flow at (1) and discharge at (2), the force balance is

$$p_1 A - p_2 A + \tau_i a_i - \tau_c a_c = 0 \qquad 8.22$$

in which A = unobstructed flow cross-section area

a_i = impeller area acting on the fluid

a_c = casing area acting on the fluid.

The shear stresses may be defined in terms of shear coefficients:

$$\tau_i = c_i \frac{w}{2g}\left(V - \frac{Q}{A}\right)^2 \qquad 8.23$$

$$\tau_c = c_c \frac{w}{2g}\left(\frac{Q}{A}\right)^2$$

In the interpretation of τ_i and τ_c, the velocity is the relative velocity between the fluid and the corresponding surface where Q/A is the average fluid velocity.

$$H = \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right) - \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) = \frac{p_2 - p_1}{w} \qquad 8.24$$

for $z_1 = z_2$ and $V_1 = V_2$.

Combining eqs 8.22, 8.23, and 8.24 gives

$$H = \frac{C_i a_i V^2}{A\, 2g}\left[\left(1 - \frac{Q}{VA}\right)^2 - \frac{C_c a_c}{C_i a_i}\left(\frac{Q}{VA}\right)^2\right] \qquad 8.24$$

The power input to the fluid from the moving surface is

$$P = \tau_i a_i U = C_i a_i \frac{w U^3}{2g}\left(1 - \frac{Q}{UA}\right)^2 \qquad 8.25$$

The efficiency of the pump is

$$e = \frac{QwH}{P} = \frac{Q}{UA}\left[1 - \frac{C_c a_c}{C_i a_i}\left(\frac{Q/UA}{1 - Q/UA}\right)^2\right] \qquad 8.26$$

Defining the ratio

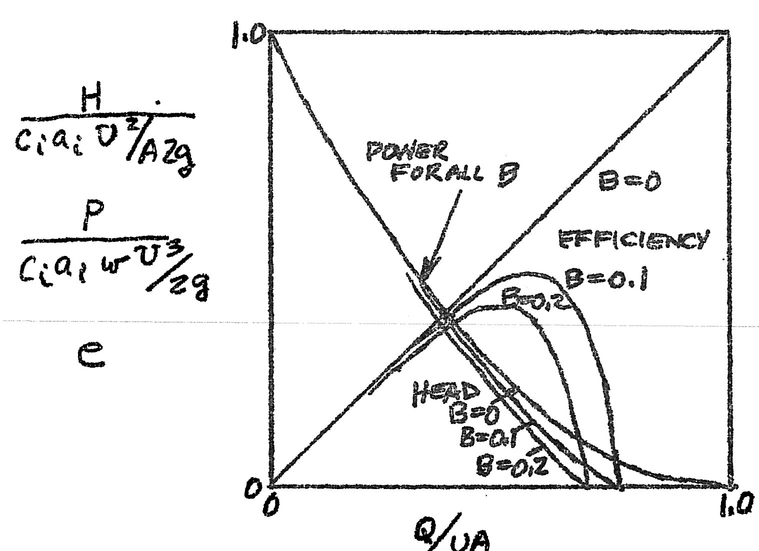
$$\frac{C_c a_c}{C_i a_i} = B \qquad 8.27$$

and normalizing

$$\frac{H}{C_i a_i U^2/A2g} = \left(1 - \frac{Q}{UA}\right)^2 - B\left(\frac{Q}{UA}\right)^2 \qquad 8.28$$

$$\frac{P}{C_i a_i w U^3/2g} = \left(1 - \frac{Q}{UA}\right)^2 \qquad 8.29$$

$$e = \frac{Q}{UA}\left[1 - B\left(\frac{Q/UA}{1 - Q/UA}\right)^2\right] \qquad 8.30$$



ADD complete performance

8.6 Model laws for turbulence drag pumps. The model laws are obtained by applying the technique described in Section 7.3. For geometrically similar pumps with a characteristic dimension D (the impeller diameter) and a rotational speed N,

$$Q \propto N D^3$$

$$H \propto N^2 D^2 \qquad (8.31)$$

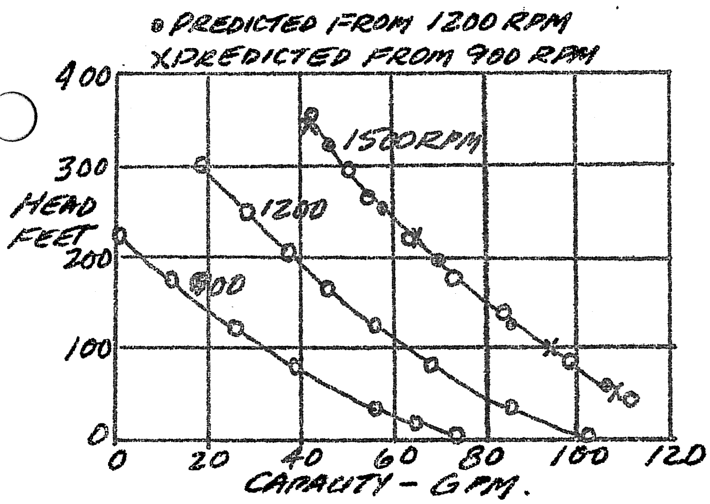
$$P \propto w N^3 D^5$$

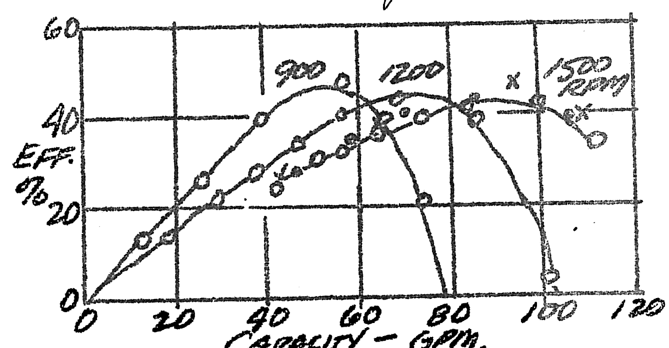
e is the same at corresponding conditions.

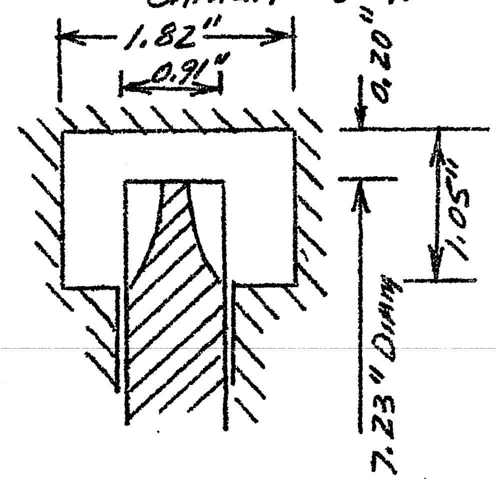
These model laws include $U \propto ND$ and $A \propto a_i \propto a_c \propto D^2$ with C_i and C_c both single-valued for the same similar geometries.

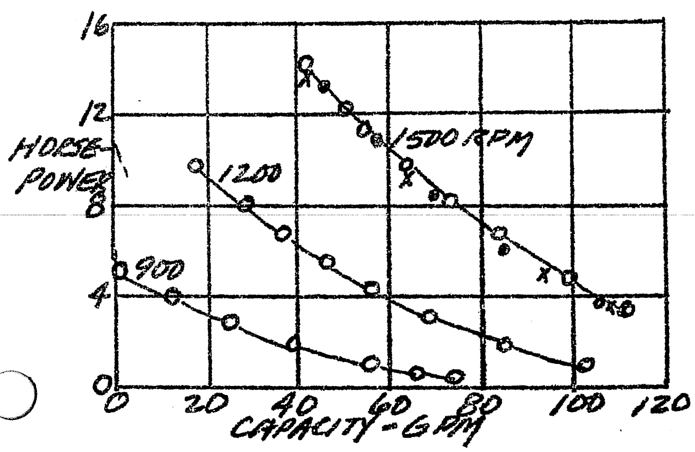
8.7. Test results.

NOTE SIMILARITY to SHAPES from ANALYSIS









FLOW CHANNEL AREA = $A = 0.00845$ SQ. FT.

$C_i a_i = 0.206$

$C_c a_c = 0.0062$

$U = 24.3$ FT/SEC AT 900 RPM.

Include effect of SIDE WALL CLEARANCE

Tests with water at 70°F.

8.7 References on Drag Pumps.

——— Screw Viscosity Pumps. Engineering, July 30, 1915 and Nov. 17, 1922, p. 606.

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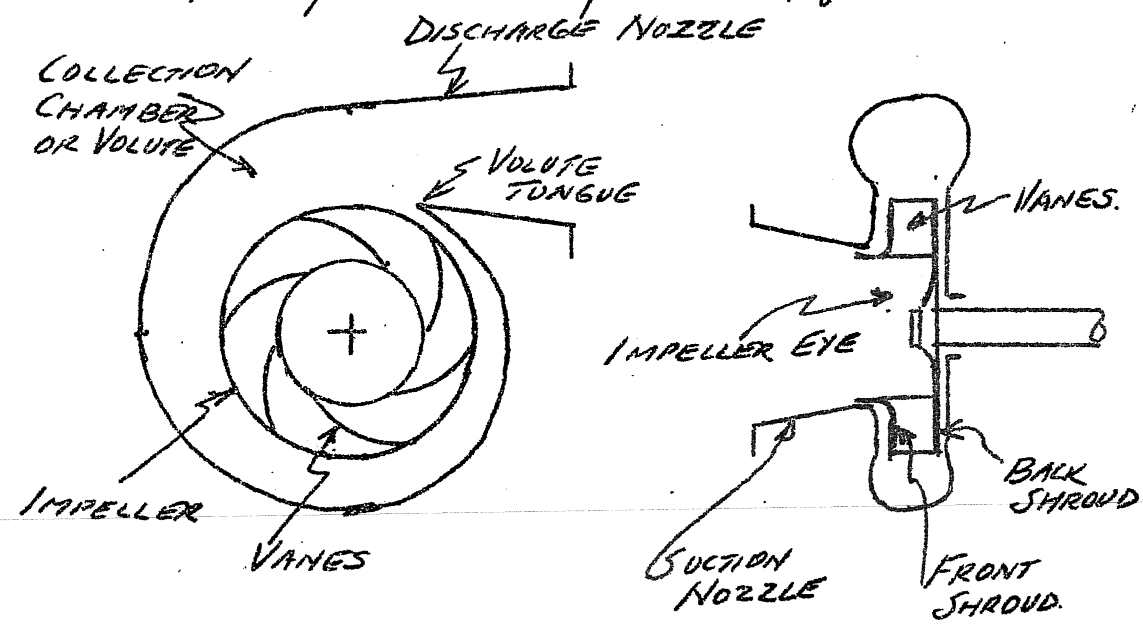
9.0 Centrifugal Pumps. (See Sections 2.2 and 5.1)

9.0 General description. Fluid transport is produced by forces on the fluid from the vanes of an impeller causing a change in the moment of momentum of the fluid. If the direction of the fluid in the vaned section of the impeller is in radial planes, the pump is called a radial flow centrifugal. If the direction of the fluid in the vaned section is at a constant radius from the axis of rotation of the vanes, the pump is called an axial flow or propeller pump. If the fluid in the vaned section has both radial and axial flow components, the pump is called a mixed flow. (Mixed flow pumps are also misnamed turbine pumps).

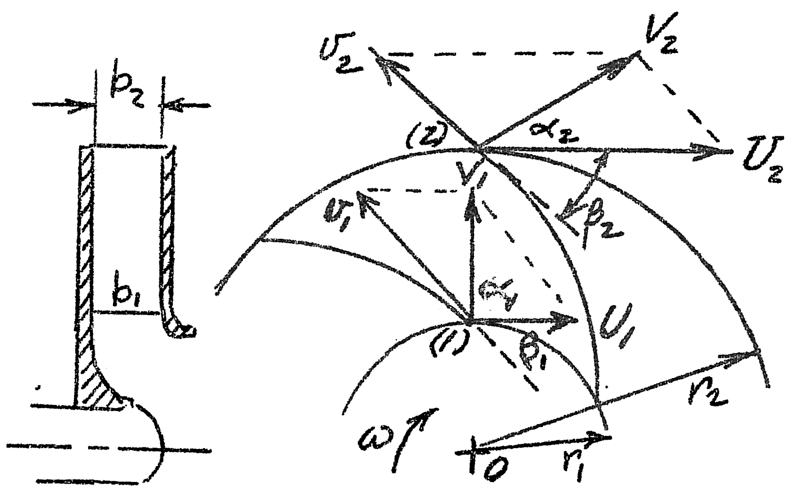
The analysis of the action of the vanes on the fluid is the same whether the flow is radial or axial or a combination of radial and axial. Other desireable or necessary features of centrifugal pumps, such as stationary guide vanes, are found from investigation of the flow patterns entering or leaving the impeller.

9.1. Description of the radial flow centrifugal pump.



The pump is the complete assembly from suction flange to discharge flange. Analysis of the pumping action includes all features of the flow between the flanges.

9.2 Analysis of the ideal radial flow impeller. Fluid enters the eye of the impeller in an axial direction but turns to a radial direction before entering the vanes. With no pre-rotation, the absolute fluid velocity at entrance to the vanes is radial. The fluid leaving the vanes must have a tangential component in the direction of the impeller movement if there is to be a force acting on the fluid and hence an energy transfer from the impeller to the fluid.



U - Impeller velocity
V - fluid absolute velocity
v - fluid velocity relative to the impeller
β - vane angle
b - vane width between shrouds.

For the fluid between (1) and (2)

$$\Sigma(\text{Moments})_0 = \Delta(\text{Moment of momentum rate})_0$$

The external moment acting on the fluid is the torque driving the impeller.

$$T = \int^{A_2} V_{2t} \cdot r_2 \, d\dot{m} - \int^{A_1} V_{1t} \, r_1 \, d\dot{m} \qquad 9.1$$

V_{2t} is the tangential component of V_2 at (2) or the component of V_2 in the direction of U_2

V_{1t} is the component of V_1 in the direction of U_1

$\dot{m}$ is the mass rate

Eq. 9.1 is general and applies for any velocity distribution at the sections (1) and (2). It also includes frictional effects between (1) and (2) since T is the torque acting on the fluid from the impeller and includes all the boundary forces of the impeller.

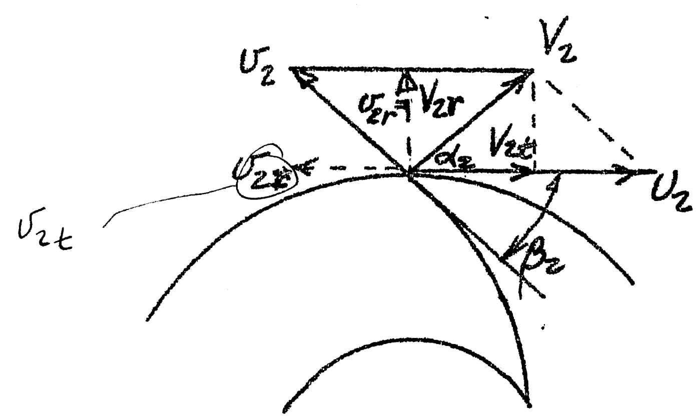
In order to integrate eq. 9.1, the velocity distribution must be known as a function of $\dot{m}$. As a first approximation to determine the possible performance, consider the ideal impeller which is defined as one with

(a) no friction
(b) an infinite number of vanes
and, (c) vanes of zero thickness

The assumption is also made that the fluid enters the impeller with no prerotation such that $V_{1t} = 0$. Then,

$$T = \int^{A_2} V_{2t}\, r_2\, d\dot{m} = V_{2t}\, r_2\, \dot{m} = V_{2t}\, r_2 \frac{Q w}{g} \qquad 9.2$$

With the ideal impeller the fluid is perfectly guided by the vanes and leaves the impeller with the relative velocity, v_2, which is parallel to the vane direction



For the ideal impeller with perfect fluid guidance and no friction, the efficiency must be 100%.

Then, $$P_\infty = Q w H_\infty = T\omega \qquad 9.3$$

Combining eqs 9.2 and 9.3

$$H_\infty = \frac{\omega V_{2t} r_2}{g} \qquad 9.4$$

also, $U_2 = \omega r_2$

Then, $$H_\infty = \frac{U_2 V_{2t}}{g} \qquad 9.5$$

From the velocity diagram,

$$V_{2t} = V_2 \cos\alpha \qquad 9.6$$

$$V_{2t} = U_2 - v_{2t} \qquad 9.7$$

$$v_{2t} = v_2 \cos\beta_2 \qquad 9.8$$

$$v_2 = v_{2r}/\sin\beta_2 = V_{2r}/\sin\beta_2 \qquad 9.9$$

Then,

$$V_{2t} = U_2 - v_{2t} = U_2 - v_2 \cos\beta_2 = U_2 - V_{2r}\frac{\cos\beta_2}{\sin\beta_2} \qquad 9.10$$

$$= U_2 - V_{2r} \cdot \cot\beta_2 \qquad 9.11$$

also,

$$Q = V_{2r} \cdot 2\pi r_2 \cdot b_2 \qquad 9.12$$

$$V_{2t} = U_2 - \frac{Q \cot\beta_2}{2\pi r_2 b_2} \qquad 9.13$$

Combining eqs. 9.5 and 9.13

EULER HEAD

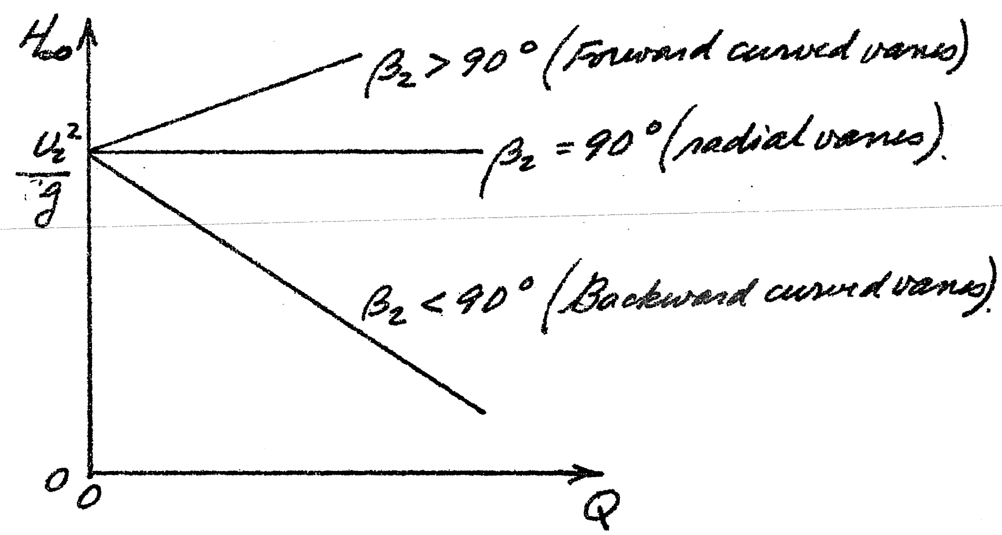
$$H_\infty = \frac{U_2^2}{g} - \frac{Q\, U_2 \cot\beta_2}{2\pi r_2 b_2 g} \qquad 9.14$$

For a single geometry and a constant speed, H is linear in Q. The effect of the vane angle may be noted

$\beta_2 > 90°$ $\cot\beta_2$ is negative

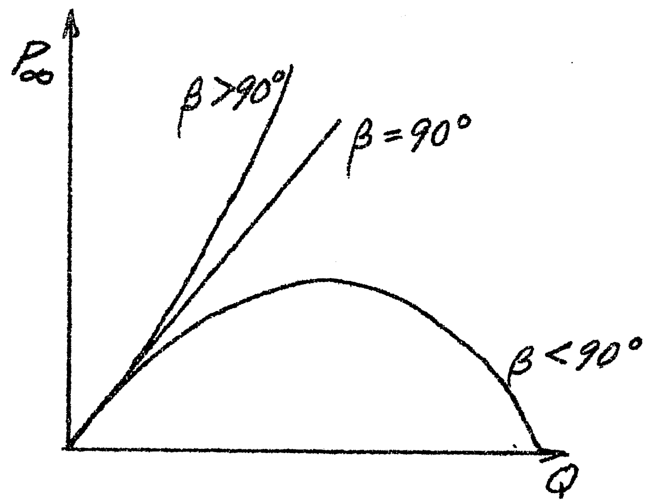
$\beta_2 = 90°$ $\cot\beta_2 = 0$

$\beta_2 < 90°$ $\cot\beta_2$ is positive.



From eqs 9.14 and 9.3

$$P_\infty = Q w \left[\frac{U_2^{\,2}}{g} - \frac{Q U_2 \cot\beta_2}{2\pi r_2 b_2 g} \right] \qquad 9.15$$



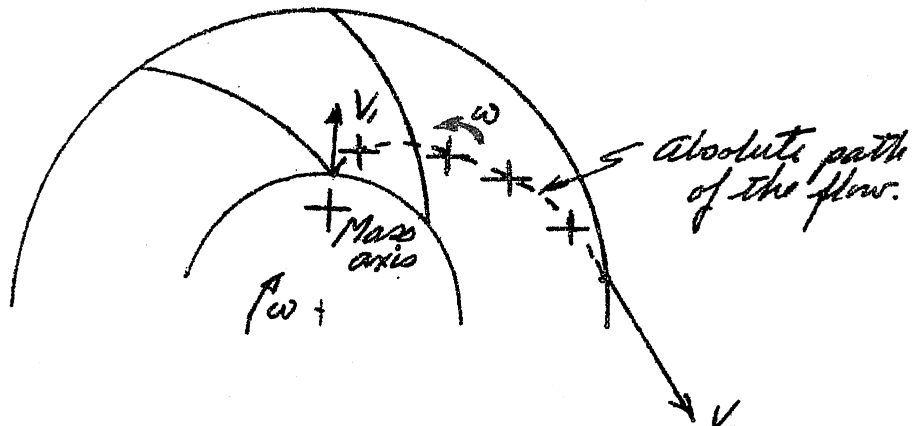
The efficiency for all β and all Q is 1.0 (or 100%)

There is one implied condition in the foregoing analysis. The fluid enters the impeller at (1) with no abrupt change in direction causing losses in head due to disturbances of the flow. Thus, for radial inflow at all Q

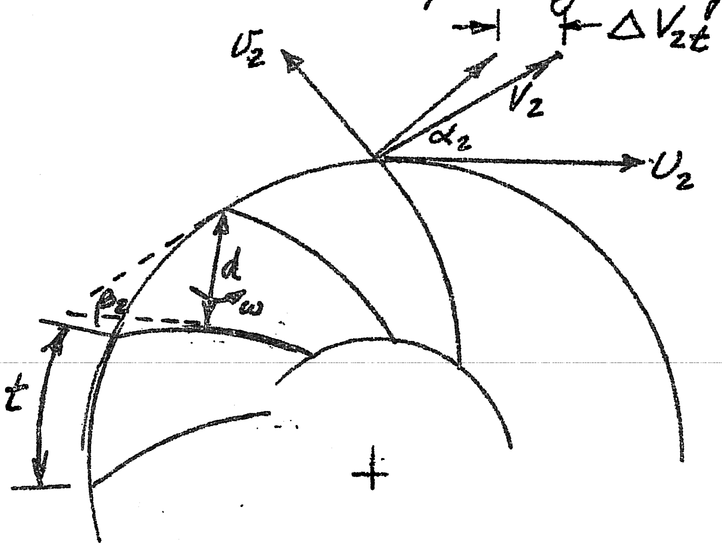
Then, $$\tan\beta_1 = \frac{V_1}{U_1} = \frac{Q}{2\pi r_1 b_1 U_1} \qquad 9.16$$

If eq. 9.16 is to hold for all flow rates, the vane angle at entrance must be different for each flow. However, an impeller has only one inlet vane angle. At flow rates other than the one flow rate for "shock-free" entrance obtained from eq. 9.16 there is the possibility of an entrance loss. This is minimized since the fluid will actually assume a pre-rotation to partially adjust itself to the required minimum entrance loss condition. The ideal impeller analysis still holds since any induced prerotation must be furnished by the action of the impeller. The inlet section (1) of eq. 9.1 could be taken anyplace ahead of the entrance to the vane without changing the results of eqs 9.14 and 9.15 as long as there was no tangential component of the fluid at the selected location.

9.3 Finite number of vanes. The fluid enters the impeller with an essentially radial velocity and acquires a tangential component on passing through the impeller. The absolute path of the flow is in the nature of a spiral.



Coupled with the throughput motion is a relative rotation between the vanes due to the inertia of fluid. The axis of a fluid mass tends to remain in the direction relative to a fixed frame of reference. Relative to the impeller, this mass tends to rotate at the value of the angular velocity of the impeller but in a sense opposite to that of the impeller. The absolute velocity, V_2, in direction and magnitude, must be modified by this effect.



The effective tangential component of the absolute velocity at (2) is

$$V_{2t}' = V_{2t} - \Delta V_{2t} \qquad 9.17$$

$$\Delta V_{2t} = K_S \frac{d}{2}\omega \qquad 9.18$$

K_S is introduced to account for imperfections in this approximate analysis.

Assume that the vane spacing, t, is small. Then the approximation of the triangle containing d and t gives

$$d \simeq t \sin\beta_2 \qquad 9.19$$

also, $t = \dfrac{2\pi r_2}{n}$ where n = number of vanes 9.20

Then,

$$\Delta V_{2t} = K_S \frac{\pi r_2 \omega \sin\beta_2}{n}$$

$$= K_S \frac{\pi U_2 \sin\beta_2}{n}$$

From eq. 9.5, with the effective tangential component of velocity,

$$H_S = \frac{U_2 V'_{2t}}{g} \qquad 9.21$$

$$= \frac{U_2 \left(U_2 - v_{2t} - \Delta V_{2t}\right)}{g}$$

$$= \frac{U_2^2}{g}\left(1 - K_S \frac{\pi \sin\beta_2}{n}\right) - \frac{U_2 v_{2t}}{g}$$

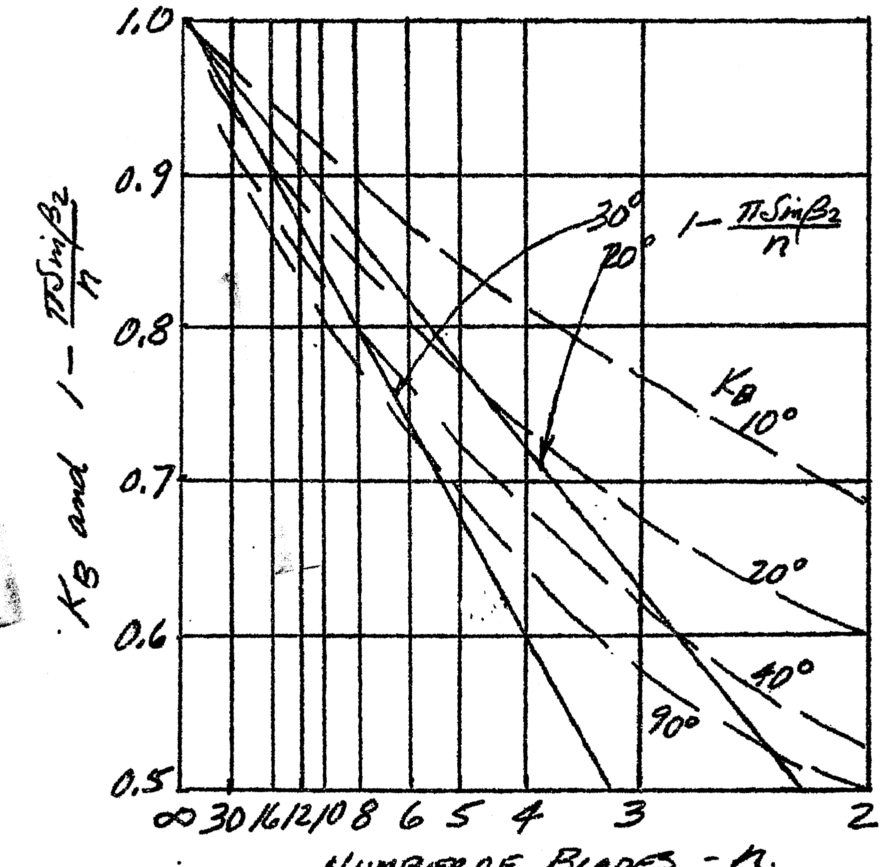
$$H_S = \frac{U_2^2}{g}\left(1 - K_S \frac{\pi \sin\beta_2}{n}\right) - \frac{Q U_2 \cot\beta_2}{2\pi r_2 b_2 g} \qquad 9.22$$

The term $K_S \dfrac{\pi \sin\beta_2}{n} \cdot \dfrac{U_2^2}{g}$ is the Stodola correction for a finite number of blades. Note that for $n < 3$ and $\beta_2 = 90$ with $K_S = 1$ the correction results in negative heads for all values of Q and is therefore not a good approximation for all vane angles and numbers of vanes. But for β_2 in the range $20°$ to $40°$ and $n > 4$ the correction gives reasonable agreement with more refined analyses.

K_S may be taken as 1.0 for $n > 4$ and $\beta_2 < 40°$.

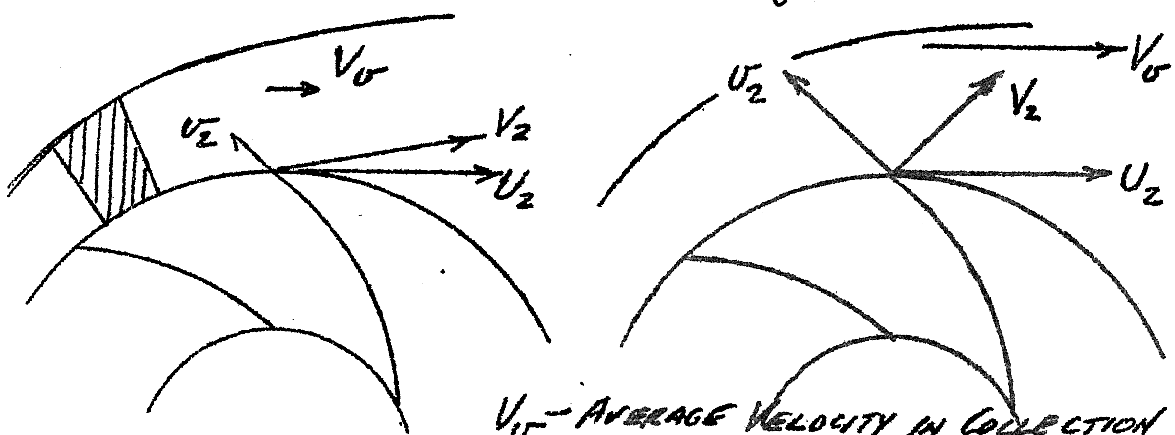
A more detailed analysis by Busemann gives values of the finite blade correction which are more realistic for all values of n and β_2. This may be defined as K_B in

$$H_i = K_B \frac{U^2}{g} - \frac{Q U_2 \cot\beta_2}{2\pi r_2 b_2 g}$$



Possible improved impeller performance with a larger number of vanes is balanced by increased friction losses of the flow through the impeller since each additional vane introduces more shear surface. The blade length to spacing ratio is also a factor in providing guidance of the fluid. Short blades with wide spacings are not as effective as long blades with narrow spacings. Optimum impeller designs have evolved from comparisons of measured performance for different numbers of blades, blade length, blade shape, and blade angles. The idealized analyses of the impeller gives a first approximation for design.

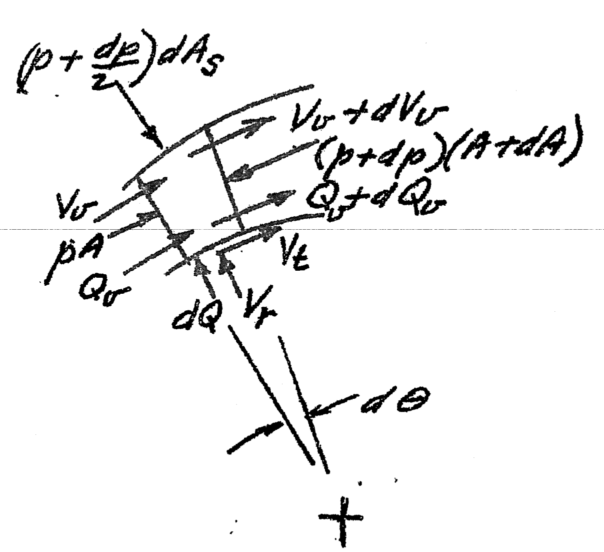
9.4 Losses in the collection chamber. The fluid discharges from the impeller and enters the collection chamber with a velocity, V_2, which is larger at low flow rates than at high flow rates. In the collection chamber the converse is true. The average velocity is small at low flow rates and large at high flow rates. In general, then, there is a mixing of two fluid streams of different velocities with a consequent mixing loss. This mixing also gives rise to a non-uniform pressure distribution around the collection chamber and at the discharge perimeter of the impeller. A radial thrust on the impeller is another consequence of the mixing.



V_v – AVERAGE VELOCITY IN COLLECTION CHAMBER

LOW FLOW RATE HIGH FLOW RATE

An approximate analysis of the mixing action with resultant effects may be made by considering an element of the collection chamber (shaded element in the above figure.)



A_S – surface area of the collection chamber

A – Cross-section area of the collection chamber

Fluid leaves the impeller with the radial velocity V_r and the tangential velocity V_t.

θ – angle from the volute tongue

r = Impeller radius.

Assumptions:
(1). The pressure is constant over the area A.
(2) The velocity is constant over the area A
(3) The collection chamber radial width is small compared to the impeller radius such that all moment arms about the central axis may be taken as the impeller discharge radius.
(4) The discharge from the impeller is uniform over the width of the impeller, and also over the circumference of the impeller
(5) Side wall friction is negligible.

The force from the pressure acting on the surface area A_s has a tangential component

$$\left(p+\frac{dp}{2}\right) dA_s \sin\alpha$$

where α is an angle related to the increase of the collection chamber area in the direction θ.

A moment balance about the axis of rotation of the impeller and in the direction θ is

$$A p r - (A+dA)(p+dp) r + \left(p+\frac{dp}{2}\right) dA_s (\sin\alpha) r$$
$$= \rho (Q_v + dQ_v)(V_v + dV_v) r - \rho Q_v V_v r - \rho V_t r \, dQ \qquad 9.23$$

also, $(\sin\alpha)\, dA_s = dA$ 9.24

Expanding eq. 9.23, neglecting second order terms, and using eq. 9.24 gives,

$$-A\,dp = \rho V_v \, dQ_v + \rho Q_v \, dV_v - \rho V_t \, dQ \qquad 9.25$$

By assumption (2),

$$V_v = \frac{Q_v}{A} \quad \text{and} \quad dV_v = \frac{dQ_v}{A} - \frac{Q}{A^2} dA \qquad 9.26$$

w/no int. recirc.

Eq. 9.25 then becomes, with a mass balance giving $dQ = dQ_v$

$$-dp = \rho\left(2\frac{Q_v}{A^2} dQ_v - \frac{Q^2}{A^2} dA - \frac{V_t \, dQ}{A^2}\right) \qquad 9.27$$

?

In order to integrate eq. 9.27, the variation of A with θ must be known. A common design condition is

$$A = A_0 + K_1 \theta \qquad 9.28$$

in which A_0 is the clearance area between the tip of the tongue and the impeller periphery and K_1 is a constant.

There is a short-circuit flow through A_0 in addition to the flow from the impeller between the tongue at $\theta = 0$ and the location of A at θ.

$$Q_v = Q_0 + K_2 \theta \qquad 9.29$$

where
$$K_2 = \frac{Q}{2\pi} \qquad 9.30$$

Substituting eqs. 9.28, 9.29, and changing the independent variable to θ, and with the condition

$$p = p_e \quad \text{at} \quad \theta = 2\pi$$

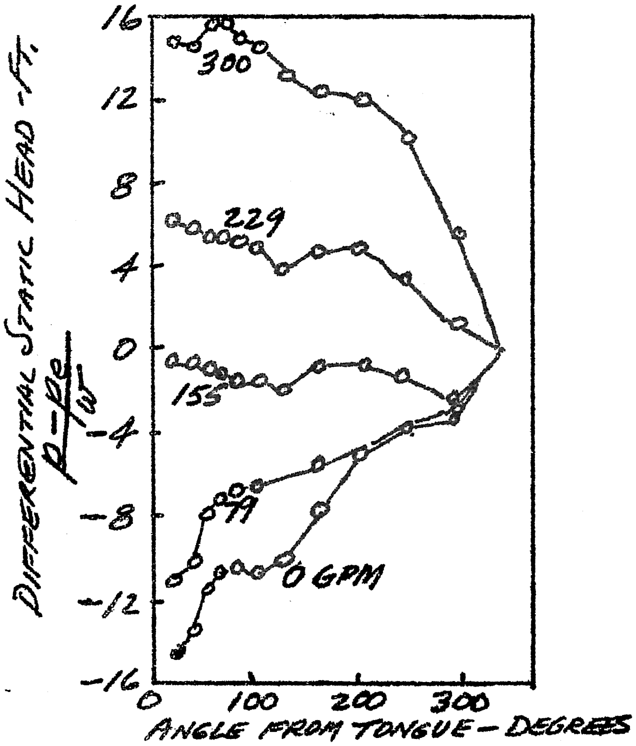
in eq. 9.27 and integrating yields

$$\frac{p - p_e}{\rho} = \left[V_t \frac{K_2}{K_1} - \left(\frac{K_2}{K_1}\right)^2 \right] \ln \left(\frac{A_0 + K_1 \theta}{A_0 + 2\pi K_1} \right)$$
$$+ \frac{(K_2 A_0 - K_1 Q_0)^2}{2 K_1^2} \left[\frac{1}{(A_0 + 2\pi K_1)^2} - \frac{1}{(A_0 + K_1 \theta)^2} \right] \qquad 9.31$$

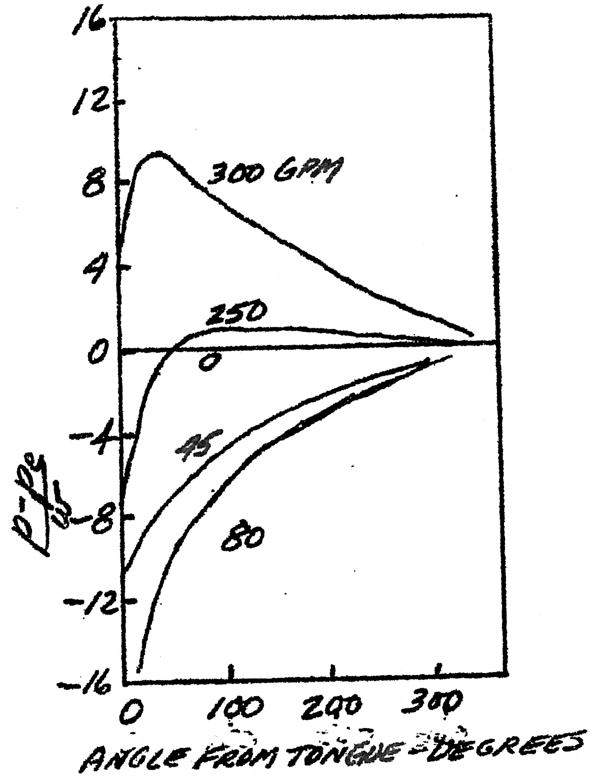
The pressure variation around the collection chamber given in eq. 9.31 was compared to measured pressure variations of a test pump which had the following constants:

$A_0 = 0.00244$ sq. ft
$K_1 = 0.00346$ sq. ft. per rad.
$U_2 = 44.3$ ft. per sec. (at 1000 RPM)
$\beta_2 = 22.5°$ $\qquad n = 9$ vanes
$r = 0.423$ ft
$b_2 = 0.043$ ft

Although A_0 of the test pump was not zero, Q_0 was taken as zero in the numerical evaluation of eq. 9.31.



MEASURED STATIC-HEAD DISTRIBUTION

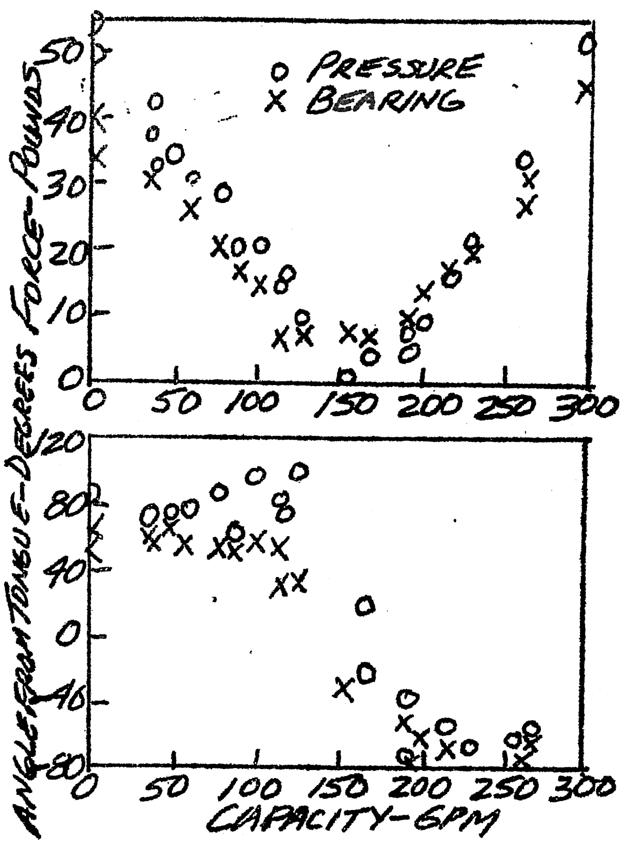


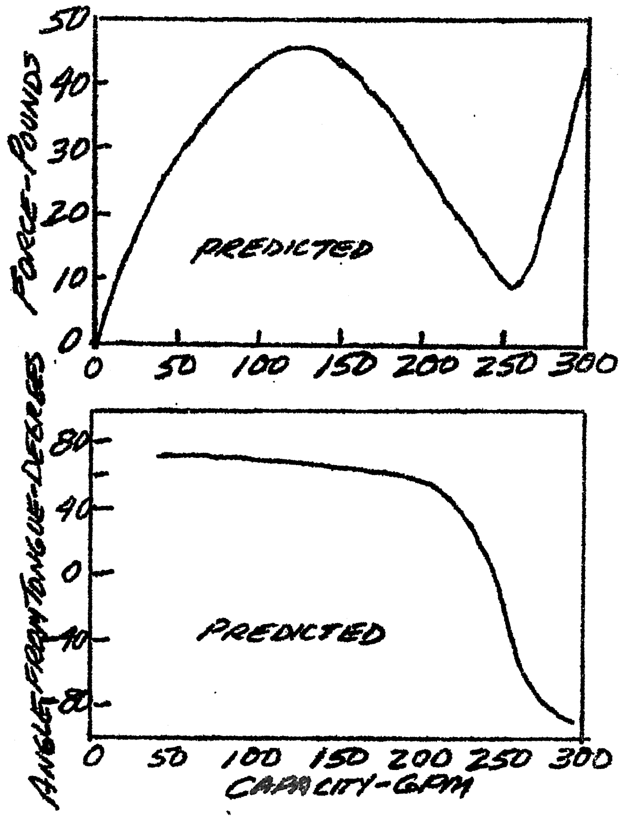
PREDICTED STATIC-HEAD DISTRIBUTION

The test pump had large wearing ring clearances between the impeller and casing. Thus, there was considerable internal recirculation. The flow through the impeller was finite and of appreciable magnitude even when the pump capacity was zero. The analysis shows a qualitative comparison with the measured pressure distribution around the collection chamber.

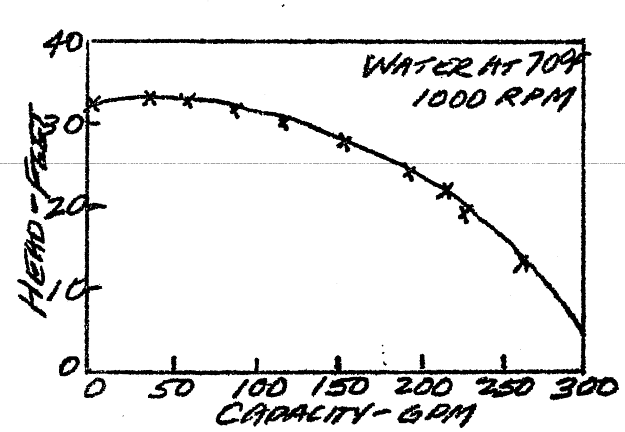
The non-uniform pressure distribution in the collection chamber produces a net radial force on the impeller which is transmitted to the shaft and bearings. This force is obtained by integration of right angle components of the pressure acting on the total perimeter of the impeller. Resolution of the components gives both total force magnitude and direction.

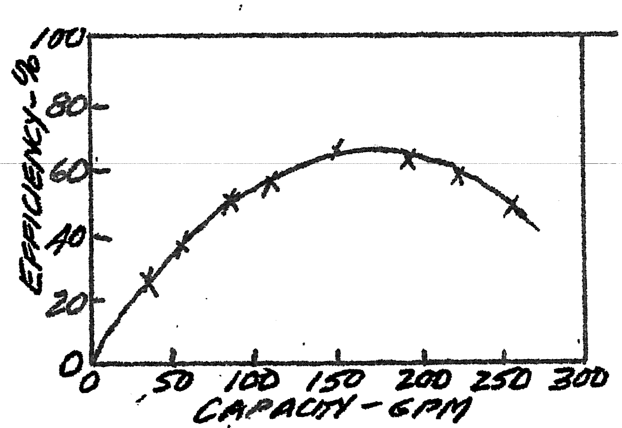
In the test pump the bearing reactions were also measured. Comparison of the force from the pressure distribution and the radial force from the bearing reaction shows reasonable agreement.





The predicted force must be interpreted in terms of the capacity corrected for internal recirculation. If this is done, the predicted curves shift to the left with a much larger shift in the low capacity (or high head) range than in the high capacity (or low head) range. Qualitatively the predicted and measured forces agree. The maximum and minimum values show quantitative agreement.





The mechanical energy loss rate in the mixing process in the collection chamber is obtained from the mechanical energy balance on the element of the volute.

$$dE_m = Q_v\left[p+\rho\frac{V_v^2}{2}\right]+dQ\left[\left(p+\frac{dp}{2}\right)+\rho\frac{(V_t^2+V_r^2)}{2}\right]$$

$$-(Q_v+dQ_v)\left[(p+dp)+\rho\frac{(V_v+dV_v)^2}{2}\right] \qquad 9.32$$

Expanding and reducing,

$$dE_m = \rho\frac{(V_t^2+V_r^2)}{2}dQ - Q_v dp - \rho Q_v V_v dV_v - \rho\frac{V_v^2}{2}dQ_v \qquad 9.33$$

Substituting for the variables in terms of θ and integrating from $\theta = 0$ to $\theta = 2\pi$ yields the total energy loss rate due to mixing.

$$E_m = \frac{2\pi K_2 \rho}{2}\left[V_t^2+V_r^2+\frac{(Q_0K_1-K_2A_0)^2}{K_1^2A_0(A_0+2\pi K_1)}\right.$$

$$+(K_1Q_0-K_2A_0)\left(\frac{K_2}{\pi K_1^3}-\frac{V_t}{\pi K_1^2}\right)\ln\frac{A_0+2\pi K_1}{A_0}$$

$$\left.+\frac{K_2^2}{K_1^2}-2V_t\frac{K_2}{K_1}\right] \qquad 9.34$$

The head loss in the collection chamber is

$$H_m = \frac{E_m}{Q\rho g} = \frac{E_m}{2\pi K_2\rho g} \qquad 9.35$$

The pump head is

$$H = H_s - H_m \qquad 9.36$$

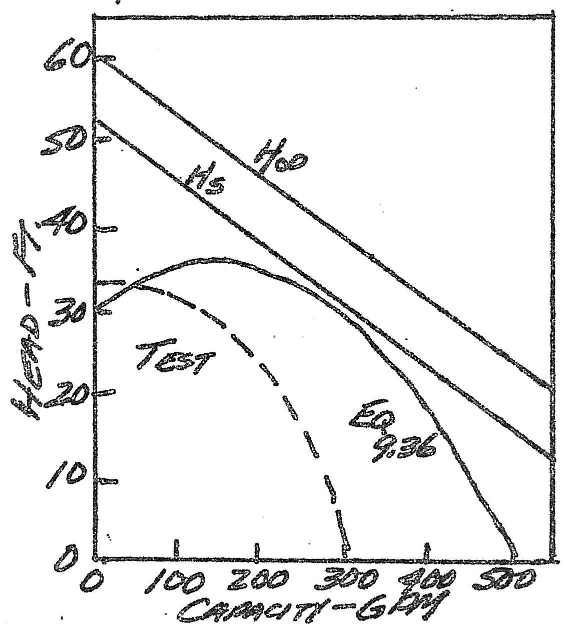
The significance of the mixing loss analysis may be seen for the conditions $K_2 = 0$ (or $Q = 0$), A_0 is small but finite, $Q_0 = 0$, and n is large making $H_s \simeq H_0$

Then $$H = H_0 - H_m = \frac{U_2^2}{g} - \frac{V_t^2}{2g} \qquad 9.37$$

at $Q = 0$; $V_t = U_2$ and

$$H = \frac{U_2^2}{2g}$$, a value in general agreement with test values.

The predicted head from eq. 9.36 for the test pump shows the expected form of the head-capacity function.



Flow friction losses within the pump will reduce the head at all flow rates by an amount proportional to the flow rate squared thus bringing the prediction more in line with the actual measured performance.

also Internal LEAKAGE

9.5. Model laws for radial flow centrifugal pumps. The model laws are obtained from the behavior equations. Eq. 9.36 in complete form is

$$H = \frac{U_2^2}{g}\left(1 - K_s\frac{\pi \sin\beta_2}{n}\right) - \frac{Q U_2 \cot\beta_2}{2\pi r_2 b_2 g}$$

$$- \frac{1}{2g}\left[V_t^2 + V_r^2 + \frac{(Q_0K_1 - K_2A_0)^2}{K_1^2A_0(A_0+2\pi K_1)}\right.$$

$$+ (K_1Q_0 - K_2A_0)\left(\frac{K_2}{\pi K_1^3} - \frac{V_t}{\pi K_1^2}\right)\ln\frac{A_0+2\pi K_1}{A_0}$$

$$\left. + \frac{K_2^2}{K_1^2} - 2V_t\frac{K_2}{K_1}\right]$$

Then,

$$b_H = \frac{b_{U_2}^2}{b_g} = \frac{b_{U_2}^2\, b_{K_S}\, b_{\sin\beta_2}}{b_g\, b_n} = \frac{b_Q\, b_{U_2}\, b_{\cot\beta_2}}{b_{r_2}\, b_{b_2}\, b_g} = \frac{b_{V_t}^2}{b_g} = \frac{b_{V_r}^2}{b_g} =$$

$$= \frac{b_{Q_0}^2\, b_{K_1}^2}{b_{K_1}^2\, b_{A_0}^2\, b_g} = \frac{b_{Q_0}^2\, b_{K_1}^2}{b_{K_1}^2\, b_{A_0}\, b_{K_1}\, b_g} = \text{etc.} -$$

The last two equalities give $b_{A_0} = b_{K_1}$. This is a condition of geometrical similarity. A complete representation of all areas in terms of linear dimension would show that geometrical similarity is a necessary condition.

From the first two equalities

$$b_H = b_U^2 \quad \text{since} \quad b_g = 1$$

also $\quad U \propto ND \quad$ or $\quad b_U = b_N b_D$

Then $\quad b_H = b_N^2 b_D^2$

Or $\quad H \propto N^2 D^2 \qquad$ 9.38

With geometrical similarity $b_{\sin\beta_2} = 1$; $b_{\cot\beta_2} = 1$; $b_{r_2} = b_D$; $b_{b_2} = b_D$; $b_{K_S} = 1$. From the third and fourth equalities,

$$b_{U_2}^2 = \frac{b_Q b_{U_2}}{b_D^2}$$

And, $\quad b_Q = b_{U_2} b_D^2 = b_N b_D^3$

Or, $\quad Q \propto ND^3 \qquad$ 9.39.

The power input to the impeller is

$$P = Q w H_S$$

Then $\quad b_P = b_Q b_w b_H = (b_N b_D^3) b_w (b_N^2 b_D^2)$

$$= b_w b_N^3 b_D^5$$

Or $\quad P \propto w N^3 D^5 \qquad$ 9.40

It follows that $\quad b_e = 1.0$

or $\quad e \propto 1.0 \qquad$ 9.41

Flow friction produces head losses proportional to the square of the flow rate for high Reynolds numbers and follow the same model laws as expressed above. Bearing and shaft seal losses modify the power input to the impeller to give the power input at the shaft

$$P_{sh} = P_{imp} + P_{bearing,\ etc}$$

Bearing and seal losses do not model with the same proportionality as the fluid effects given in eq. 9.40. However, the bearing and seal losses are usually a small proportion of the total power input. Neglecting this difference is not usually serious in predicting power requirements for different speeds.

Summarizing, the model laws are:

$$H \propto N^2 D^2 \qquad 9.38$$
$$Q \propto N D^3 \qquad 9.39$$
$$P \propto \omega N^3 D^5 \qquad 9.40$$
$$e \propto 1.0 \qquad 9.41$$

9.6. Specific Speed. A useful "type constant" of a pump is obtained from the model laws. Instead of expressing the model laws in the proportional forms, the proportionality constants are used.

$$H = K_H N^2 D^2 \qquad 9.42$$

$$Q = K_Q N D^3 \qquad 9.43$$

Eliminating D from eqs 9.42 and 9.43 gives

$$Q = K_Q N \left(\frac{H^{1/2}}{K_H^{1/2} N}\right)^3$$

$$\frac{K_Q}{K_H^{3/2}} = \frac{QN^2}{H^{3/2}}$$

Or,

$$\left(\frac{K_Q}{K_H^{3/2}}\right)^{1/2} = \frac{NQ^{1/2}}{H^{3/4}}$$

If the performance at maximum efficiency is taken as the condition for selection of Q and H, then $K_Q^{1/2}/K_H^{3/4}$ is a single value representing the effects of the complete geometry of a pump. This is defined as the specific speed.

$$\text{Specific speed} = N_S = \left(\frac{K_Q}{K_H^{3/2}}\right)^{1/2}_{e_{max}} = \left(\frac{NQ^{1/2}}{H^{3/4}}\right)_{e_{max}} \qquad 9.44$$

Since the specific speed is a combination of the constants K_H and K_Q which are obtained at the head and capacity at maximum efficiency, the value of the specific speed represents the geometry of a pump. Also, since the specific speed was obtained from the model laws, it has the same value for all geometrically similar pumps. Thus the specific speed may be used as a correlating modulus to compare different radial flow centrifugal pumps. These comparisons will be considered later since axial and mixed flow centrifugal pumps will be shown to have the same model laws as radial flow centrifugals and consequently will have the specific speed as a correlating modulus. All correlations will be given for the complete centrifugal pump class including radial flow, mixed flow, and axial flow.

The usual practices in determining the numerical value of the specific speed is to use

N - RPM
Q - GPM
H - Ft.

With these units, specific speed has dimensions of

$$\frac{RPM\,(GPM)^{1/2}}{(ft)^{3/4}}$$

However, if g was retained in the development of the model laws, and if consistent units were used, the specific speed would be dimensionless.

$$N_S = \frac{\omega\, Q^{1/2}}{H^{3/4}\, g^{3/4}} \qquad \frac{\frac{1}{T}\left(\frac{L^3}{T}\right)^{1/2}}{L^{3/4}\left(\frac{L}{T^2}\right)^{3/4}} \quad \text{UNITS CANCEL} \qquad 9.45$$

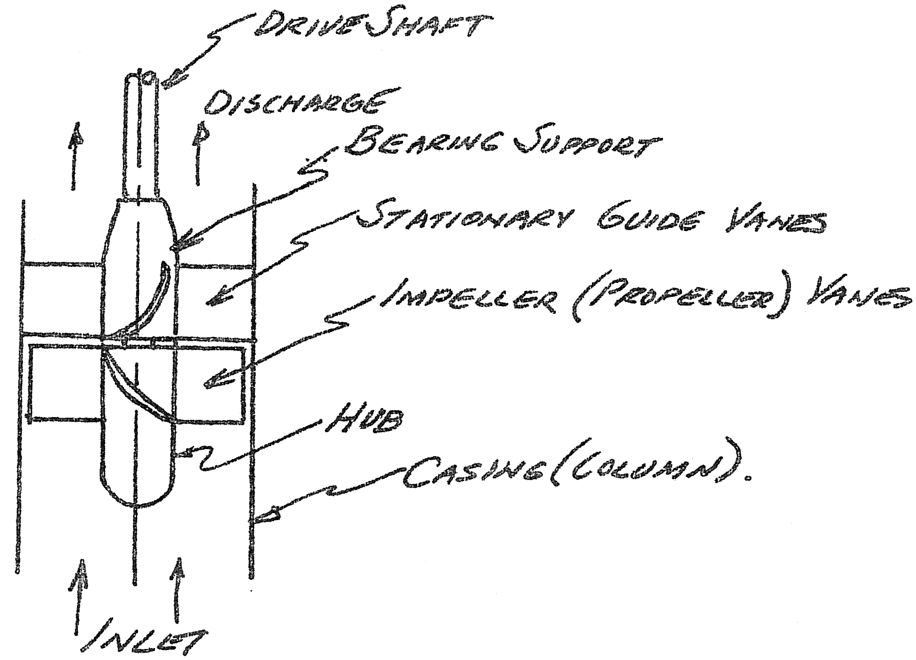
There is a trend in the modern literature to the dimensionless representation of specific speed with ω in rad/sec, Q in cubic feet per second, H in feet, and g in ft per second squared in eq. 9.45.

— OPEN, SEMI-OPEN, CLOSED IMPELLER

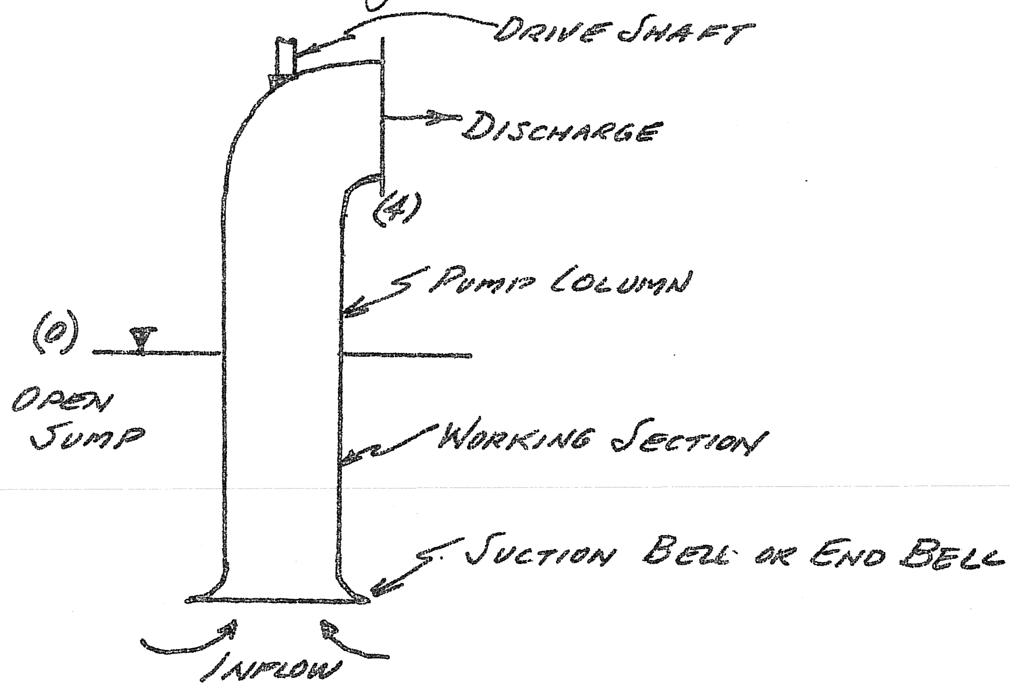
DOUBLE VOLUTE RADIAL THRUST

MODIFIED VOLUTE

9.7 Description of the axial flow pump.

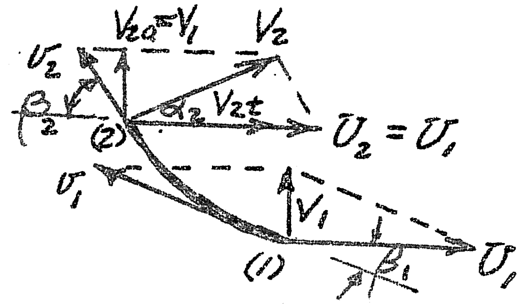


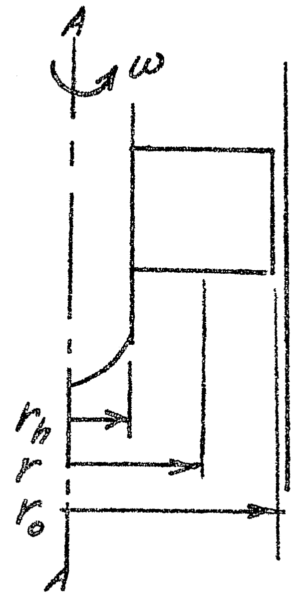
The working section of the pump consists of the impeller and the stationary guide vanes. Very often this type of pump is applied to take suction flow from an open sump. The pump then includes the suction or end bell and the working section. In some cases the pump also includes the discharge column and the discharge elbow.



In this arrangement the pump is considered as all effects from 0 to 4.

9.8 Analysis of the ideal axial flow impeller. Fluid enters the impeller in the annulus between the hub and the casing. With no prerotation the absolute velocity at entrance to the vanes is axial. The fluid leaving the vanes must have a tangential component in the direction of the impeller movement if there is to be a force acting on the fluid and hence an energy transfer from the impeller to the fluid.





U – impeller velocity

V – fluid absolute velocity

v – fluid velocity relative to the impeller

β – vane angle

r – radius from the axis of rotation

Subscript h refers to the conditions at the radius of the hub. Subscript o refers to the conditions at the radius of the outer edge of the vane

For the fluid between (1) and (2)

$$\Sigma (\text{Moments})_A = \Delta (\text{Moment of momentum rate})_A$$

For a mass $d\dot{m}$ at a radius r and with axial inflow

$$dT = V_{2t}\, r\, d\dot{m} \qquad 9.46$$

Also $dP = \omega\, dT = \omega V_{2t}\, r\, d\dot{m}$ 9.47

With no friction $dP = dQ\, w\, H = g\, d\dot{m}\, H$ 9.48

Combining eqs 9.47 and 9.48 gives the head added to the fluid at a radius r

$$H = \frac{\omega V_{2t} r}{g} \qquad 9.49$$

An ideal impeller is defined as one with an infinite number of vanes of zero thickness. With the ideal impeller the fluid is perfectly guided by the vanes and leaves the impeller with the relative velocity, v_2, which is parallel to the vane direction or with the angle β_2.

$$V_{2t} = U_2 - v_2 \cos\beta_2 \qquad 9.50$$

$$v_2 \sin\beta_2 = V_{2a} = V_1 \qquad 9.51$$

$$V_{2t} = U_2 - V_1 \frac{\cos\beta_2}{\sin\beta_2} = U_2 - V_1 \cot\beta_2 \qquad 9.52$$

Substituting eq. 9.52 in eq. 9.49 gives the head added to the fluid at a radius, r.

$$H = \frac{r\omega}{g}\left(U_2 - V_1 \cot\beta_2\right) \qquad 9.53$$

also $U_2 = r\omega$ and $V_1 = \frac{Q}{A_1}$ for axial inflow

and where $A_1 = \pi(r_o^2 - r_h^2)$

Then $$H = \frac{1}{g}\left(r^2\omega^2 - r\omega\frac{Q}{A_1}\cot\beta_2\right) \qquad 9.54$$

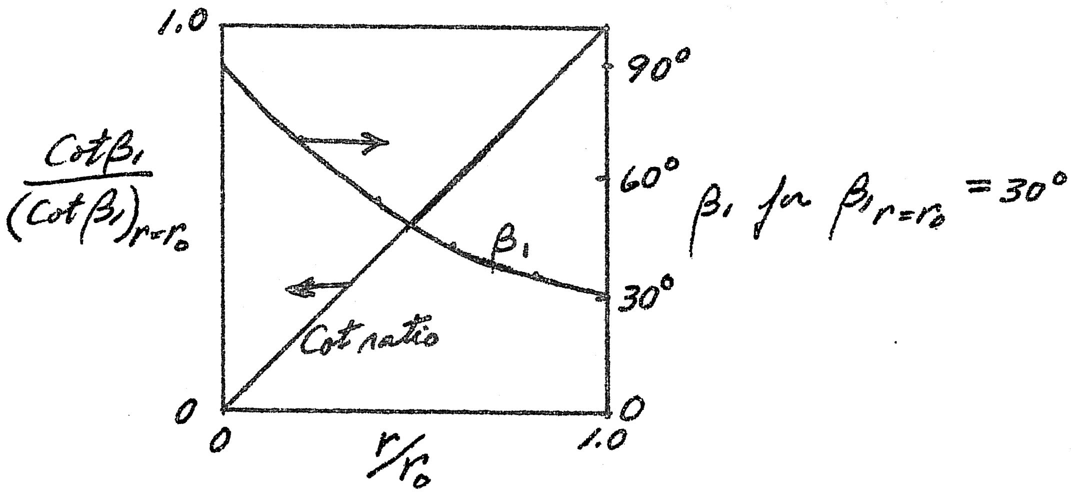
The head is a function of both r and β_2. Since the complete flow through the pump has the same head added to all portions of the flow and this head is added by the impeller vanes, then this constant head condition gives the relationship between r and β_2. The relationship of all the angles of the vane can best be seen by taking one angle as a reference. The angle selected is the inlet vane angle at the outer edge of the vane.

at the inlet, $$\cot\beta_1 = \frac{U_1}{V_1} = \frac{r\omega A_1}{Q} \qquad 9.55$$

$$\left(\cot\beta_1\right)_{r=r_o} = \frac{r_o\omega A_1}{Q} \qquad 9.56$$

Then $$\frac{\cot\beta_1}{\left(\cot\beta_1\right)_{r=r_o}} = \frac{r}{r_o} \qquad 9.57$$

For purposes of showing the nature of the angle variations β_1 at $r = r_o$ is taken as $30°$.



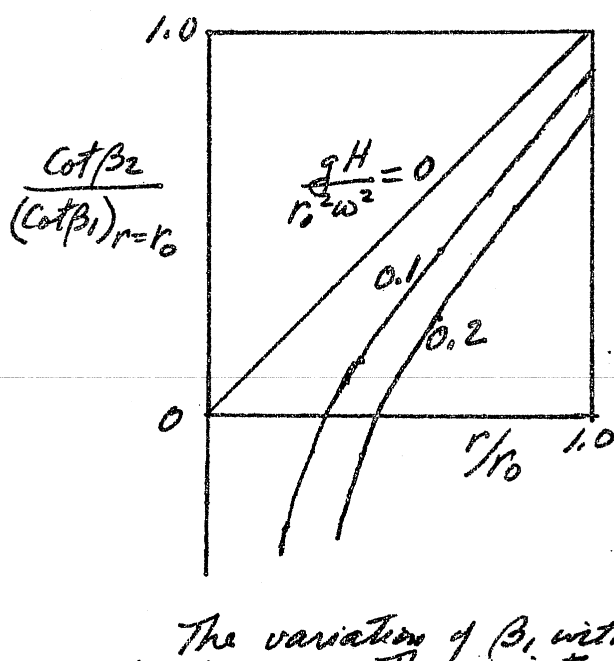
From eq. 9.54

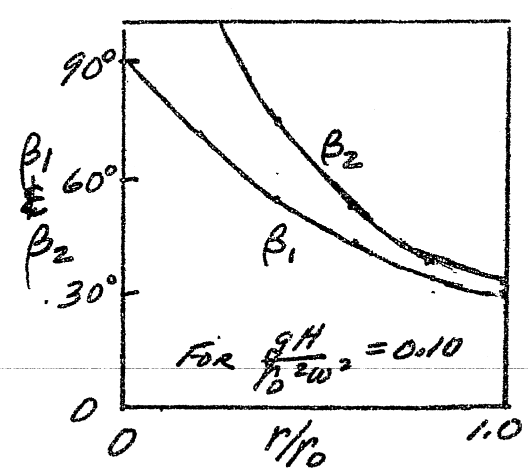
$$\cot\beta_2 = \frac{A_1}{Q}\left(r\omega - \frac{gH}{r\omega}\right) = \frac{r_0\omega A_1}{Q}\left(\frac{r}{r_0} - \frac{gH}{r_0^2\omega^2}\frac{r_0}{r}\right)$$

Using eq. 9.56.

$$\frac{\cot\beta_2}{(\cot\beta_1)_{r=r_0}} = \frac{r}{r_0} - \frac{gH}{r_0^2\omega^2}\frac{r_0}{r} \qquad 9.58$$

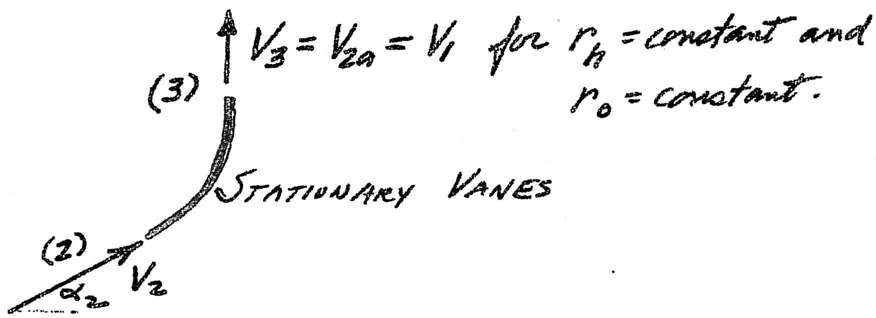
In eq 9.58 the grouping $\frac{gH}{r_0^2\omega^2}$ with the condition of H = constant, is a constant parameter for ω = constant.





The variation of β_1 with r shows the nature of the twist of the vanes. The variation of β_2 compared to β_1 shows the warp of the vanes.

Since the fluid must leave the vanes with a tangential component of velocity in order for there to be any head addition, a set of stationary vanes is required to redirect the flow in the axial direction. The leading edge of these stationary vanes should have the direction α_2 for smooth entry of the flow from the impeller into the straightening vanes. The discharge angle of the stationary vanes should be in the axial direction

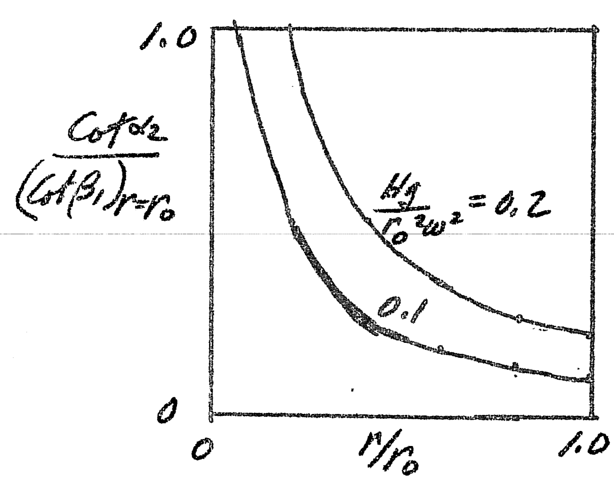


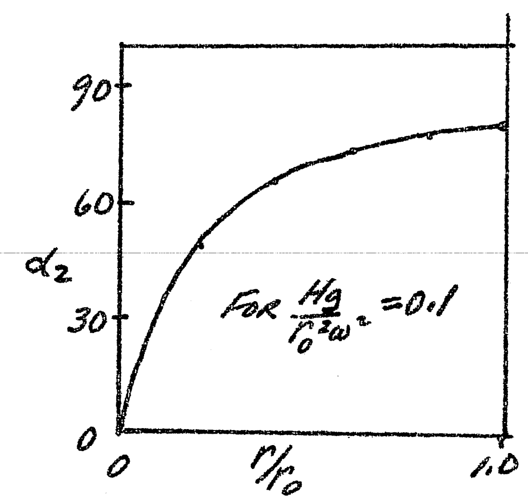
$$\tan\alpha_2 = \frac{V_{2a}}{V_{2t}} = \frac{V_1}{Hg/\omega r} = \frac{V_1 \omega r}{Hg} \qquad 9.59$$

$$\cot\alpha_2 = \frac{Hg A_1}{Q r_0 \omega}\left(\frac{r_0}{r}\right) \qquad 9.60$$

Using $(\cot\beta_1)_{r=r_0}$ as the reference angle as before,

$$\frac{\cot\alpha_2}{(\cot\beta_1)_{r=r_0}} = \frac{Hg}{r_0^2\omega^2}\left(\frac{r_0}{r}\right) \qquad 9.61$$



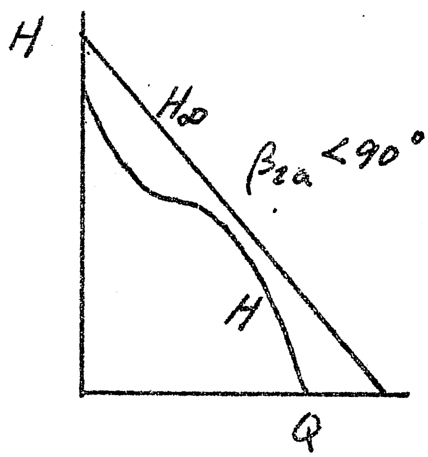


The ideal head-capacity function is given by eq. 9.54 for an element of the vane. However, the vane angles are related for all blade elements to give the same head, at least at the design flow rate. Thus eq. 9.54 can be considered to represent the head-capacity function of the complete impeller by using a representative average radius and a representative average blade angle.

$$H_o = \frac{1}{g}\left(r_a^2 \omega^2 - r_a \omega \frac{Q}{A_1} \cot \beta_{2a}\right) \qquad 9.62$$

The subscript a identifies the representative averages.

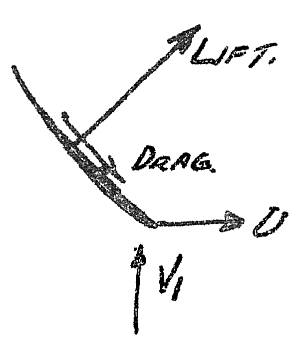
Eq. 9.62 shows H_o is a linear function of Q.



The actual head-capacity function is also shown. Its shape depends upon departures from the ideal flow patterns and includes flow friction and separation losses. It also includes the effects of a finite number of vanes. The separation losses are not easily determined by analysis.

It should be recognized that "shock-free" entrance into the impeller and also into the stationary guide vanes occurs only at the design flow rate since this is the flow rate at which all angles are matched to the relative velocity directions.

9.9 <u>Axial flow pump performance from airfoil characteristics.</u> The lift and drag of an airfoil section may be used to obtain the head-capacity performance of a section of an axial flow pump impeller vane.



The axial components of L and D produce the head added to the fluid. The tangential components produce the force applied to the fluid from the drive and hence give the needed power input.

This approach gives a better performance prediction than that from the ideal impeller since both friction and separation are included. However, the ideal impeller analysis is sufficient to show many of interrelated features of the axial flow pump and the airfoil theory will not be described here.

9.10. Model laws of axial flow pumps. From eq. 9.54 and eq. 9.48 the model laws are

$$Q \propto ND^3$$

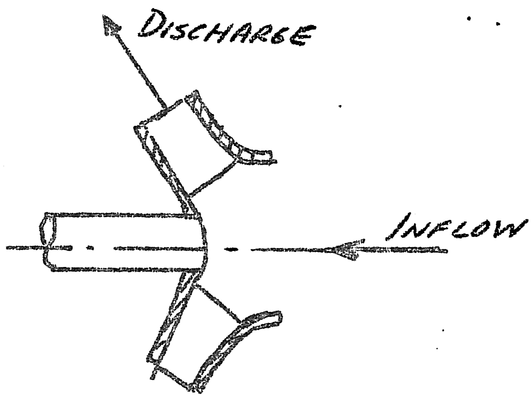
$$H \propto N^2 D^2 \qquad 9.62$$

$$P \propto \omega N^3 D^5$$

$$e \propto 1.0$$

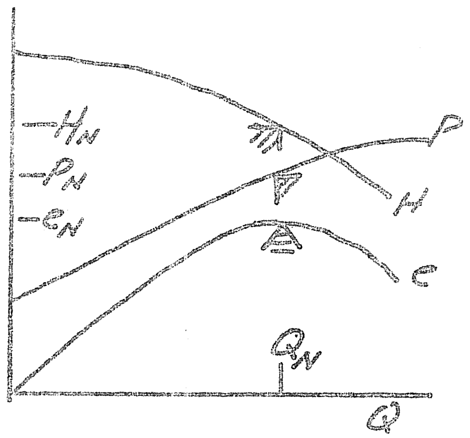
These are the same as for the radial flow centrifugal pump. Thus, the "type constant" or the specific speed applies to the axial flow pump as well as for the radial flow pumps and it has the same significance.

9.11. Mixed flow centrifugal pumps. The mixed flow pump impeller has passages in the vaned section of the impeller in which the fluid has both axial and radial components. Thus it has performance functions which combine the features of both the radial and the axial flow analyses. The same model laws apply, as does the concept of the specific speed.



9.12 Centrifugal pump performance correlations. The specific speed represents the relative geometry of the impeller, and of the associated casing passages which match the impeller conditions at the design flow rate. By relative geometry is meant the ratio of the impeller width to the impeller diameter. Impellers of the same relative geometry regardless of absolute size should give pumps with the same specific speed, at least for pumps of best design.

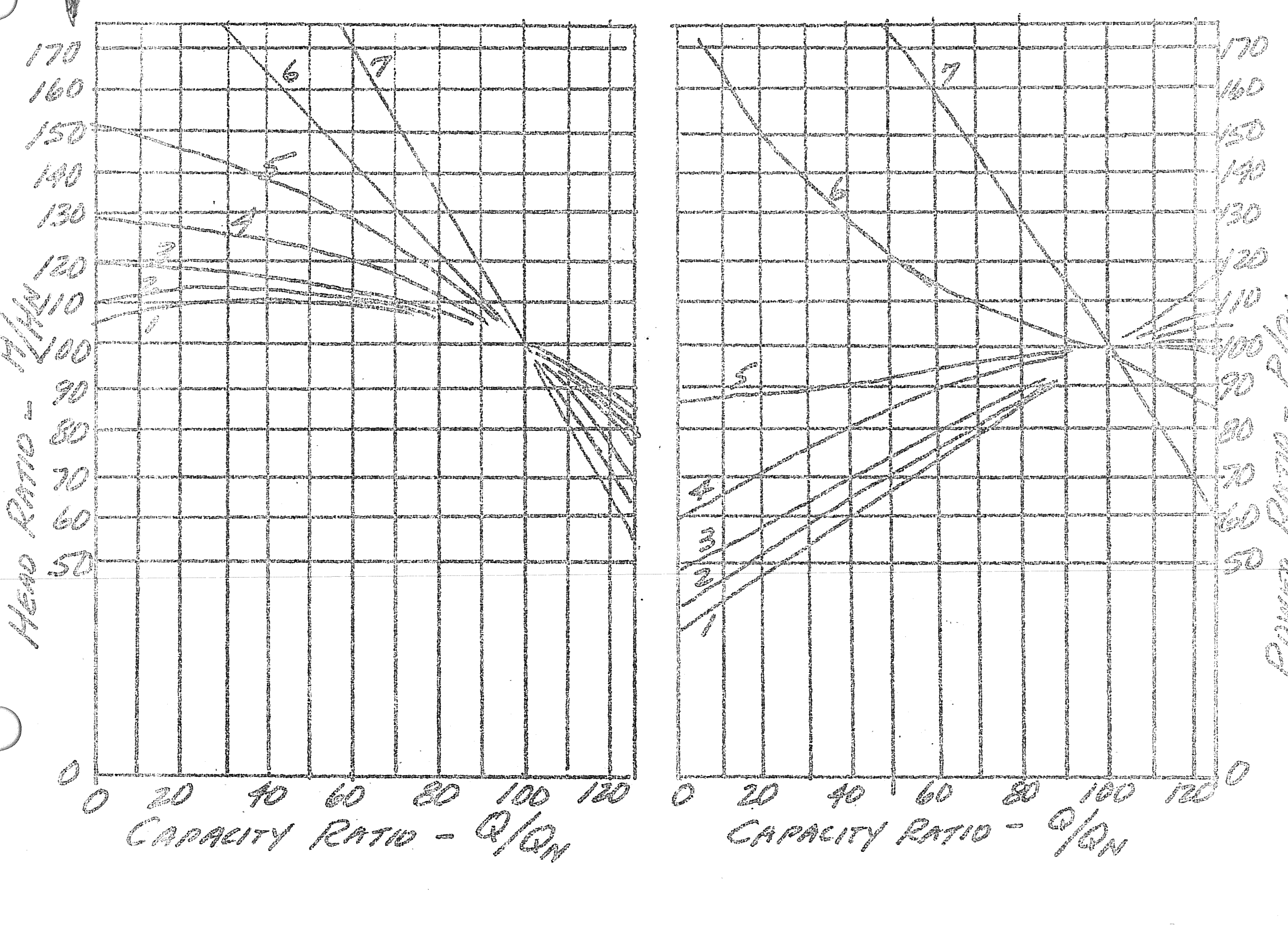
The shape of the performance curves is then a function of specific speed. Comparison of pumps of different size is possible by normalizing the performance using the capacity, head, power, and efficiency at the best efficiency point as the base value.



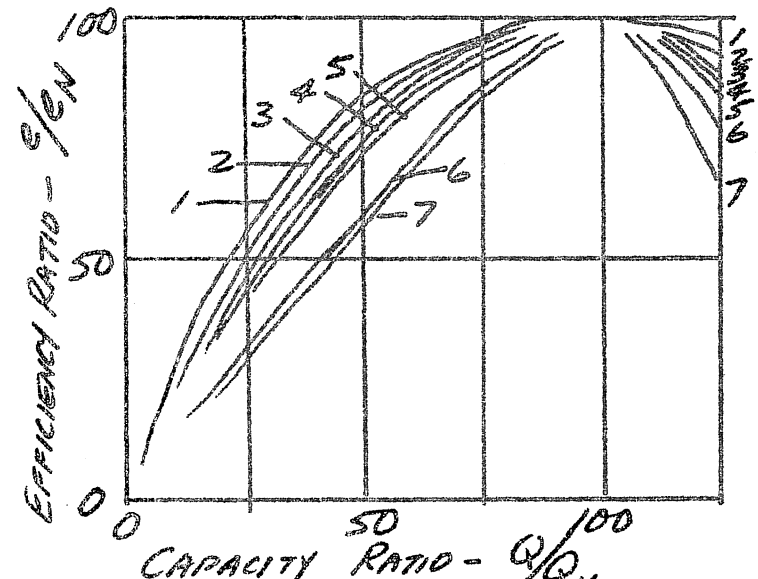
The ratios $\frac{Q}{Q_N}$, $\frac{H}{H_N}$, $\frac{P}{P_N}$ and $\frac{e}{e_N}$ are the normalized performance values into which the performance curves can be transformed for comparisons between pumps.

Curve	Single Suction Spec. Speed	Curve	Sing. Suc. Spec. Speed
1	650	5	2700
2	1100	6	5700
3	1550	7	9200
4	2100		

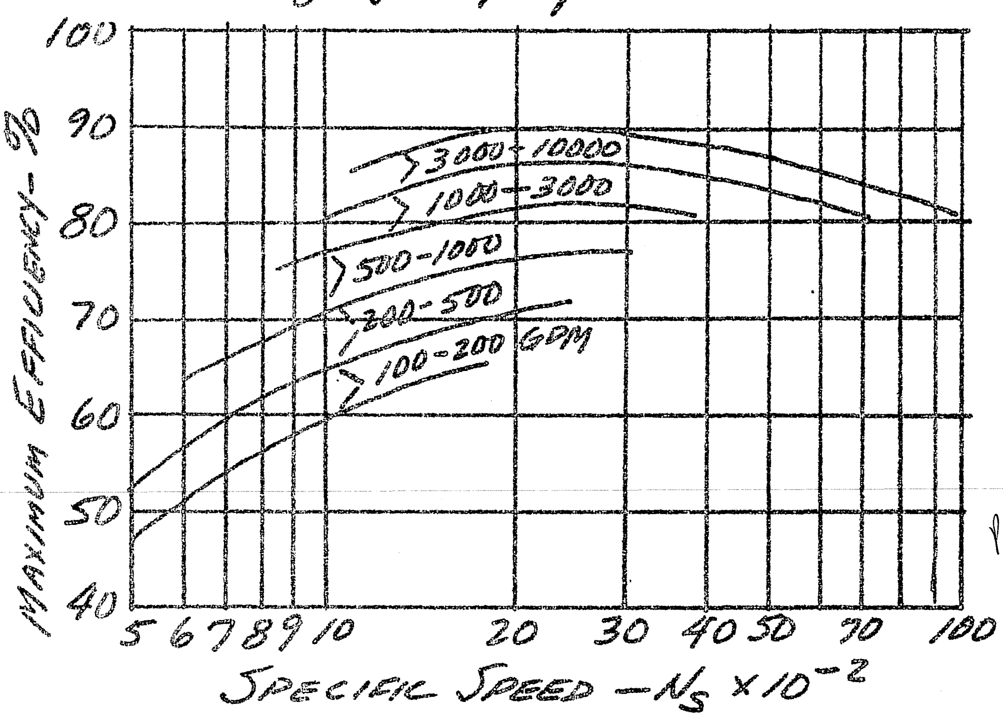
?



Pumps of normal design



Efficiencies of pumps of good design are correlated on the basis of specific speed. A size factor must be included since smaller pumps are generally not as efficient as larger pumps. This is partly due to larger friction factors in the smaller pumps and partly due to proportionately larger bearing and seal losses. The flow rate at maximum efficiency is a measure of the relative size of the pump.



PUMPING WATER

PUMPS / NORMAL DESIGN

9.13 <u>Centrifugal pump performance with fluids of different viscosities.</u> The correlations of Section 9.12 are for pumps handling water at normal 60°–100°F temperatures. If the measured performance of a pump handling water is known an estimate may be made of the performance handling a more viscous fluid such as an oil ~~may be made~~ from correlations established from test results.

A Reynolds number of a pump is used which may be defined as follows:

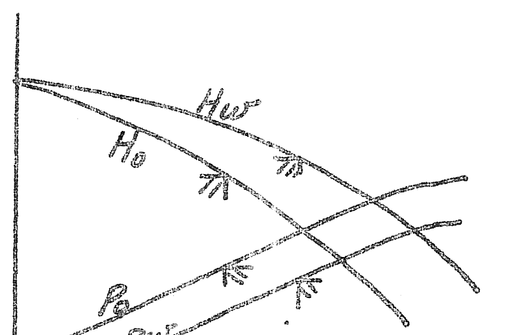
$$Re = \frac{(\text{Characteristic flow passage dimension})(\text{charac. veloc})}{(\text{Fluid kinematic viscosity})}.$$

The characteristic flow passage dimension is the distance between the shrouds taken at the impeller perimeter or b_2. The characteristic velocity is the flow rate divided by the impeller discharge area or

$$V \propto \frac{Q}{b_2 D_2}$$

Then $$Re = \frac{b_2\left(\frac{Q}{b_2 D_2}\right)}{\nu} = \frac{Q}{D_2 \nu} \qquad 9.63$$

A typical set of test performance curves is shown.



It is found that, at maximum efficiency

$$F_H = \frac{H_0}{H_W} = \varphi(Re) \qquad 9.64$$

$$F_e = \frac{e_0}{e_W} = \varphi(Re) \qquad 9.65$$

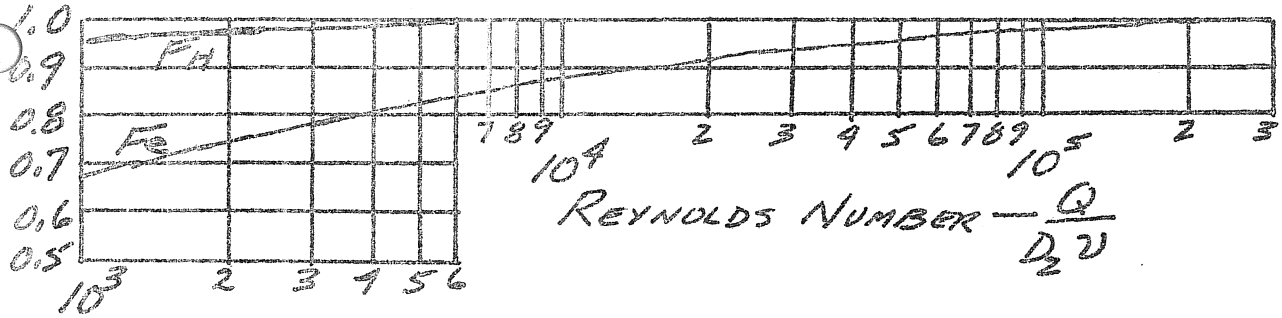
$$N_{S_0} = \frac{N_0 Q_0^{1/2}}{H_0^{3/4}} = N_{SW} = \frac{N_W Q_W^{1/2}}{H_W^{3/4}} \qquad 9.66$$

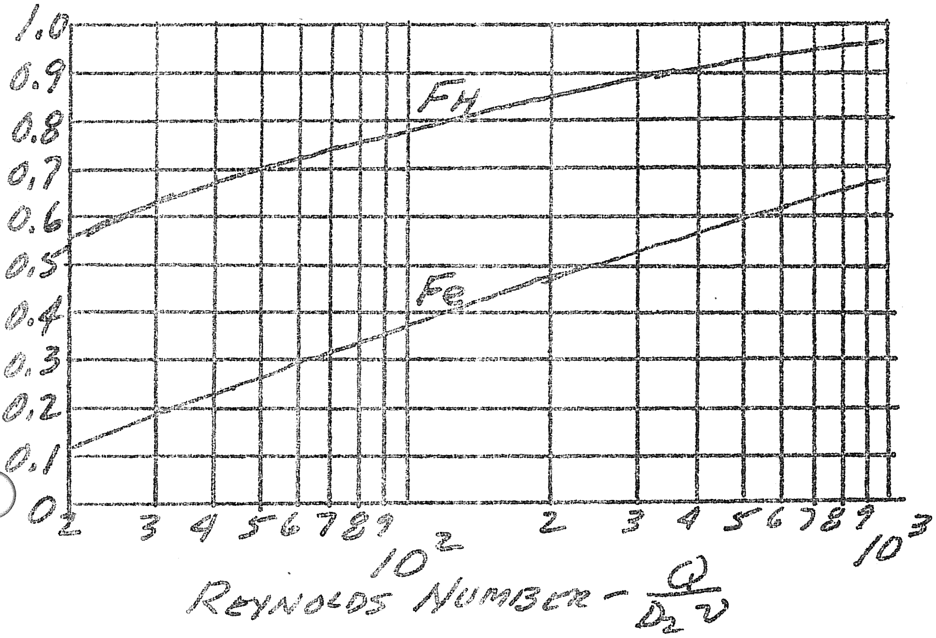
Also,

$$H_{0_{Q=0}} = H_{W_{Q=0}}. \qquad 9.67$$

$P_0 - P_W = \text{constant}.$
(Power curves are parallel).

F_H and F_e are correlated from test results as functions of the Reynolds number defined in eq. 9.63





Q — Flow rate at best efficiency — CFS.

D_2 — Impeller diameter — Ft.

ν — Fluid kinematic viscosity — Ft^2/sec.

Pumps of Normal Design

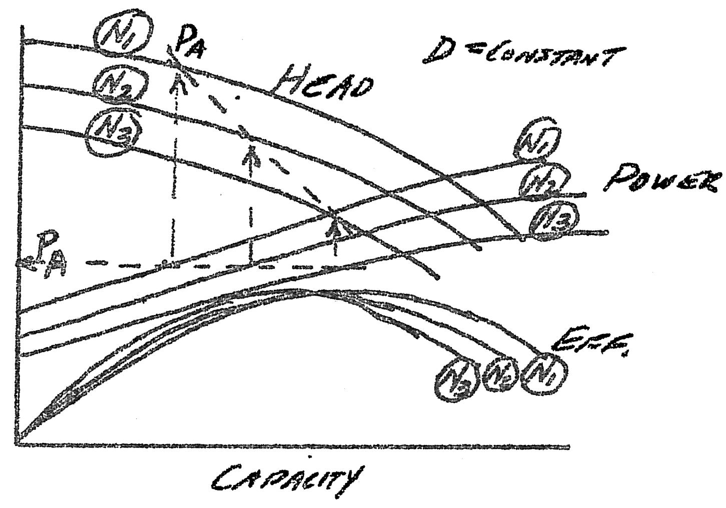
If the performance of a centrifugal pump handling water is known, then the performance of the same pump handling a more viscous fluid may be estimated. Eqs. 9.64 and 9.66, when solved simultaneously using the correlation curves, give the head and capacity at maximum efficiency when pumping the more viscous fluid. The shut-off head for both fluids is the same. The rest of the head-capacity curve for the more viscous fluid may be estimated by fairing in a curve through these two points. The correlation of the efficiency factor, eq. 9.65, gives the maximum efficiency of the pump handling the more viscous fluid from which the power required at the maximum efficiency flow rate may be obtained from

$$P_o = \frac{Q_o w_o H_o}{e_o} \qquad 9.68$$

A curve through this point and parallel to the power curve for water gives the power-capacity performance. The complete efficiency curve is found from eq. 9.68.

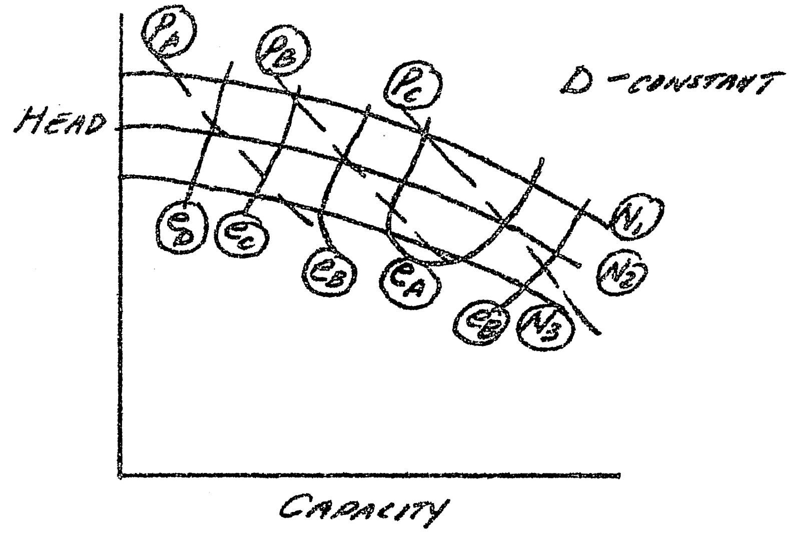
9.14 Performance representation. Manufacturers produce a limited number of different pump sizes for reasons of economy. However, each pump may be run at any speed up to a maximum speed limited by the allowable shaft torque or the allowable safe working pressure of the casing. Thus a single pump can be rated at different speeds with the corresponding different performance curves. In addition, the radial flow centrifugal impeller can be trimmed in diameter, starting with the maximum diameter that can be accomodated in a casing, to give corresponding performance curves for each diameter. Trimming is also possible to an extent with an axial flow pump. Speed and impeller diameter are parameters for the usual performance presentation of head, power, efficiency as function of the capacity.

For one diameter the performance curves for different speeds may be presented on the same plot.



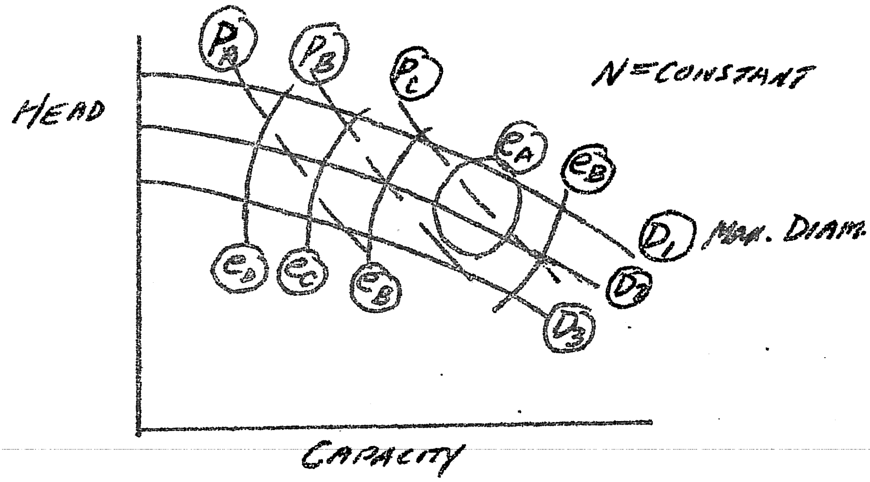
It is apparent that rapid and easy use of this type of performance representation leaves much to be desired particularly if a pump is to be selected for a given head and capacity and the selection is to be made of the pump with the best efficiency. The performance representation can be simplified by noting that at a particular head and capacity there is a unique corresponding shaft input power and a unique efficiency. Iso-power and iso-efficiency contours can then be located on the head-capacity curves. One such iso-power contour at the power value P_A is shown on the figure.

The families of iso-power and iso-efficiency curves on the head-capacity curves give a complete performance picture that is easy and rapid to use



The region of high efficiency is located at a glance. At a given head and capacity the corresponding power and efficiency are determined easily. Note that the model laws predict iso-efficiency lines that are parabolas with vertices at zero head and zero capacity. Part of the iso-efficiency lines approximate the parabolic shape. Departures are due to bearing, shaft seal, and fluid flow friction losses which predict differently than the model laws of the pumping action.

A similar representation may be made for different diameters of a trimmed impeller.



The usual manufacturer's performance file includes a set of curves for a casing and impeller combination with the impeller diameters as parameters on the head-capacity curves and speed as a parameter for the different plots.

9.15 Correlation of impeller geometry with pump performance.
In Sections 9.2 and 9.8 the head and capacity of centrifugal pumps are shown to be functions of the geometry of the discharge region of the impeller. If all the other features of the complete pump are arranged to match the impeller discharge geometry for optimum performance then it is possible to correlate centrifugal pump performance on the basis of r_2, b_2, and β_2.

Eq. 9.14 is $$H_\infty = \frac{U_2^2}{g} - \frac{Q U_2}{2\pi r_2 b_2 g} \cot\beta_2$$

Or, $$\frac{H_\infty}{U_2^2/g} = 1 - \frac{Q}{A_2 U_2} \cot\beta_2 \qquad 9.69$$

Where, $$A_2 = 2\pi r_2 b_2$$

Then, $$\frac{H_\infty}{U_2^2/g} = \text{Function of}\left(\frac{Q}{U_2 A_2}, \beta_2\right) \qquad 9.70$$

But $H = f(H_\infty)$ and A_2 for an actual pump is the net discharge area taking into account the vane thickness.

Then $$\frac{H}{U_2^2/g} = f\left(\frac{Q}{U_2 A_2}, \beta_2\right) \qquad 9.71$$

Define $$H/\frac{U_2^2}{g} = \psi \quad \text{and} \quad \phi = \frac{Q}{U_2 A_2} \qquad 9.72$$

also note that ψ and ϕ are the dimensionless head and capacity coefficients of the model laws in the form of eqs. 9.42 and 9.43. This leads to the dimensionless specific speed

$$\omega_S = \frac{\phi^{1/2}}{\psi^{3/4}} \qquad 9.73$$

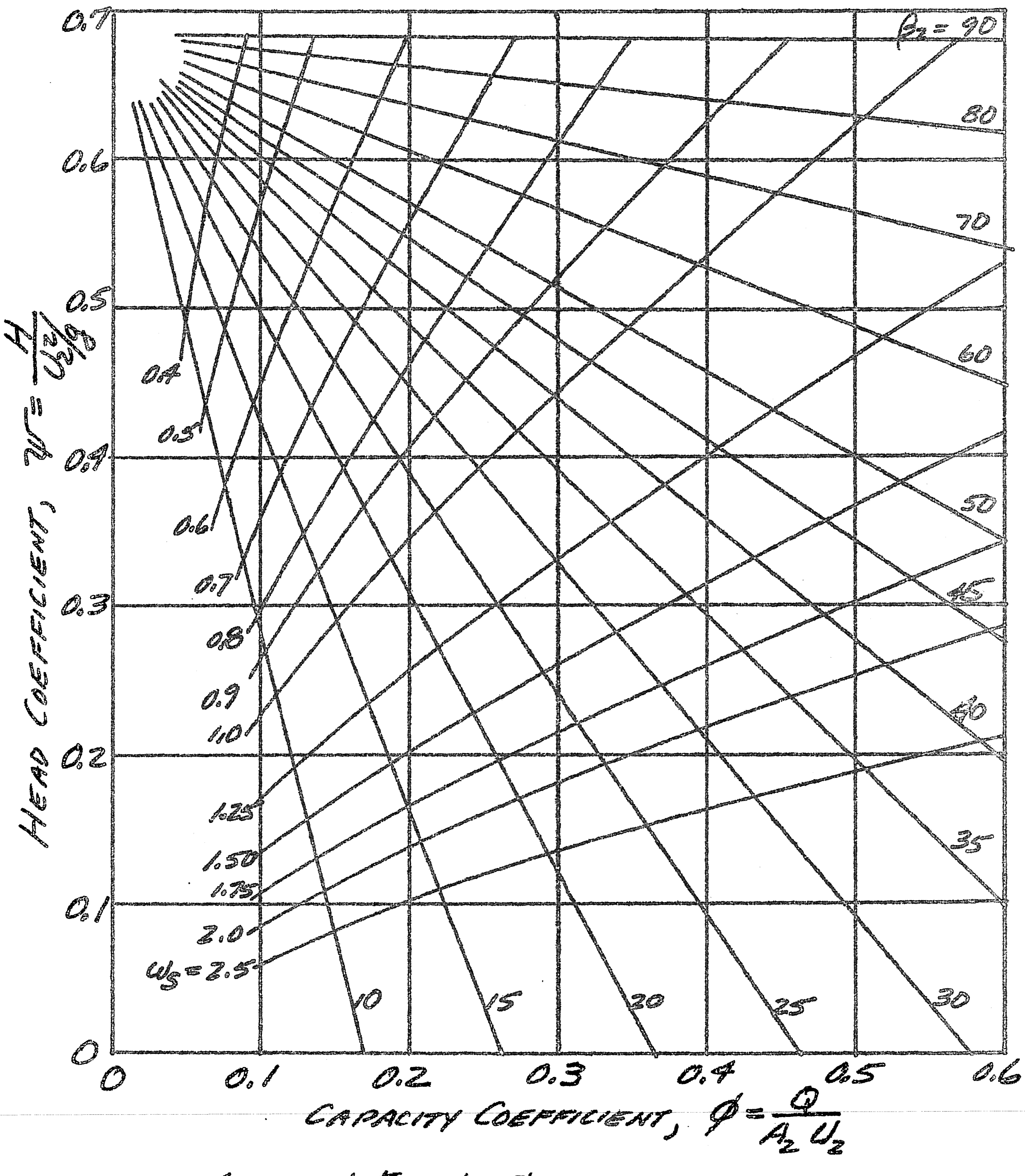
It can also be shown that

$$N_S = 9675\left(\frac{b_2}{D_m}\right)^{1/2}\left(\frac{D_{ave}}{D_m}\right)^{1/2} \omega_S \qquad 9.74$$

Where $$D_m = \left[\frac{D_{2o}^2 + D_{2h}^2}{2}\right]^{1/2} \quad \text{and} \quad D_{av} = \frac{D_{2o} + D_{2h}}{2} \qquad 9.75$$

Eqs 9.75 give the mean diameter and the average diameter for mixed flow and axial flow pumps.

Stepanoff* presents the correlation for head and capacity at maximum efficiency.



This correlation, together with eq. 9.74 permits an estimate of Q and H at best efficiency from D_2, A_2 and β_2 for a given speed N.

* STEPANOFF, A.J. "CENTRIFUGAL AND AXIAL FLOW PUMPS", JOHN WILEY & SONS, 1957.

Stepanoff states that each value of ω_s corresponds to a unique value of b/D. By eq. 9.74 each value of N_s corresponds to a unique value of b/D. Stepanoff's dimensionless correlation can be put into a more convenient form for estimation of a centrifugal pump performance having only the impeller geometry.

Geometrically similar pumps are described for $Q_e = 1000$ GPM and $H_e = 100$ Ft. Then

$$|N_s| = \frac{N Q^{1/2}}{H^{3/4}} = \frac{N (1000)^{1/2}}{(100)^{3/4}} = N$$

These pumps have a rotational speed that is numerically equal to the specific speed. It follows that

$$Q = Q_e\left(\frac{N\,D^3}{N_s\,D_e^3}\right) \quad \text{and} \quad H = H_e\left(\frac{N\,D}{N_s\,D_e}\right)^2$$

The correlation of b/D and D_e with the specific speed were obtained from Stepanoff's dimensionless correlation.

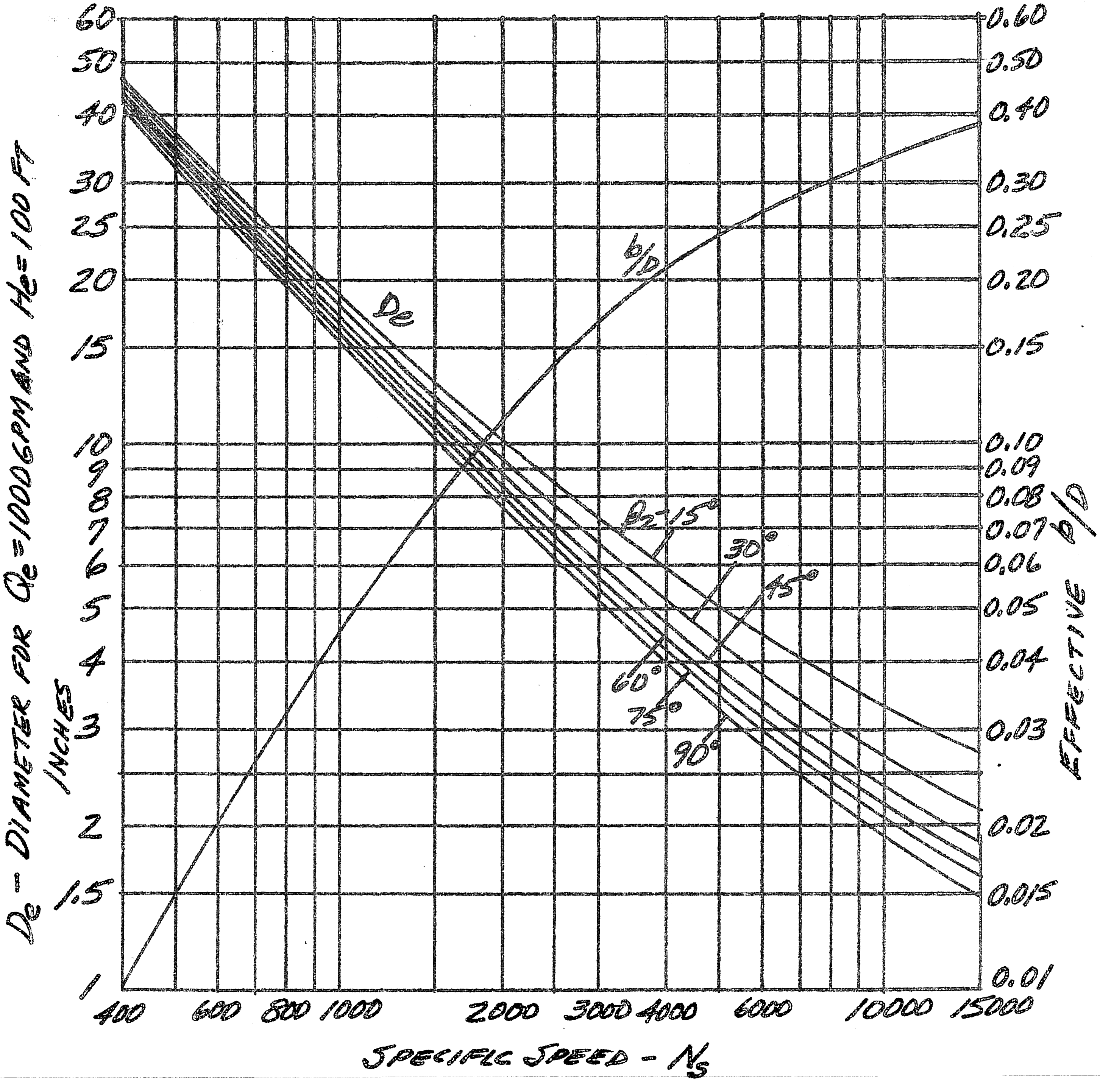
To take into account the reduced impeller discharge area due to vane thickness b/D is described as

$$\frac{b}{D} = \frac{\text{Single suction discharge area}}{\pi\left[\frac{D_{2o}^2 + D_{2h}^2}{2}\right]}$$

If the vanes have negligible thickness, and if $D_{2o} = D_{2h} = D_2$ then,

$$\frac{b}{D} = \frac{b_2}{D_2}$$

It should be emphasized that all correlation are for pumps of normal, or optimum, design with all features of inlet nozzle, impeller, collection chamber, etc. matched to give the best hydraulic flow conditions. Even for normal designs the correlations are not perfect. However, they are useful for estimates of features relating to pumps application particularly in preliminary system design considerations.


60
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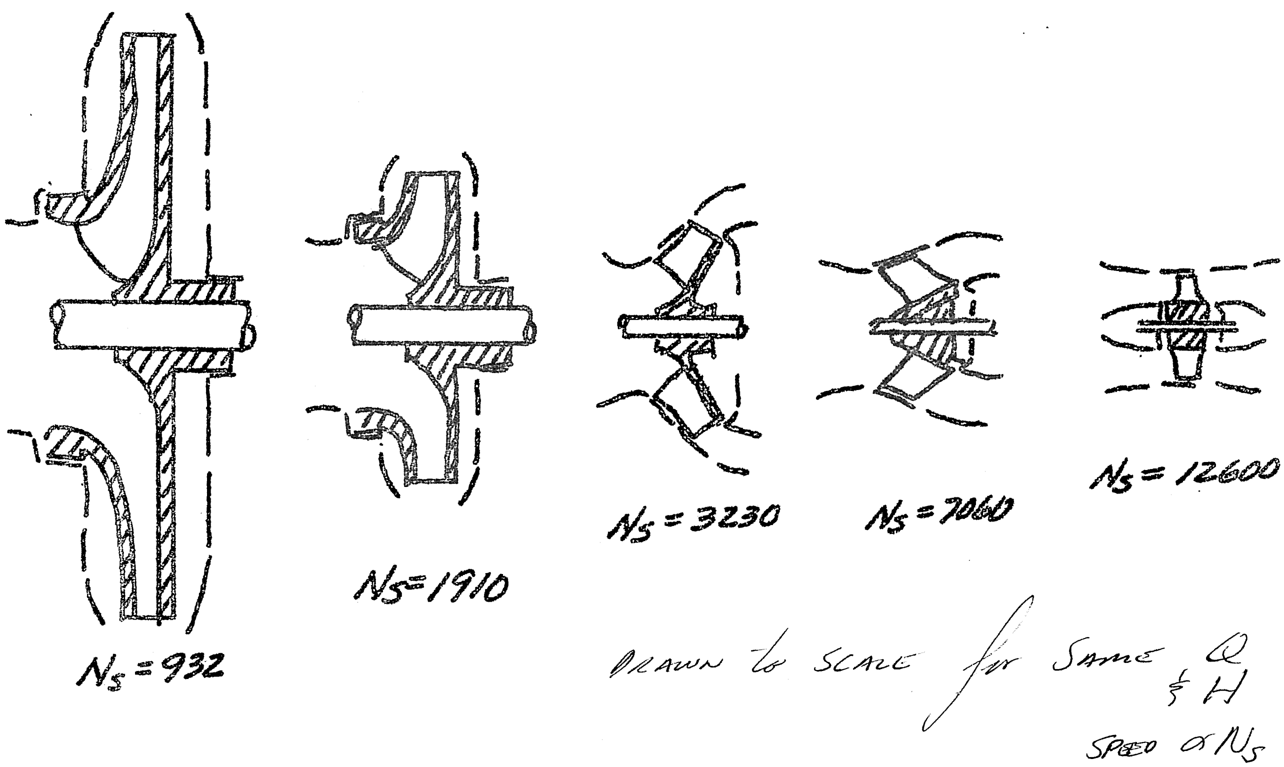
4000
6000
10000
15000
b/D
De
β2=15°
30°
45°
60°
75°
90°
De - DIAMETER FOR Qe=1000 GPM AND He=100 FT INCHES
EFFECTIVE b/D
SPECIFIC SPEED - Ns

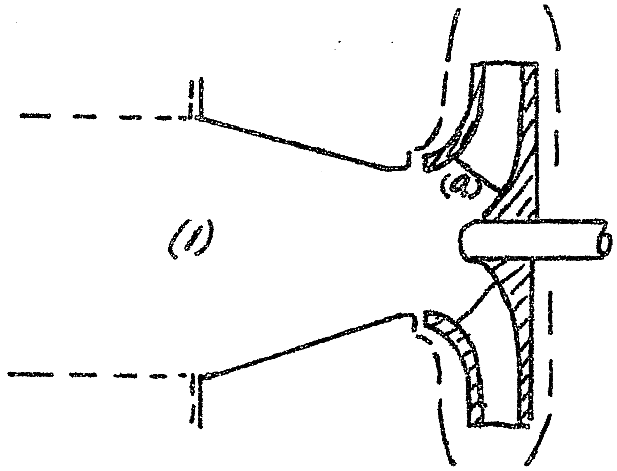
The implications of the correlation may be seen from the comparison of impeller sections as a function of specific speed.



9.16 Cavitation. When the local pressure in a fluid becomes the pressure corresponding to the saturation, or boiling, temperature the liquid will vaporize. In fluid machinery, and in other flow conditions with no heat transfer, this vaporization is called cavitation.

In a pump the lowest pressure region is on the suction where the velocities are greatest, usually at the entrance edge of the vanes. If the supply pressure is too low and cavitation occurs there will be a reduction in performance. The primary effect is a limitation of capacity. Another effect is a head loss on liquifaction of the vapor as the flow passes to a pressure higher than the vapor pressure as it moves on through the impeller.

Cavitation conditions may be investigated by considering the flow from the pump suction flange to the location in the impeller that has the lowest pressure. The impeller shape at inlet gives the flow pattern and this is related to the pump specific speed. An applicable cavitation criterion should show a correlation with specific speed.



FROM (1) to (a) $$\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} = \frac{p_a}{w} + z_a + \frac{V_a^2}{2g} + h_{\substack{loss\\1-a}} \qquad 9.76$$

Referred to the centerline of the flow and with a symmetrical impeller such that the lowest pressure is at the same radial location all around the axis of the impeller, then $z_1 = z_a$

$$\left(\frac{p_1}{w} + \overset{V_1^2/2g}{\cancel{z_1}}\right) - \frac{p_a}{w} = \frac{V_a^2}{2g} + h_{\substack{loss\\1-a}} \qquad 9.77$$

when cavitation occurs p_a = vapor pressure = p_v

also $V_a = \dfrac{Q}{A_a}$

For turbulent flow $h_{\substack{loss\\1-a}} = \left(\dfrac{Q}{A}\right)^2 \dfrac{1}{2g} \cdot K_\ell$

where K_ℓ is the head loss coefficient.

Then eq. 9.77 is

$$\left(\frac{p_1}{w} + \overset{V_1^2/2g}{\cancel{z_1}}\right) - \frac{p_v}{w} = Q^2\left[\frac{1}{A_a^2 2g} + \frac{K_\ell}{A^2 2g}\right] \qquad 9.78$$

The bracketed term on the right contains factors which are related to the geometry of the pump only since K_ℓ in turbulent flow is essentially a constant.

The right hand side of eq. 9.78 is independent of the properties of the liquid. Then,

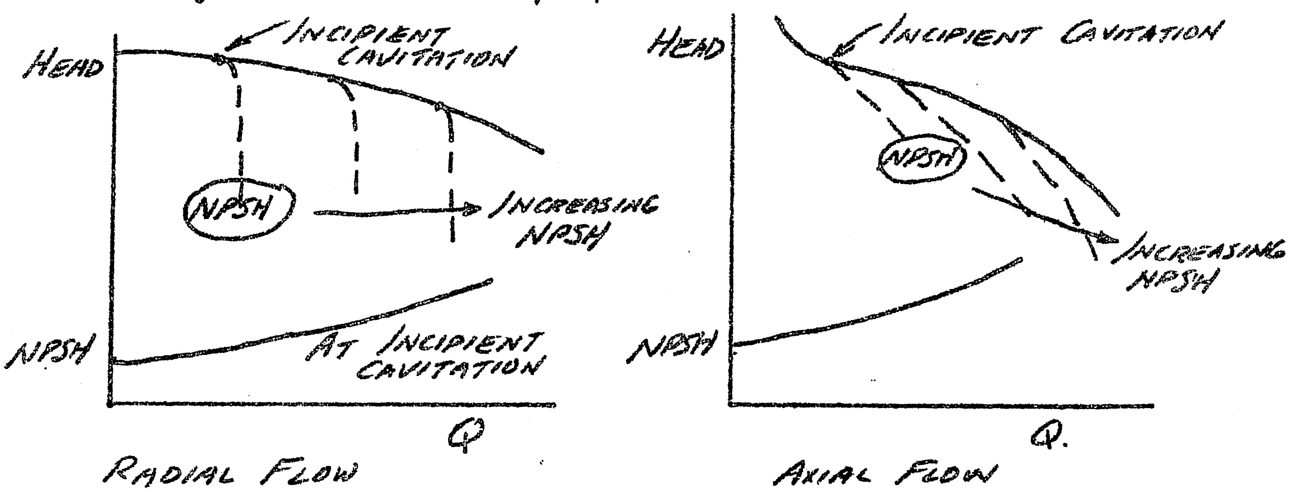
$$\left(\frac{p_1}{w} + z_1\right) - \frac{p_v}{w} = \phi(Q, \text{pump geom}) \qquad 9.79$$

The left hand side of eq. 9.78 is the difference between the head at the suction flange of the pump and the vapor pressure head of the liquid. At cavitation at (a) this difference is a Net Positive Suction Head at the suction flange when the pressures are both absolute pressures. The Net Positive Suction HEAD ~~Pressure~~ is abbreviated NPSH.

Then $$NPSH = \left(\frac{p_1}{w} + \frac{V_1^2}{2g}\right) - \frac{p_v}{w} = \phi(Q, geom.) \qquad 9.80$$

For a given pump, NPSH is a function of Q and is determined from a test of the pump. When determined for one liquid for the condition of incipient cavitation, the performance of $NPSH = \phi(Q)$ is the same for all liquids as long as K_e is the same.

Typical cavitation performances are shown.



A correlation for centrifugal pumps is, at max. efficiency,

$$\frac{NPSH}{H} = \frac{6.3}{10^6} N_s^{4/3} \qquad 9.81$$

Pumps may be designed for minimum NPSH by enlarging the impeller eye to reduce velocities of the flow. However, this usually results in a reduced pump efficiency.

The flow head at (1) is furnished by the upstream system and is independent of the pump. For cavitation free operation

$$(NPSH)_{system} \geq (NPSH)_{pump} \qquad 9.82$$

Eq. 9.82 gives the condition for system design up to the pump in order to prevent the occurrence of cavitation in the pump.

Maximum suction lift (or suction lift) is sometimes used to describe incipient cavitation conditions. Suction lift is related to the vapor pressure of the liquid for which the values of suction lift are specified and therefore is not directly generally applicable for all liquids as is NPSH.

$$\text{Suction lift} = S.L. = \frac{p_a}{w} - \left(\frac{p_1}{w} + \frac{V_1^2}{2g}\right) \qquad 9.83$$

where p_a = atmospheric pressure

Combining eqs. 9.83 and 9.80

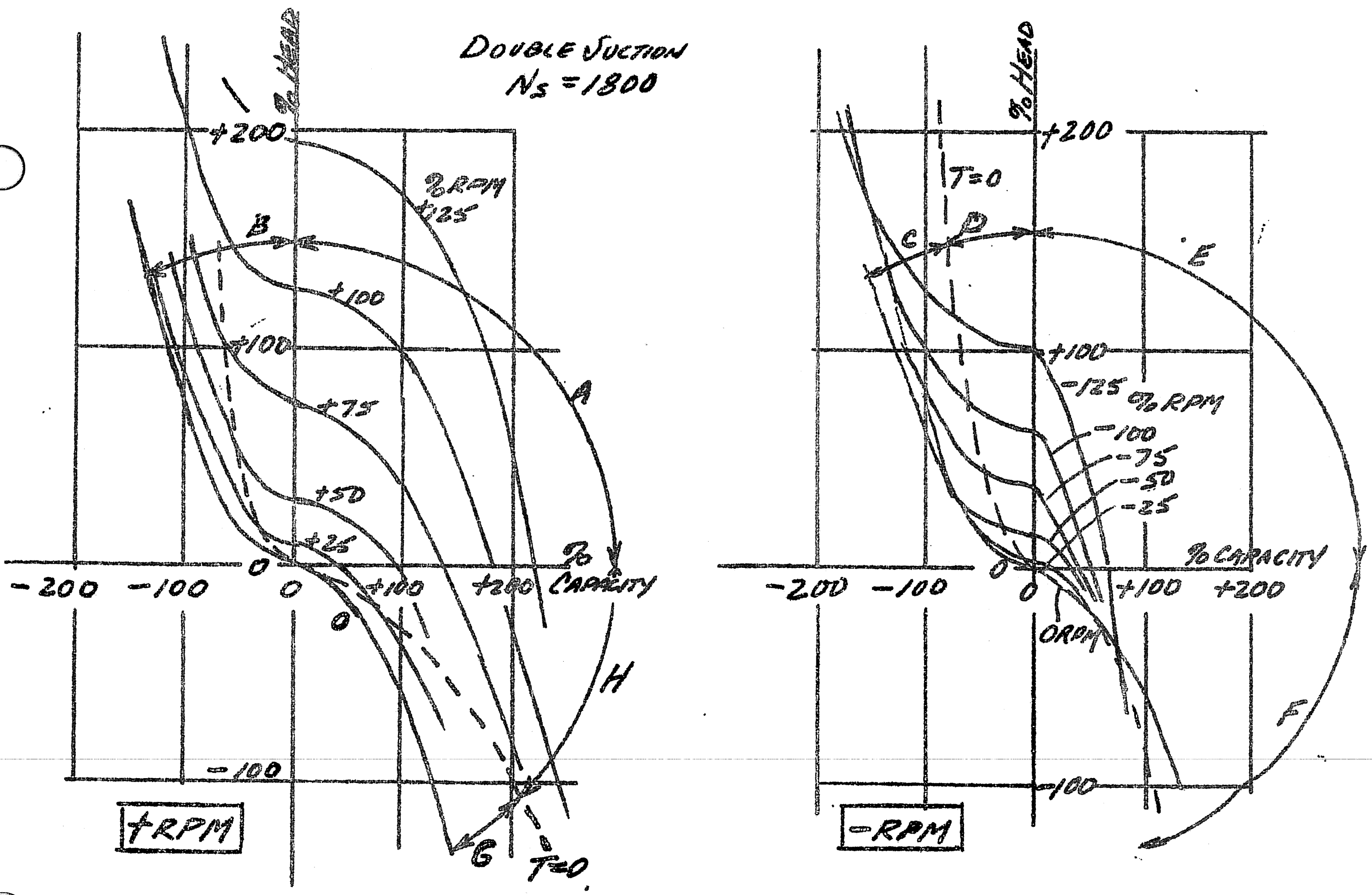
$$S.L. = \frac{p_a}{w} - (NPSH) + \frac{p_v}{w} \qquad 9.84$$

In determining S.L. or NPSH, p_1 is the pressure at the center line of the shaft in horizontal shaft pumps and is the pressure measured at the suction flange of the pump and referred to the plane of the highest elevation of the inlet edge of the vanes for vertical shaft pumps.

or the inlet plane of the impellor

SUMP DESIGN

9.17 Complete centrifugal pump characteristics. The normal pumping range of a centrifugal pump is in the quadrant bounded by Q = 0 and H = 0 and with the impeller rotating in the design direction. However, heads may be applied to the pump which are greater than the shut-off head, or the impeller may rotate opposite to the design direction, or other non-normal conditions may be imposed. Complete performance of representative centrifugal pumps has been obtained by test. All heads, capacities, and speeds are given as percentages of the pump rating at design speed. Heads are positive when the discharge flange head is greater than the inlet flange head. Capacities are positive when the flow is from inlet flange to discharge flange. Speeds are positive when the impeller rotates in the normal pumping direction. Heads, capacities, and speed are negative when the opposite conditions are applied.

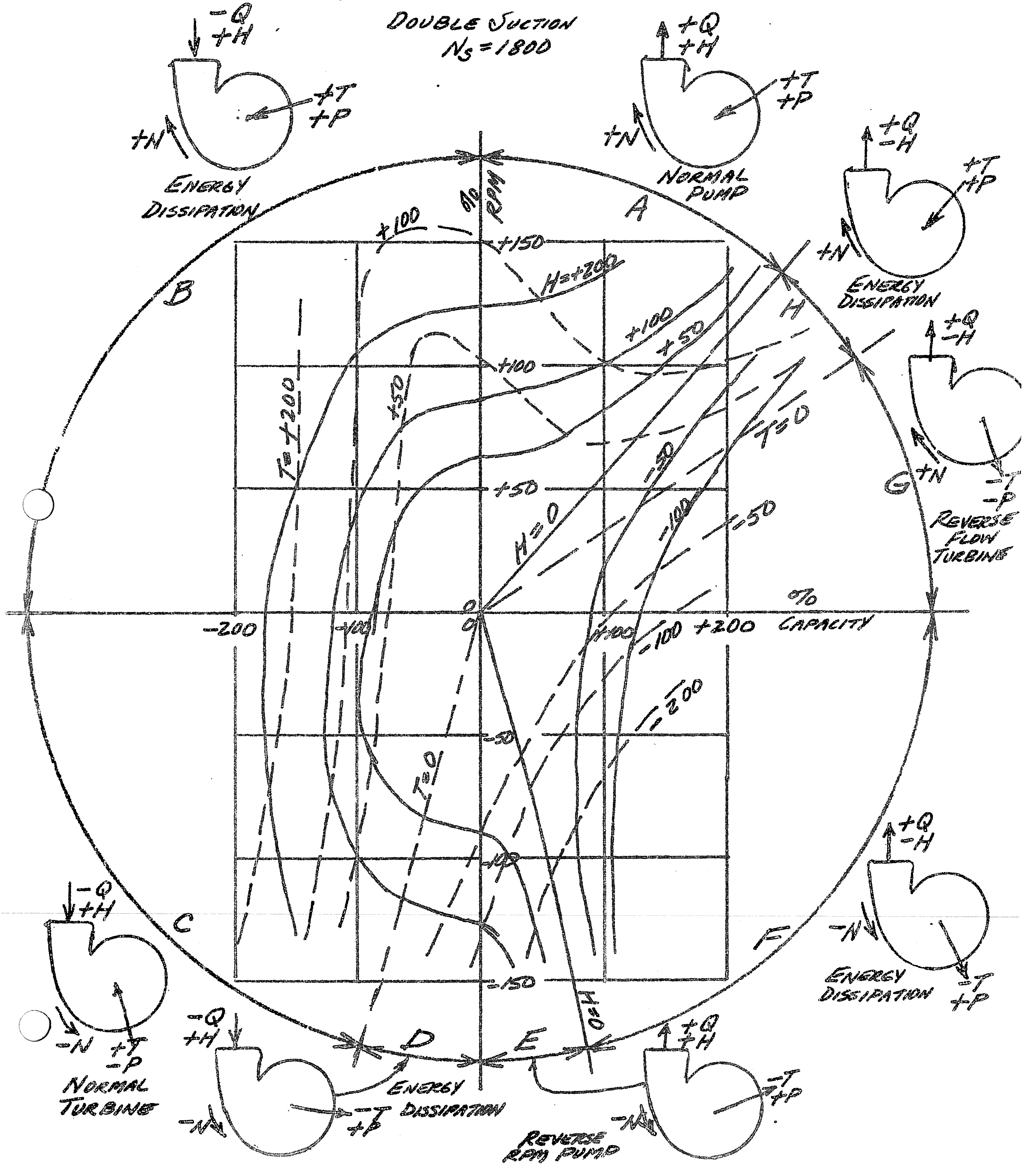


REGIONS:
A - NORMAL PUMP
B - ENERGY DISSIPATION
C - NORMAL TURBINE
D - ENERGY DISSIPATION
E. REVERSE RPM PUMP
F. ENERGY DISSIPATION
G. REVERSE FLOW TURBINE
H. ENERGY DISSIPATION

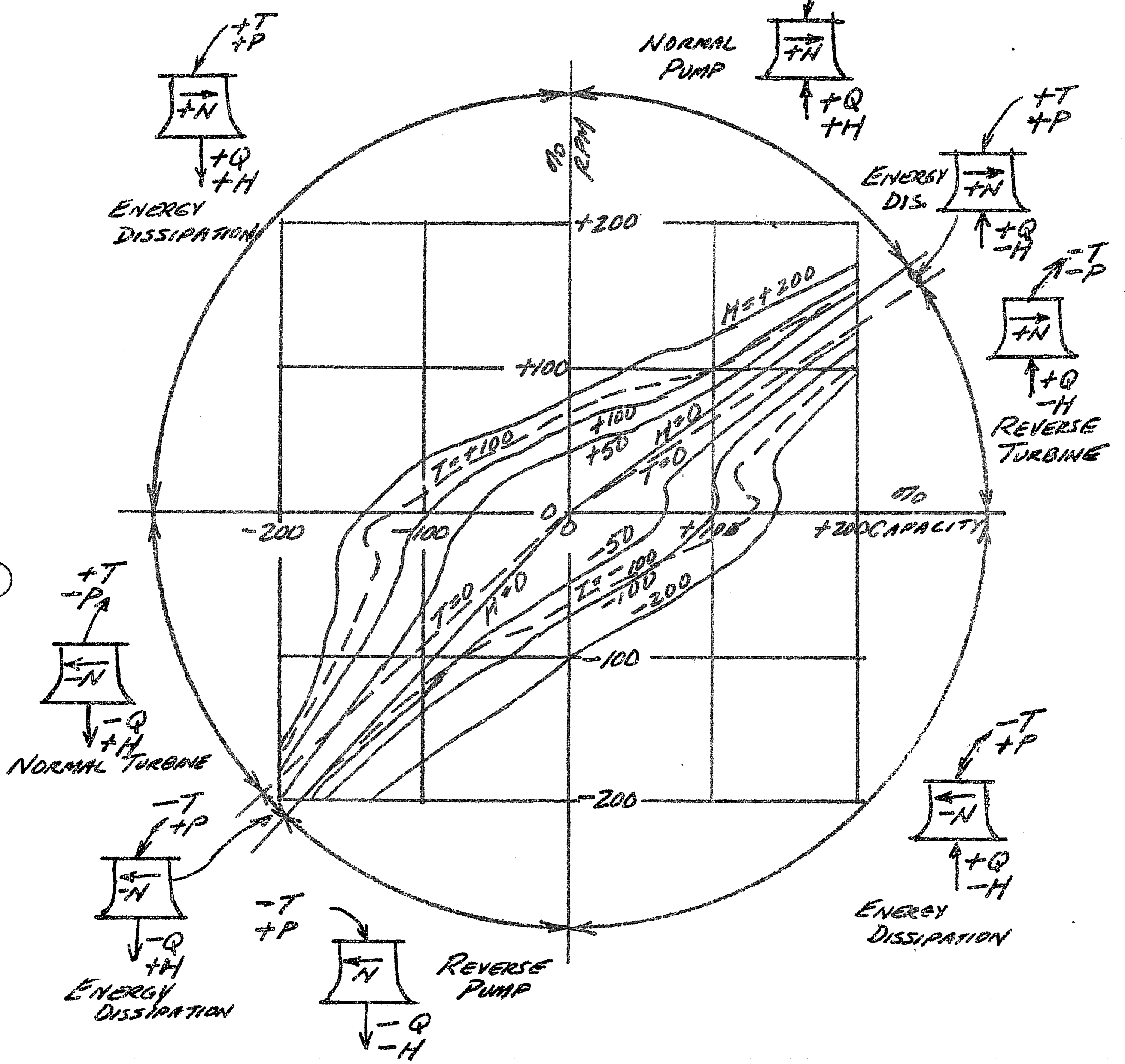
FROM: R.T. KNAPP. TRANS. ASME NOV. 1937.

The complete characteristics are represented on a single plot using capacity and speed as the coordinates.

DOUBLE SUCTION
$N_S = 1800$

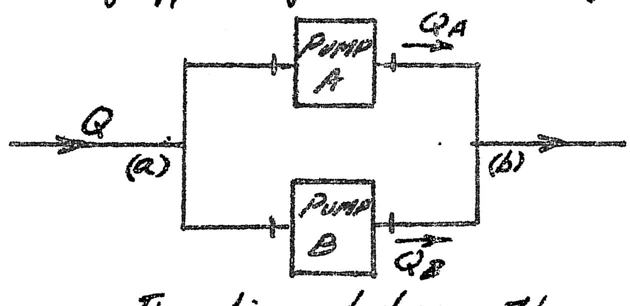


AXIAL FLOW
$N_S = 13,500$



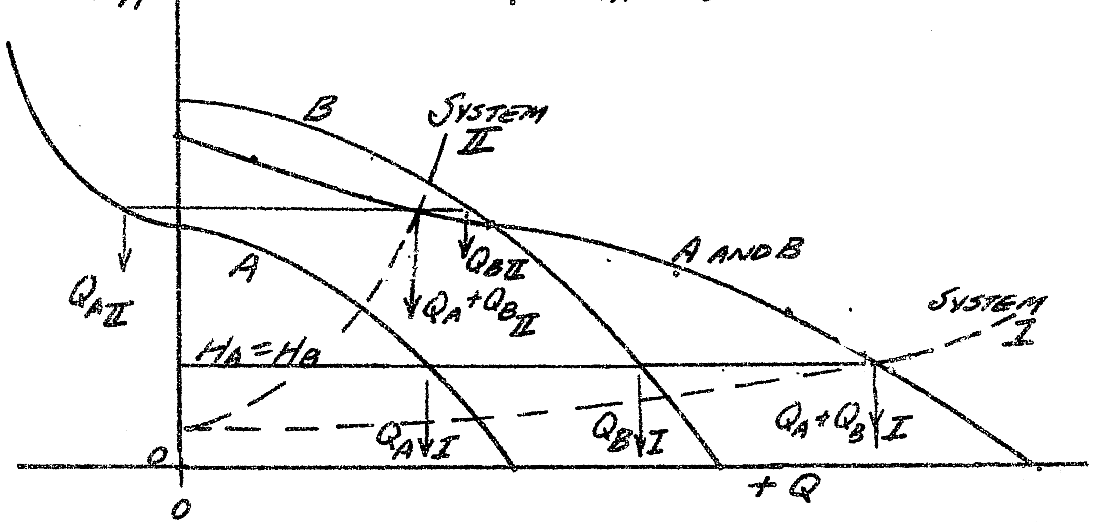
FROM: SWANSON. TRANS ASME, VOL. 75, pp 819-826, 1953

9.18 <u>Centrifugal Pumps in Parallel.</u> Two or more pumps may be placed in parallel in a flow system to provide the desired maximum flow rate. This arrangement also provides some flow in the event of a failure of one of the pumps. In systems with a variable flow demand the pumps may be sequentially operated to obtain maximum pumping efficiency over wide ranges of flow rates.



If the connecting lines between the pump flanges and the system locations (a) and (b) are short, then each pump operates at the same head $H_S = H_a - H_b$. The governing conditions are

$$H_A = H_B$$
$$Q = Q_A + Q_B$$



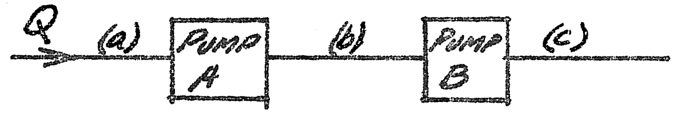
Consider two systems with system performance curves labeled I and II.

System I. Flow rate increased over that obtainable by either pump alone.

System II. Flow rate less than that obtainable from Pump B alone. Pump A is dissipating energy. (Note that if A and B are identical pumps no energy dissipation is possible and both pumps will contribute to the desired flow.)

Power failure to one pump of a parallel combination results in a steady state condition of essentially zero torque for the powerless pump. The head-capacity curve for zero torque can be obtained from the complete characteristics of the pump. When combined with the still operating pump the system flow rate is found as is the operating point of the powerless pump. Of importance is the reverse speed of the powerless pump since a large overspeed may result in mechanical damage.

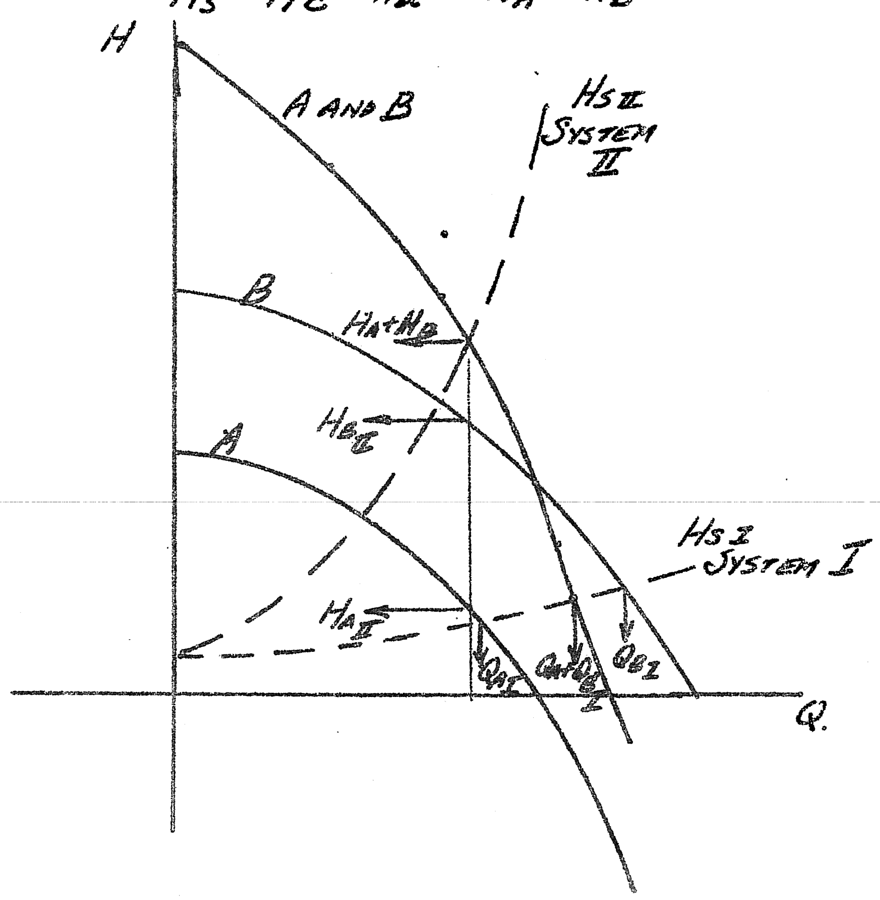
9.19 Centrifugal pumps in series. Two or more pumps may be placed in series in a flow system to provide the desired maximum flow rate.



Each pump operates with the same flow rate. The pump heads are additive to give the head between (a) and (c). The governing conditions are

$$Q_A = Q_B = Q$$

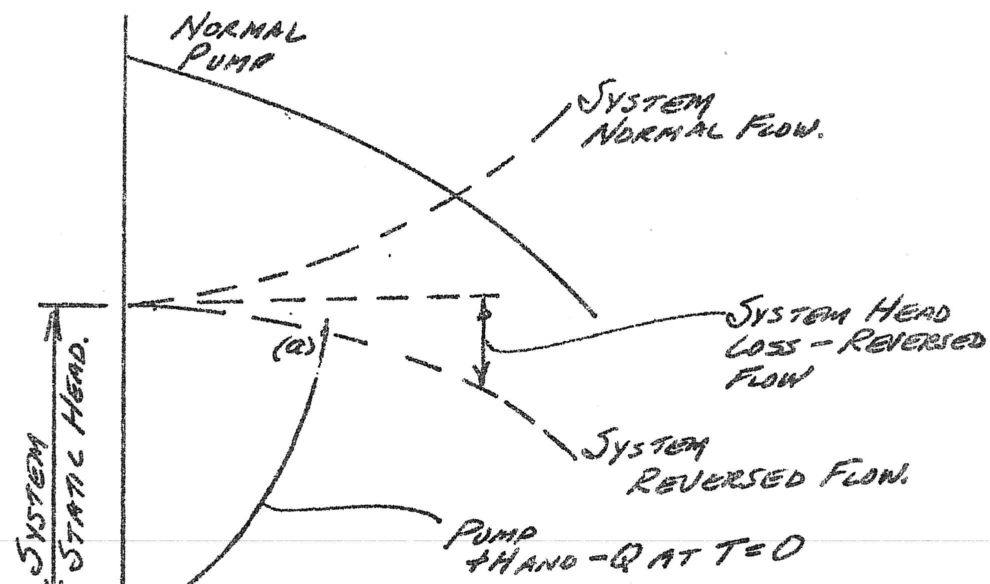
$$H_S = H_c - H_a = H_A + H_B$$



For System I, the flow rate $(Q + Q_B)_I$ is less than that which would be obtained with pump B alone. Pump A operates at a negative head and is an energy dissipator.

Power failure to one pump of a series combination results in that pump being driven by the flow as a free-running, or zero torque, turbine. The head-capacity performance of the free-running pump can be determined from the complete performance characteristics of that pump and combined with the system performance to give the operating conditions. Of particular interest is the free-running speed which will be attained.

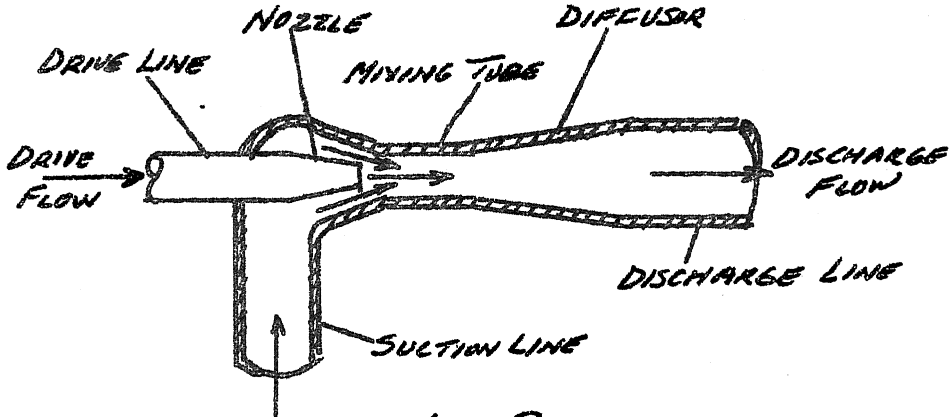
9.20 Power failure to a single pump in a system. If there are no check valves in a system, power failure to the drive of a pump results in the system operating static head difference applied to reverse the flow. The pump becomes a zero torque turbine with reverse rotation. Maximum speed is obtained from the complete pump characteristics matched with the reversed flow system performance.



The intersection of the pump performance at zero torque and the reversed flow system performance, point (a) gives the steady state operating condition from which the pump reversed speed is found.

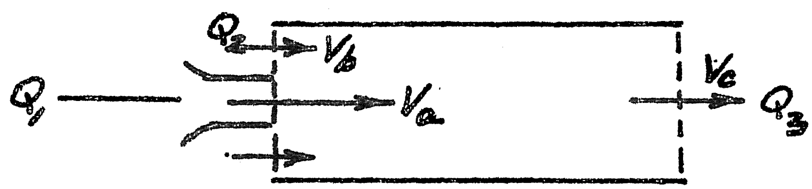
10.0 JET PUMPS.

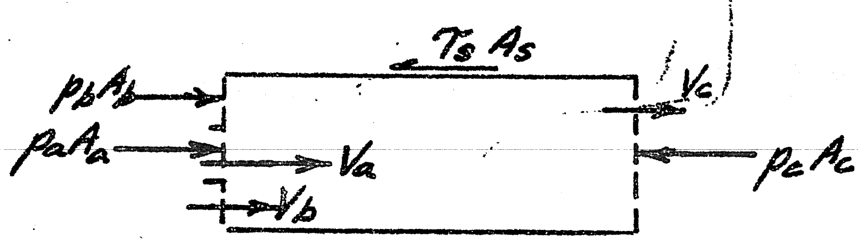
10.1 General description. Transport of the pumped fluid is produced by a jet of drive fluid which is injected into the pumped fluid and mixes with the pumped fluid to move the pumped fluid.



JET PUMP

10.2 Analysis of the pumping action. Flow patterns in the usual jet pump are complex and are not easily analyzed. The pumping action may be described analytically by considering a simplified, or cylindrical, jet pump.





Complete mixing is assumed between a and b and c.
A force balance on this cylindrical mixing chamber gives

$$p_a A_a + p_b A_b - p_c A_c - \tau_s A_s = \rho_c Q_3 V_c - \rho_a Q_1 V_a - \rho_b Q_2 V_b \qquad 10.1$$

The pressures in two parallel streams are the same.

$$p_a = p_b \qquad 10.2$$

The shear force is represented by a coefficient referred to the velocity at (c).

$$\tau_s A_s = \frac{1}{2} \rho_c K_c V_c^2 A_s \qquad 10.3$$

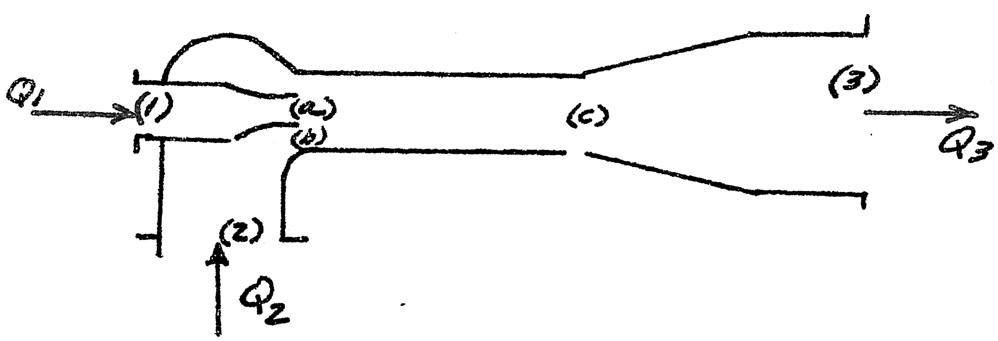
Also, in this cylindrical mixing tube

$$A_a + A_b = A_c \qquad 10.4$$

Eq. 10.1 becomes,

$$p_b - p_c = p_a - p_c = \rho_c \frac{Q_3}{A_c} V_c - \rho_a \frac{Q_1}{A_c} V_a - \rho_b \frac{Q_2}{A_c} V_b + \rho_c K_c \frac{V_c^2 A_s}{2 A_c} \qquad 10.5$$

"Flange-to-flange" performance of the complete pump may be taken into account by using appropriate loss coefficients.



For incompressible flow

$$\frac{p_1}{w_1} + z_1 + \frac{V_1^2}{2g} = \frac{p_a}{w_a} + z_a + \frac{V_a^2}{2g} + K_j \frac{V_a^2}{2g} \qquad 10.6$$

$$\frac{p_2}{w_2} + z_2 + \frac{V_2^2}{2g} = \frac{p_b}{w_b} + z_b + \frac{V_b^2}{2g} + K_s \frac{V_b^2}{2g} \qquad 10.7$$

$$\frac{p_c}{w_c} + z_c + \frac{V_c^2}{2g} = \frac{p_3}{w_3} + z_3 + \frac{V_3^2}{2g} + K_d \frac{V_c^2}{2g} \qquad 10.8$$

$$w_1 = w_a \qquad 10.9$$

$$w_2 = w_b \qquad 10.10$$

$$w_c = w_3 \qquad 10.11$$

A further simplification may be made in eq. 10.5 by combining K_c, A_s, and A_c into a single coefficient.

$$K_c \frac{A_s}{A_c} = K_t \qquad 10.12$$

also, $$\frac{Q_3}{A_c} = V_c \;;\; Q_1 = A_a V_a \;;\; Q_2 = A_b V_b \qquad 10.13$$

With the elevation datum taken as the centerline axis of the mixing tube

$$z_a = z_b = z_c = 0 \qquad 10.14$$

All the relationships are now available to describe the pumping action if the fluids are incompressible. These may be generalized for certain cases.

Take the case of a jet pump in which the pumped fluid and the drive fluid have the same properties such as water pumping water.

Then, $$\rho_1 = \rho_2 = \rho_3 = \rho_a = \rho_b = \rho_c = \rho = \frac{w}{g} \qquad 10.15$$

Divide eq. 10.5 by w and substitute eqs. 10.13.

$$\frac{p_b}{w} - \frac{p_c}{w} = \frac{p_a}{w} - \frac{p_c}{w} = \frac{V_c^2}{g} - \frac{V_a^2}{g}\frac{A_a}{A_c} - \frac{V_b^2}{g}\frac{A_b}{A_c} + K_t \frac{V_c^2}{2g} \qquad 10.16$$

$$\left(\frac{p_b}{w} + \frac{V_b^2}{2g}\right) - \left(\frac{p_c}{w} + \frac{V_c^2}{2g}\right) = \frac{V_b^2}{2g} + \frac{V_c^2}{2g} - \frac{V_a^2}{g}\frac{A_a}{A_c} - \frac{V_b^2}{g}\frac{A_b}{A_c} + K_t \frac{V_t^2}{2g} \qquad 10.17$$

$$\left(\frac{p_a}{w} + \frac{V_a^2}{2g}\right) - \left(\frac{p_c}{w} + \frac{V_c^2}{2g}\right) = \frac{V_a^2}{2g} + \frac{V_c^2}{2g} - \frac{V_a^2}{g}\frac{A_a}{A_c} - \frac{V_b^2}{g}\frac{A_b}{A_c} + K_t \frac{V_t^2}{2g} \qquad 10.18$$

Use eqs. 10.6, 10.7, and 10.8 with eq 10.14 to put eqs. 10.17 and 10.18 in terms of conditions at the flanges.

Define $$M = \frac{Q_2}{Q_1} \qquad 10.19$$

$$R = \frac{A_a}{A_c} \qquad 10.20$$

$$h = \frac{p}{w} + z + \frac{V^2}{2g} \quad \left(\text{i.e. } h_1 = \frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) \qquad 10.21$$

Also, $$Q_1 + Q_2 = Q_3 \qquad 10.22$$

The dimensionless head difference ratio is formed.

$$H = \frac{h_3 - h_2}{h_1 - h_3} = \frac{1-N}{N+M} \qquad 10.23$$

where,

$$N = \frac{1 + K_j + (1+K_s)M^3\left(\frac{R}{1-R}\right)^2 + (1+K_d+K_t)R^2(1+M)^3 - 2R(1+M) - \frac{2M^2R^2(1+M)}{(1-R)}}{1 + K_j - (1+K_s)M^2\left(\frac{R}{1-R}\right)^2}$$

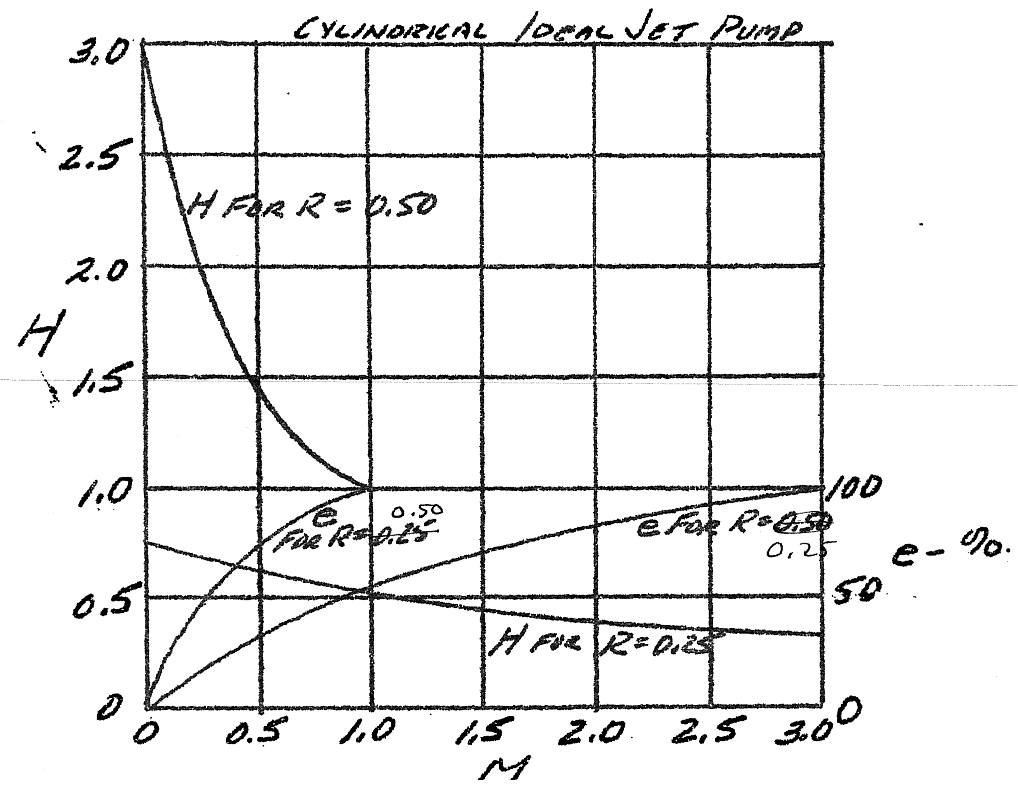
In eq. 10.23, $h_3 - h_2$ is the head added to the pumped fluid and $h_1 - h_3$ is the head used by the drive fluid.

The efficiency is obtained from

$$e = \frac{\text{Energy rate added to the pumped fluid}}{\text{Energy rate used by the drive fluid}}$$

$$e = \frac{Q_2\, w\,(h_3 - h_2)}{Q_1\, w\,(h_1 - h_3)} = MH \qquad 10.24$$

The nature of the performance given by eqs 10.23 and 10.24 may be seen by taking an ideal jet pump with $K_j = K_s = K_d = K_t = 0$. H is found as a function of M using R as a parameter.

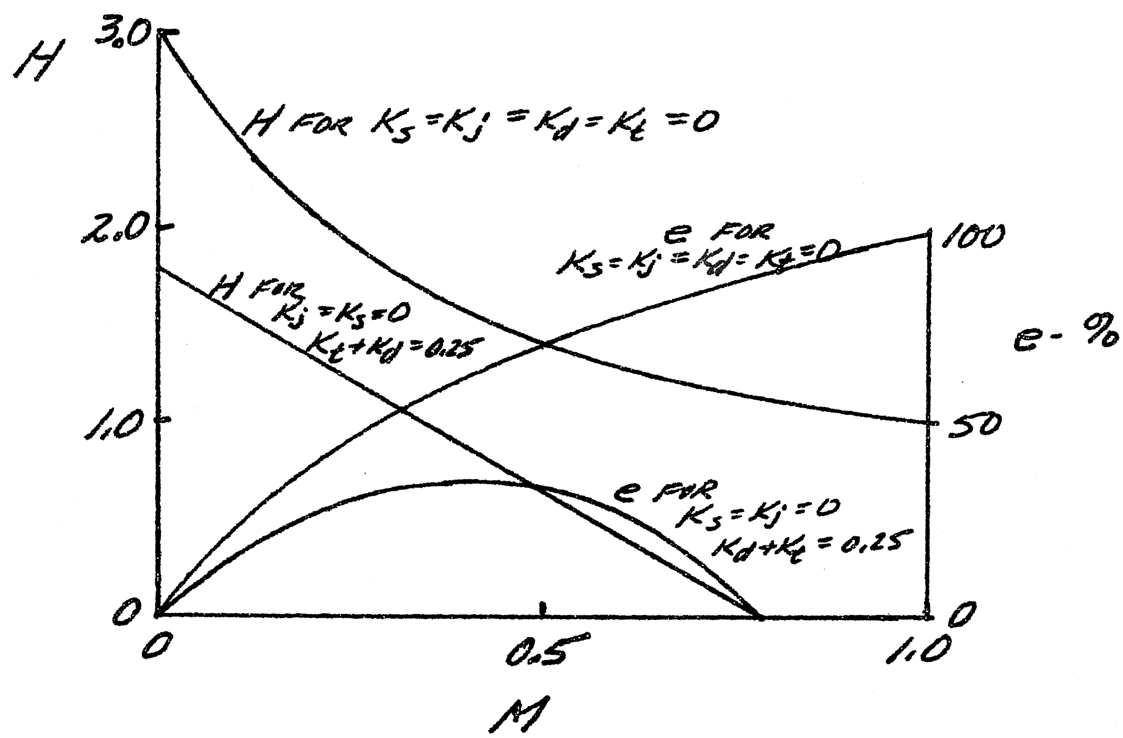


At the values of M and H where the efficiency becomes 100%, the conditions are

$$V_a = V_b = V_c .$$

No head is added to the pumped fluid nor is any head used by the drive fluid.

The influence of finite magnitudes of the loss coefficients may be seen by taking $K_t + K_d = 0.25$ $K_s = 0$ and $K_j = 0$. These values are selected for a well designed nozzle and suction inflow passage. The largest shear surface area is in the mixing chamber and diffuser with shear forces that are of appreciable magnitude.



Eqs. 10.23 and 10.24 give the normalized performance of the jet pump. All cylindrical jet pumps which are geometrically similar and which have the same loss coefficients will have ~~the~~ same normalized performance. Performance magnitudes for different sizes can be represented from eqs. 10.6 and 10.7. Subtracting eq. 10.7 from eq. 10.6 gives

$$h_1 - h_2 = \left(\frac{V_a^2}{2g} + K_j \frac{V_a^2}{2g}\right) - \left(\frac{V_b^2}{2g} + K_s \frac{V_b^2}{2g}\right) \qquad 10.25$$

Eq. 10.25 may be rearranged to the form

$$Q_1 = (h_1 - h_2)^{1/2}\left[2g A_a^2 \frac{1}{\left\{1+K_j - M^2\left(\frac{R}{1-R}\right)^2(1+K_s)\right\}}\right]^{1/2} \qquad 10.26$$

or, $$Q_1 = K(h_1-h_2)^{1/2} \qquad 10.27$$

where $K = \phi(A_a, K_j, K_s, M, R)$

Note that K is not dimensionless.

10.3 The actual jet pumps. The pumping action analysis of Section 10.2 shows the performance possibilities of the jet pump and also the form for presentation of the performance of any jet pump including those which do not have the ideal cylindrical shape.

From eqs. 10.23, 10.24 and 10.27

$$H = \phi(M, R, K_j, K_s, K_d, K_t) \qquad 10.28$$

$$e = MH \qquad 10.29$$

$$Q_1 = K(h_1-h_2)^{1/2} \text{ in which } K = \phi(M, R, K_j, K_s, A_a) \qquad 10.30$$

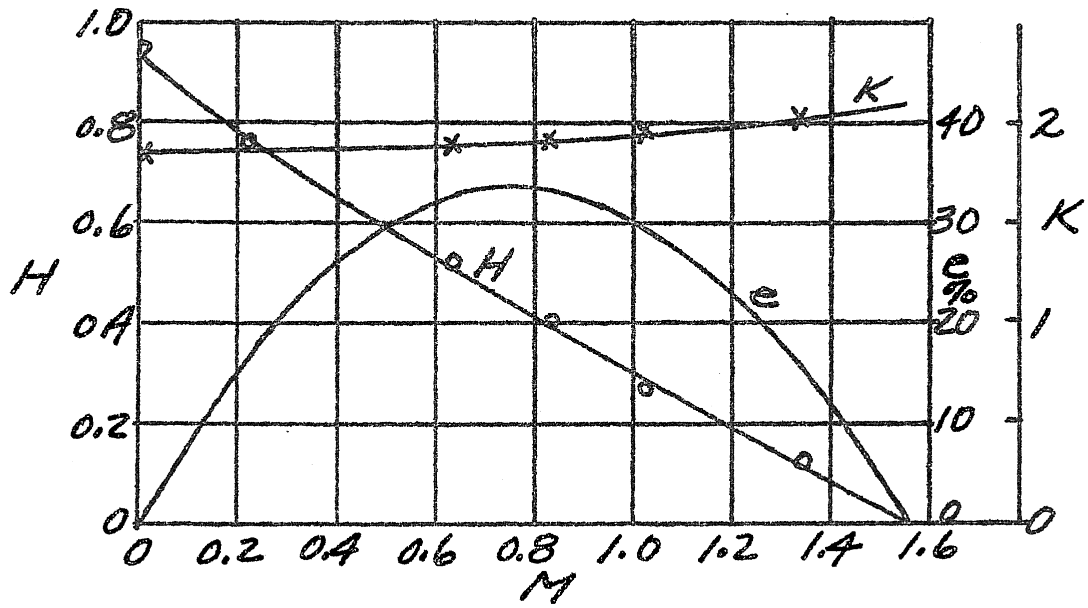
With turbulent flow in the usual application range of jet pumps with water, the loss coefficients for a given pump are constants. The geometry of a given pump is fixed and any factor based on the geometry only, such as R, is single-valued. Then

$$H = \phi(M) \qquad 10.31$$

$$e = MH \qquad 10.32$$

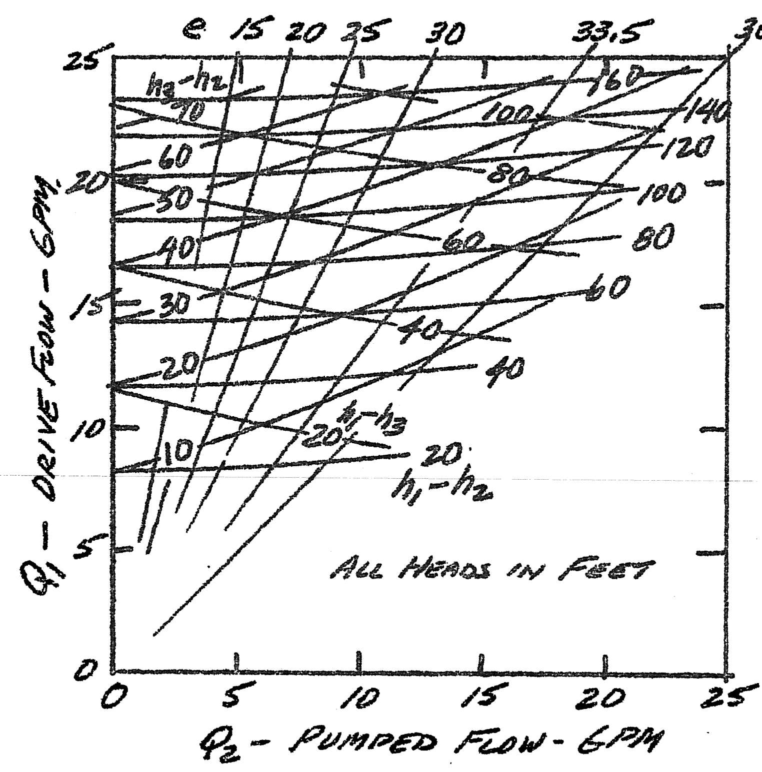
$$Q_1 = K(h_1-h_2)^{1/2} \text{ or } K = \frac{Q_1}{(h_1-h_2)^{1/2}} = \phi(M) \qquad 10.33$$

Test results from a jet pump with turbulent flows should give single-valued functions when shown according to eqs 10.31, 10.32, and 10.33. The pump may be tested within any range of heads and flow rates to determine the performance and the performance predicted with confidence for any particular set of operating conditions even though these conditions were not in the tested range.



TEST RESULTS OF A JET PUMP.
WATER PUMPING WATER

Complete performance curves may be developed from the characteristic curves. The prime variables are Q_1 and Q_2



PERFORMANCE CURVES OF A JET PUMP

∴ The performance curves are obtained from the characteristic curves as follows:

1. Lines of constant $h_1 - h_2$. Select a convenient value of $h_1 - h_2$. Select any value of M. Obtain K from the characteristic test curve. Compute Q_1 from eq. 10.30. Compute Q_2 from $Q_2 = MQ_1$. This locates a point in the realm of Q_1 and Q_2 for the selected $h_1 - h_2$. Repeat for values of M over the complete range of M from M=0 to M at H=0 to obtain the complete curve of constant $h_1 - h_2$. Repeat for other selected values of $h_1 - h_2$ to obtain the family of $h_1 - h_2$ curves.

2. Lines of constant $h_3 - h_2$. Select one of the values of $h_1 - h_2$ from the family of curves of constant $h_1 - h_2$. Select a value of $h_3 - h_2$. Compute H from the definition of $H = (h_3 - h_2)/h_1 - h_3$ transformed into

$$\frac{h_3 - h_2}{h_1 - h_2} = \frac{H}{1+H}$$

Obtain the value of M corresponding to H from the characteristic test curves. At any convenient Q_1 compute Q_2 to establish the line of constant M. A line of constant M is a straight line passing through $Q_1 = 0$ and $Q_2 = 0$. The intersection of this line with the curve of the selected $h_1 - h_2$ is a point on the curve of constant $h_3 - h_2$. Repeat for the same value of $h_3 - h_2$ and other values of $h_1 - h_2$ to establish the line of constant $h_3 - h_2$. Repeat for other values of $h_3 - h_2$ to obtain the family of curves of constant $h_3 - h_2$.

3. Lines of constant $h_1 - h_3$. From the intersections of the $h_1 - h_2$ and the $h_3 - h_2$ curves locate the family of curves of $h_1 - h_3$ since $h_1 - h_3 = (h_1 - h_2) - (h_3 - h_2)$

4. Lines of constant efficiency. Select a value of efficiency. Obtain the corresponding value of M from the characteristic curves. Select any value of Q_1. Compute Q_2 from $Q_2 = MQ_1$. A straight line through this point Q_1, Q_2 and the origin is the line of constant efficiency. Repeat for other values of efficiency.

10.4. Cavitation in jet pumps. The lowest pressure region in the jet pump is at the nozzle discharge. When the pressure at the nozzle tip reaches the vapor pressure at the temperature of the pumped fluid, the limit of Q_2 is reached.

The cavitation limit is described from the the flow equation applied between the suction flange and the region b.

$$\left(\frac{p_2}{w}+\frac{V_2^2}{2g}\right)-\left(\frac{p_b}{w}+\frac{V_b^2}{2g}\right)=K_s\frac{V_b^2}{2g} \qquad 10.34$$

in which p_2 is referred to the center-line of the region b.

At incipient cavitation $p_b = p_a = p_v$ where p_v is the vapor pressure of the pumped fluid.

$$\frac{p_2}{w}+\frac{V_2^2}{2g}-\frac{p_v}{w}=\frac{V_b^2}{2g}(1+K_s) \qquad 10.35$$

The left side of eq. 10.35 is the Net Positive Suction Head of the jet pump

$$NPSH=\frac{V_b^2}{2g}(1+K_s)=\frac{Q_b^2}{2gA_b^2}(1+K_s) \qquad 10.36$$

$$NPSH=\Phi(Q_b, A_b, K_s) \qquad 10.37$$

NPSH values may be computed from eq 10.36 if A_b and K_s are known. NPSH is determined from test results since the performance with cavitation will show a drop in H at a given M below the non-cavitating characteristic curve.

10.5 Model Laws for jet pumps. The characteristic curves of K and H as functions of Q are representations of the model laws for a given jet pump since the complete performance for any operating range is represented by these unique curves. This is for the condition of large enough Reynolds numbers for all flow regions such that all loss coefficients are essentially constant.

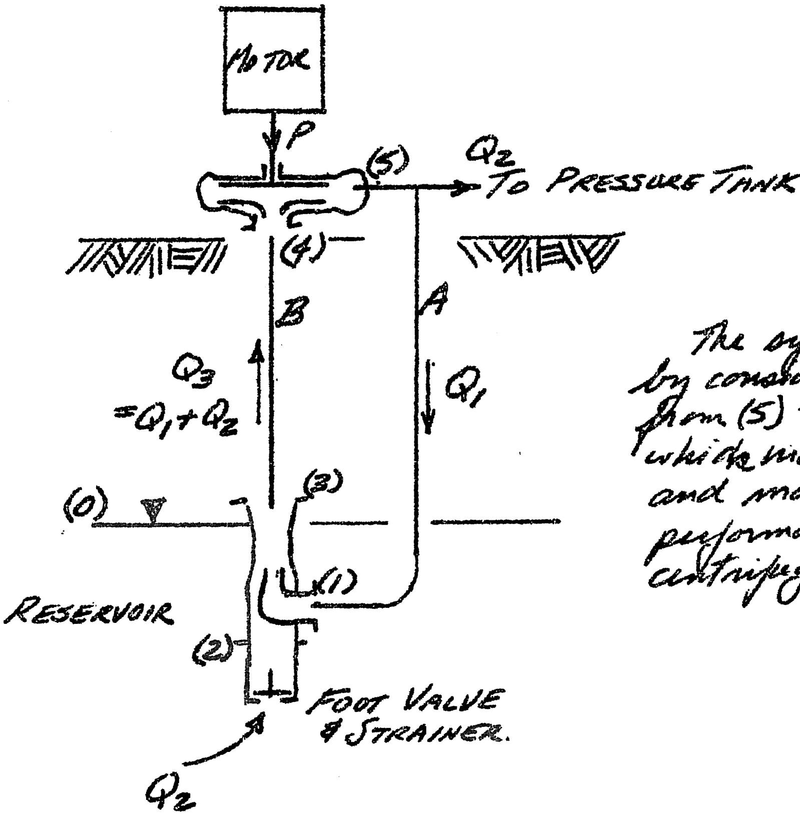
The curve of $H=\Phi(M)$ holds for all geometrically similar pumps. The curve of K is proportional to the square of the length ratio for geometrically similar pumps. Thus the characteristic curves of all geometrically similar jet pumps may be found from results on one pump. The performance curves of geometrically similar jet pumps may be formed merely by changing the flow rate scales by the factor of the square of the length ratio.

10.5 Centrifugal — Jet-pump Combination. Suction lift limitations with centrifugal pumps limit the pump placement above the suction reservoir as dictated by the NPSH rating of the pump. In pumping from underground reservoirs, it may be necessary to place a centrifugal pump below surface ground level to insure sufficient system NPSH. Large diameter wells with pump servicing difficulties result. For high flow rates where efficiency is important, the deep-well turbine type pump with the centrifugal pump impellers submerged in the reservoir and the impellers driven by long shafting from the motor on the ground surface is used in spite of the servicing difficulties. For small domestic water supplies the centrifugal — jet-pump combination is preferred.

In the combination, the jet pump is placed in the reservoir. The drive liquid is supplied by a centrifugal at ground surface. The discharge from the jet pump returns to the suction of the centrifugal. There is a head boost from the reservoir to the centrifugal pump to produce pressures at the centrifugal pump suction that are greater than the pressure required by the limiting NPSH of the centrifugal. The supply to the centrifugal consists of the fluid pumped from the reservoir and the drive fluid. The drive fluid, in a sense, circulates from the centrifugal to the jet and back to the centrifugal. The net discharge from the combination is the fluid pumped from the reservoir. The net discharge in domestic water supply systems goes to a pressure tank. The unit is controlled by an on-off pressure switch in the motor circuit which is set to regulate on the maximum and minimum pressures desired in the pressure tank.

Another application of the centrifugal — jet pump combination is to place the jet adjacent to the centrifugal with essentially no length to the connecting piping. This close-coupled arrangement produces a higher head than can be produced by the centrifugal alone but at a smaller flow rate than obtainable from the centrifugal alone and at a lower efficiency than the best efficiency of the centrifugal.

10.6 Analysis of the centrifugal—jet-pump combination.



The system is analyzed by considering the circuit from (5) to (1) to (3) to (4) which includes the jet pump and matching this circuit performance with the centrifugal pump performance.

From (5) to (1)
$$\frac{p_5}{w}+z_5+\frac{V_5^2}{2g}=\frac{p_1}{w}+z_1+\frac{V_1^2}{2g}+f_A\frac{L_A}{D_A}\frac{V_A^2}{2g} \qquad 10.38$$

Define
$$h=\frac{p}{w}+z+\frac{V^2}{2g}$$
$$f\frac{L}{D A^2 2g}=K$$

Then eq 10.38 is
$$h_5=h_1+f_A\frac{L_A}{D_A}\frac{Q_1^2}{A_A^2 2g}=h_1+K_A Q_1^2 \qquad 10.39$$

Similarly,
$$h_4=h_3-K_B Q_3^2 \qquad 10.40$$

For the jet pump,
$$H=\frac{h_3-h_2}{h_1-h_3}$$
$$h_3=\frac{H h_1+h_2}{1+H} \qquad 10.41$$

Then
$$h_5-h_4=h_1-h_3+K_A Q_1^2+K_B Q_3^2 \qquad 10.42$$
$$=h_1-\frac{H h_1+h_2}{1+H}+K_A Q_1^2+K_B Q_3^2$$

$$h_5 - h_4 = \frac{h_1 - h_2}{1+H} + K_A Q_1^2 + K_B Q_3^2 \qquad 10.43$$

For the jet $\quad Q_1 = K(h_1 - h_2)^{1/2} \qquad 10.44$

Then $$h_5 - h_4 = \frac{Q_1^2}{K^2(1+H)} + K_A Q_1^2 + K_B Q_3^2$$

$$= Q_3^2 \left[\frac{1}{(1+M)^2 K^2 (1+H)} + \frac{K_A}{(1+M)^2} + K_B \right] \qquad 10.45$$

Eq. 10.45 was developed from consideration of the piped and jet pump circuit. The centrifugal pump head is also $h_5 - h_4$ and the centrifugal pump flow rate is Q_3. Thus matching eq. 10.45 with the centrifugal pump performance gives the complete system performance.

The head and capacity of the combination are $h_5 - h_0$ and Q_2.

$$h_5 - h_0 = h_5 - h_4 + h_4 - h_2 + h_2 - h_0$$

$$= h_5 - h_4 + h_3 - K_B Q_3^2 - h_2 + h_0 - K_F Q_2^2 - h_0$$

where $\quad h_2 = h_0 - K_F Q_2^2$ and K_F is the loss coefficient of the strainer and foot valve.

$$h_5 - h_0 = h_5 - h_4 + h_3 - h_2 - K_B Q_3^2 - K_F Q_2^2$$

$$h_3 - h_2 = H(h_1 - h_3)$$

$$h_1 = h_5 - K_A Q_1^2$$

$$h_3 = h_4 + K_B Q_3^2$$

Then, $$h_5 - h_0 = (h_5 - h_4)(1+H) - H K_A Q_1^2 - (H+1) K_B Q_3^2 - K_F Q_2^2 \qquad 10.46$$

also, $$Q_2 = \frac{M Q_3}{1+M} \qquad Q_1 = \frac{Q_3}{1+M} \qquad 10.47$$

Equations 10.46 and 10.47 give the combined complete system performance of head and capacity.

The overall efficiency comes from the power input to the centrifugal and the combined head and capacity.

$$e = \frac{(h_5 - h_0)\, w\, Q_2}{P} \qquad 10.48$$

The complete system performance is predicted using eqs. 10.45, 10.46, 10.47 and 10.48 for a given centrifugal pump performance, a given jet pump characteristic, and known connecting pipe lengths and sizes. In the usual application range, the friction factors may be taken as constants making K_A, K_B, and K_F constants of the system. For a selected value of M, K and H are single valued. The bracketed term in eq. 10.45 is then single valued for one value of M. Curves of $h_5 - h_4$ as a function of Q_3 are superimposed on the centrifugal pump performance curves each curve corresponding to a selected value of M. The range of M values is taken from $M=0$ to M at $H=0$.

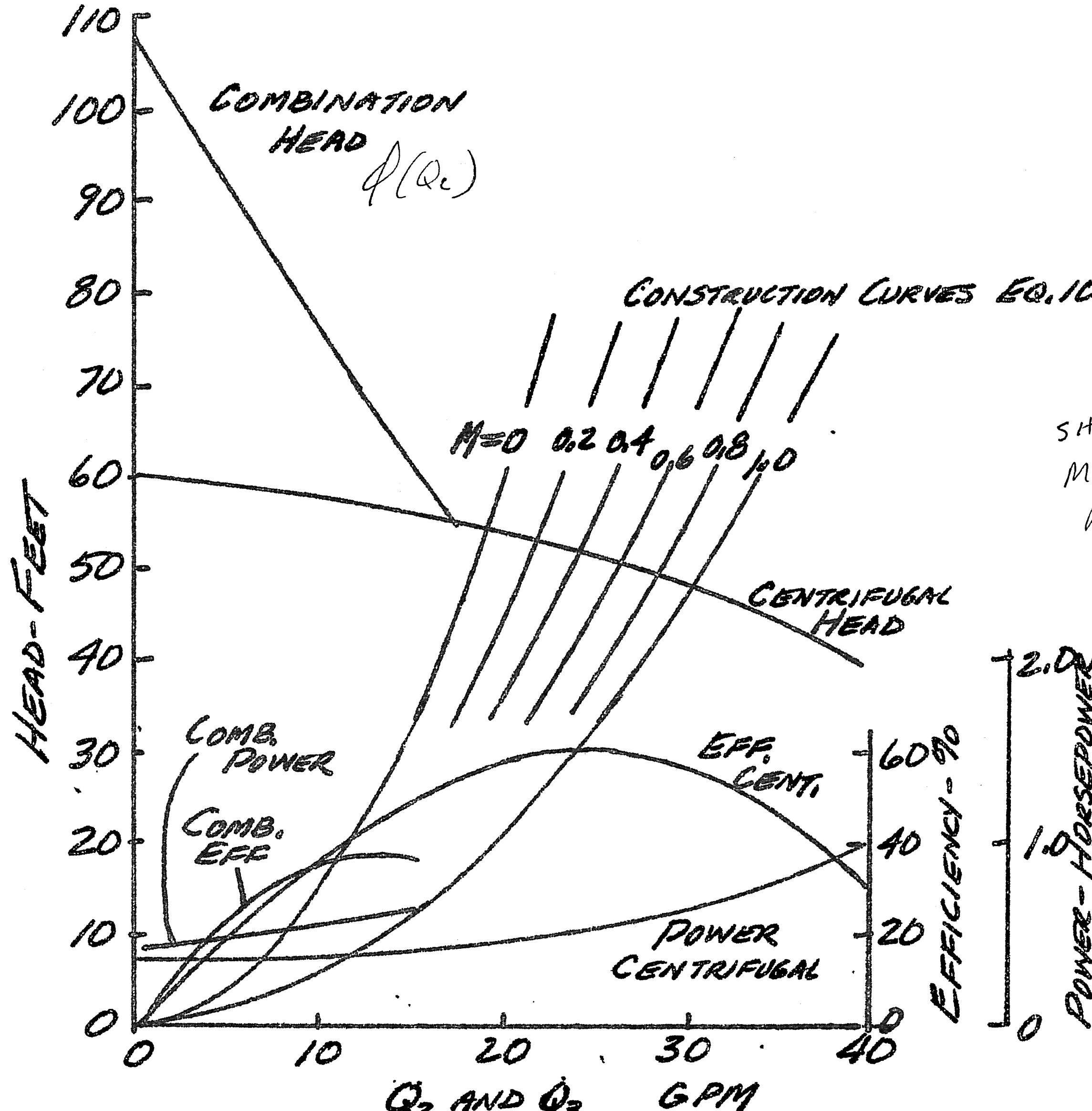
The intersection of a curve of M = constant with the centrifugal pump head-capacity curve determines Q_3 and $h_5 - h_4$ to be used with equation 10.46 to find the combined system head, $h_5 - h_0$, and with eq. 10.47 to find the combined system flow rate, Q_2. The power input in eq. 10.48 is the input to the centrifugal at Q_3.

In the close-coupled centrifugal-jet pump combination, $K_A = K_B = 0$. With no strainer and foot valve $K_F = 0$. Equations 10.45, 10.46 reduce to

$$h_5 - h_4 = Q_3^2 \left[\frac{1}{(1+M)^2 K^2 (1+H)}\right] \qquad 10.48$$

$$h_5 - h_0 = (h_5 - h_4)(1+H) \qquad 10.49$$

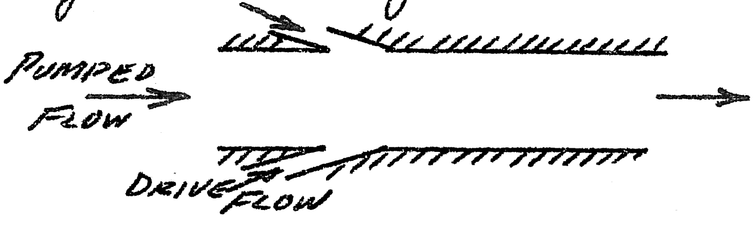
The performance of a close-coupled combination with the jet pump characteristics of p. 82 and the centrifugal pump performance as shown is presented graphically with the construction lines of constant M. The complete performance of the combination includes the head, power, and efficiency as a function of Q_2.



SHOULD include M for H=0

Q_2 for COMBINED PERF., Q_3 for CENT. PERF. & CONST. CURVES

10.7. Other forms of jet pumps. The jet nozzle can be placed in the side wall of the pump to provide a straight unobstructed passage for the pumped flow. This arrangement is advantageous when the pumped flow contains large amounts of solids such as in sand dredging applications. Usually two or more nozzles are used to give a symmetrical flow system in the mixing region. The performance analysis is modified by the inclusion of the necessary jet angle with the mixing chamber axis. Otherwise the analysis is not changed.



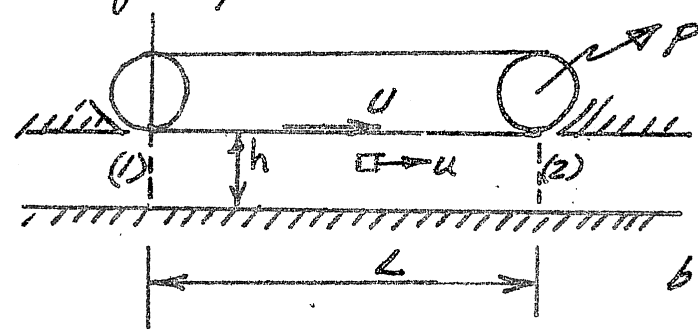
11.0 Drag Turbines.

11.1 Belt turbine. The possibility of the drag turbine has been shown in Section 8.0. Feasible turbines of this type are designed much like the drag pumps.

The analyses of Section 8.0 for the pumps may be transposed directly into turbine performance conventions of positive shaft power output and positive head reduction in the fluid between inlet and outlet. Turbine efficiency is, by definition (p. 10)

$$e = \frac{P}{QwH}$$

11.2 Analysis of the belt turbine with laminar flow.



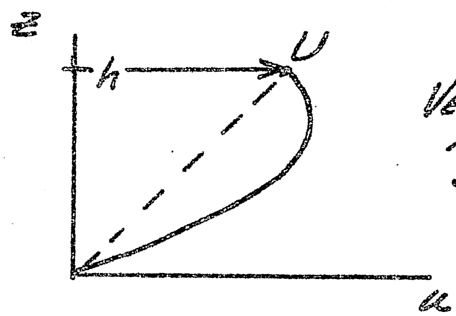
b — width perpendicular to the diagram

The moving surface is dragged by the fluid adjacent to it. The fluid head furnishing the work is

$$H = \left(\frac{p_1}{w} + z_1 + \frac{V_1^2}{2g}\right) - \left(\frac{p_2}{w} + z_2 + \frac{V_2^2}{2g}\right) \qquad 11.1$$

The analysis of the laminar flow pump applies directly by merely changing signs to conform to usual turbine sign conventions. From eq. 8.11,

$$u = \frac{-Hw}{2\mu L}\left(z^2 - zh\right) + \frac{Uz}{h} \qquad 11.2$$



VELOCITY DISTRIBUTION IN THE LAMINAR FLOW BELT TURBINE

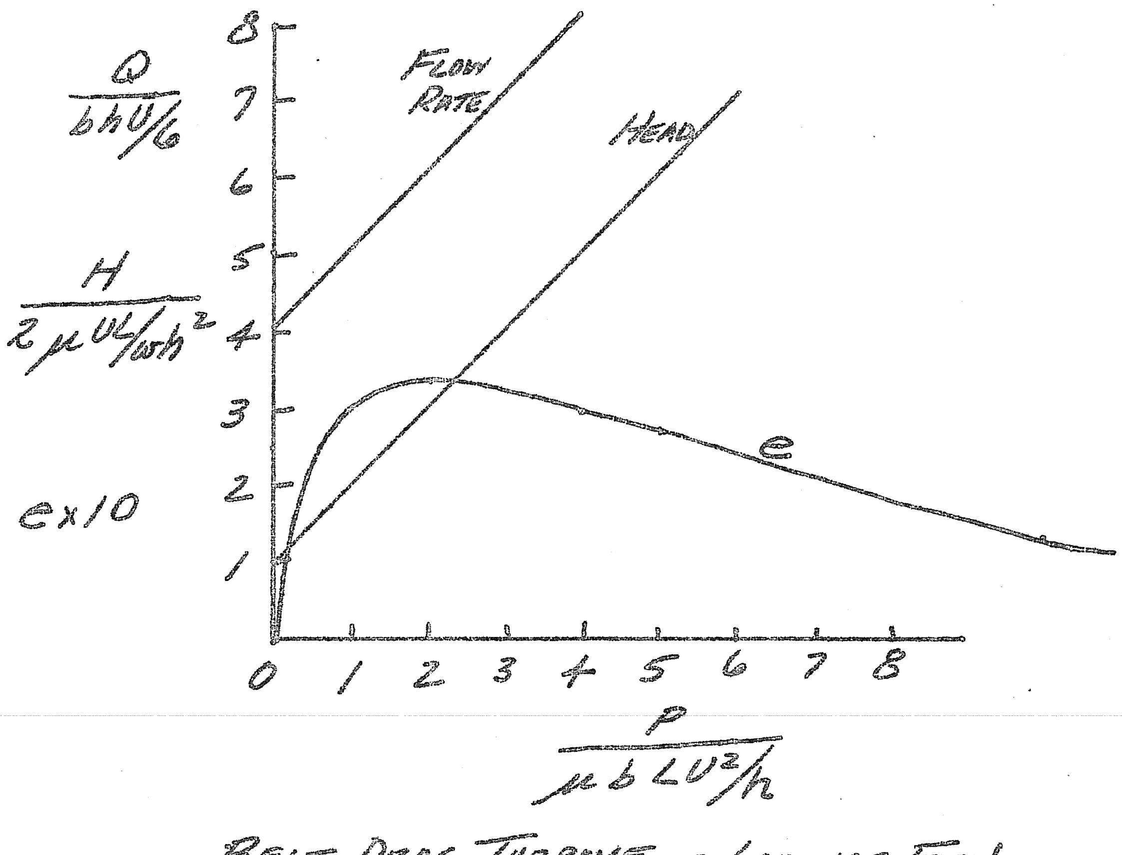
From eqs. 8.18 and 8.17 with the turbine sign convention and with P as the independent variable

$$\frac{Q}{bhU/6} = 4 + \frac{P}{\mu bLU^2/h} \qquad 11.3$$

$$\frac{H}{2\mu UL/wh^2} = 1 + \frac{P}{\mu bLU^2/h} \qquad 11.4$$

$$e = \frac{\dfrac{P}{\mu bLU^2/h}}{\dfrac{4}{3} + \dfrac{5}{3}\dfrac{P}{\mu bLU^2/h} + \dfrac{1}{3}\left(\dfrac{P}{\mu bLU^2/h}\right)^2} \qquad 11.5$$

Maximum efficiency is $\frac{1}{3}$ (or $33\frac{1}{3}$ %) at $\frac{P}{\mu bLU^2/h} = 2$



BELT DRAG TURBINE - LAMINAR FLOW.

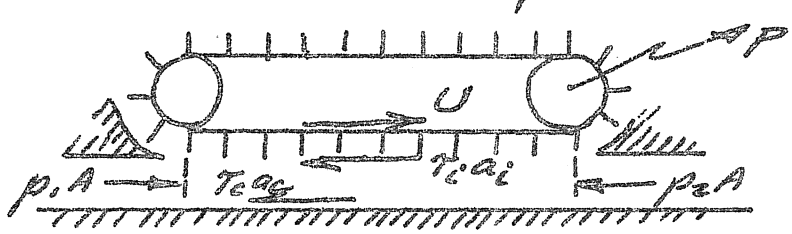
11.3 Model laws of the belt turbine with laminar flow. Since the analysis of the turbine is obtained from the same basic behavior equations as for the pump, the same model laws apply. From p. 28

$$Q \propto ND^3 \qquad 11.6$$

$$H \propto \frac{\mu N}{w} \qquad 11.7$$

$$P \propto \mu N^2 D^3 \qquad 11.8$$

11.4 Analysis of the belt turbine with turbulent flow. The turbine equations can not be taken directly from the pump equations but must be developed from the force balance with due regard to the sign of the shear stresses which are needed to produce a shaft power output.



The force balance is

$$p_1 A - p_2 A - \tau_i a_i - \tau_c a_c = 0 \qquad 11.9$$

See Section 8.5 for the definition of the symbols.

$$\tau_i = C_i \frac{w}{2g}\left(\frac{Q}{A} - U\right)^2 \qquad 11.10$$

$$\tau_c = C_c \frac{w}{2g}\left(\frac{Q}{A}\right)^2 \qquad 11.11$$

Note that Q/A must be greater than U to produce a force which will drive the belt with a power output.

$$H = \frac{p_1 - p_2}{w} \quad \text{for } z_1 = z_2 \text{ and } V_1 = V_2 \qquad 11.12$$

$$H = \frac{C_i a_i U^2}{2g A}\left[\left(\frac{Q}{UA} - 1\right)^2 + \frac{C_c a_c}{C_i a_i}\left(\frac{Q}{UA}\right)^2\right] \qquad 11.13$$

$$P = \tau_i a_i U = C_i a_i \frac{w}{2g} U \left(\frac{Q}{A} - U\right)^2 \qquad 11.14$$

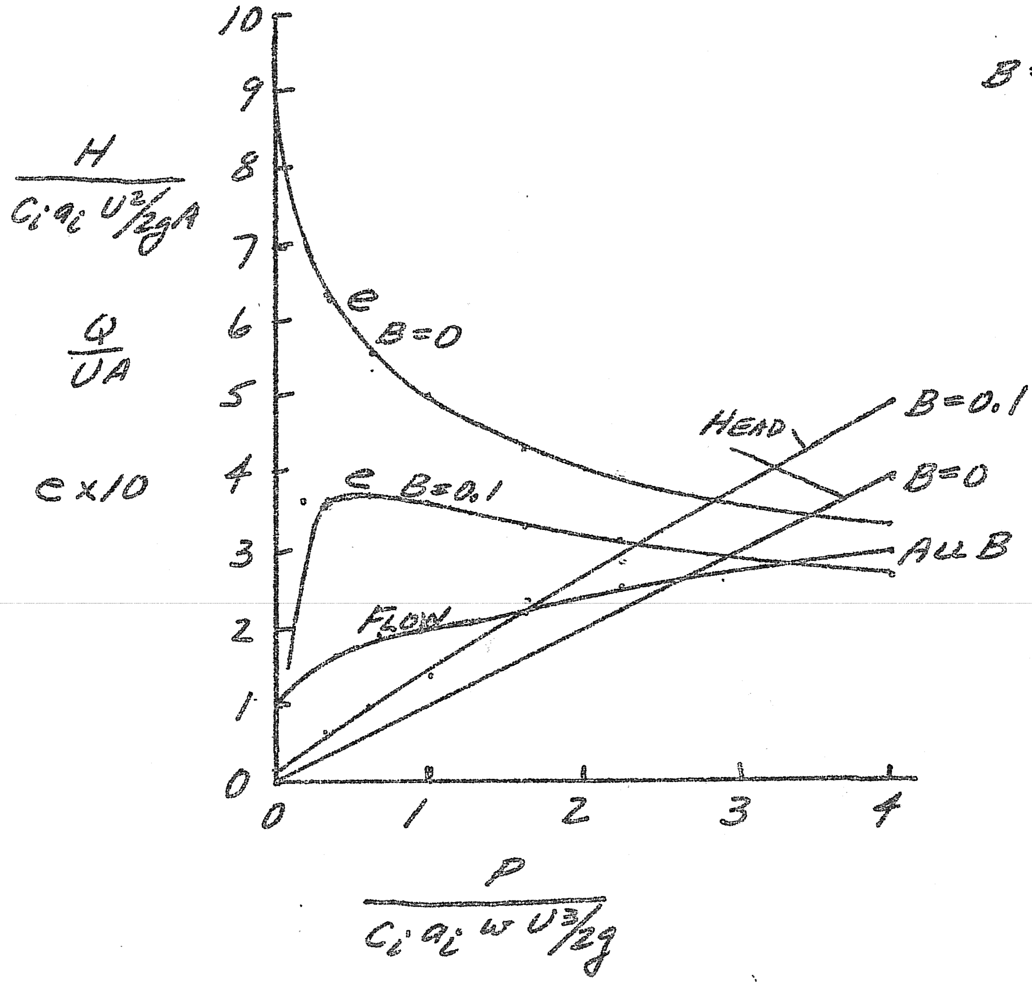
From which

$$\frac{Q}{UA} = 1 + \left(\frac{P}{C_i a_i w\, U^3/2g}\right)^{1/2} \qquad 11.15$$

$$\frac{H}{C_i a_i\, U^2/2gA} = \frac{P}{C_i a_i\, U^3 w/2g} + \frac{C_c a_c}{C_i a_i}\left\{1 + \left(\frac{P}{C_i a_i\, U^3 w/2g}\right)^{1/2}\right\}^2 \qquad 11.16$$

$$e = \frac{\dfrac{P}{C_i a_i\, w/2g}}{\left[1 + \left(\dfrac{P}{C_i a_i\, w U^3/2g}\right)^{1/2}\right]\left[\dfrac{P}{C_i a_i\, w U^3/2g} + \dfrac{C_c a_c}{C_i a_i}\left\{1 + \left(\dfrac{P}{C_i a_i\, U^3 w/2g}\right)^{1/2}\right\}^2\right]} \qquad 11.17$$

$$B = \frac{C_c a_c}{C_i a_i}$$



SHOULD take B from PUMP TEST & COMPUTE TURBINE PERFORMANCE to compare with TEST.

11.5 Regulation of drag turbines. Turbines are applied to drive other equipment at a controlled speed and very often at constant speed. The power output of the turbine is the demand of the driven equipment which may vary with time. A regulating, or control, device is a necessary part of the turbine and the complete turbine consists not only of the fluid action producing the power output but also includes the regulating device.

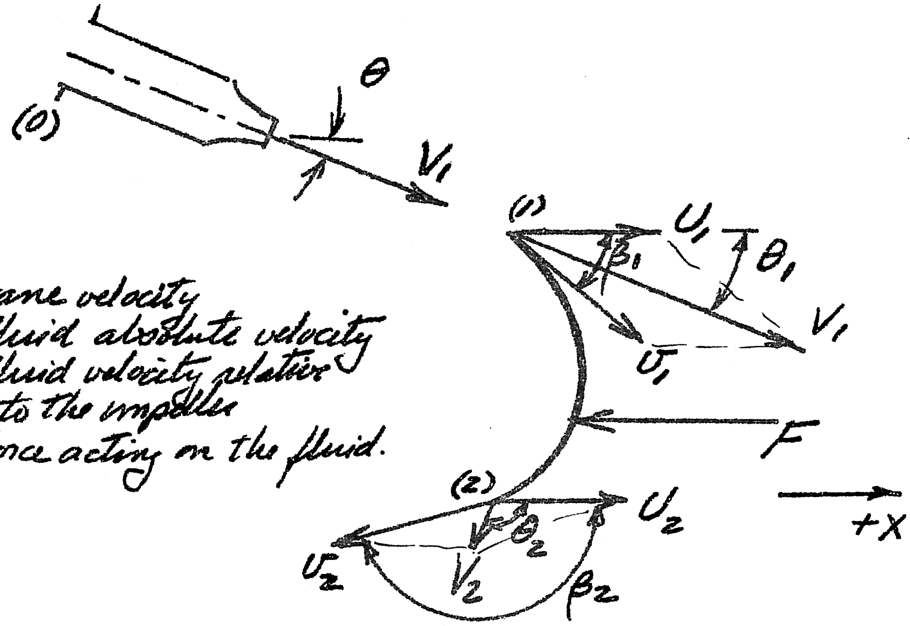
At constant speed and variable power the head and flow rate of the fluid in the active part of the turbine must vary. Normal application is with an essentially constant head supply. Constant speed control is obtained by a throttle valve in the supply line with a consequent loss of mechanical energy of the flow which must be charged to the complete turbine. This reduces the complete turbine efficiency below the values obtained for the active part of the turbine alone except for the one operating condition with the control valve wide open and with negligible valve loss.

11.6 Application range of drag turbines. The drag turbine is inherently inefficient. Its application is for small power outputs where efficiency is not important and where its small size and high speed are advantageous. Recent applications are in the aircraft and missile auxiliary equipment field.

12.0 Impulse Turbine.

12.1 Description. Impulse, by definition, is action by or on a fluid with no pressure difference between the inlet and the discharge fluid in the working sections. In the impulse turbine the head of the fluid supply is converted to kinetic energy in a nozzle. The discharge from the nozzle acts on vanes on the perimeter of a wheel to produce a torque about the axis of rotation of the wheel. The pressure field from the nozzle discharge to the final discharge of the fluid from the vanes is the same.

12.2 Analysis of the impulse turbine.



U – vane velocity
V – fluid absolute velocity
v – fluid velocity relative to the impeller
F – force acting on the fluid.

Conditions: (a) $\theta = \theta_1$
(b) v_1 is tangent to the vane at entry and has the direction of the vane angle β_1
(c) $p_1 = p_2$
(d) $z_0 = z_1 = z_2$
(e) There is a series of vanes spaced such that all the fluid issuing from the nozzle is turned through the same angle.

For the series of vanes, the force balance is

USE TORQUE ABOUT AXIS OF ROTATION, SINCE $V_1 = V_2$ IN GENERAL

$$\Sigma F_x = \Delta(\text{Momentum rate})_x$$

$$-F = \dot{m}\left(V_2 \cos\theta_2 - V_1 \cos\theta_1\right) \qquad 12.1$$

From the velocity triangles,

$$v_1 \sin\beta_1 = V_1 \sin\theta_1 \qquad 12.2$$

$$v_1 \cos\beta_1 = V_1 \cos\theta_1 - U$$

$$v_1 = \frac{V_1 \cos\theta_1 - U}{\cos\beta_1} \qquad 12.3$$

Between (1) and (2), relative to the vane

$$\frac{p_1}{w} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{w} + z_2 + \frac{v_2^2}{2g} + K_v \frac{v_2^2}{2g} \qquad 12.4$$

With $p_1 = p_2$ and $z_1 = z_2$

$$v_1^2 = v_2^2(1+K_v)$$

$$v_2 = K_B v_1 \quad \text{where } K_B = (1+K_v)^{-\frac{1}{2}} \qquad 12.5$$

Combining eqs. 12.3 and 12.5

$$v_2 = K_B \frac{V_1 \cos\theta_1 - U_1}{\cos\beta_1} \qquad 12.6$$

$$V_2 \cos\theta_2 = U_2 + v_2 \cos\beta_2$$

$$= U_2 + K_B\left(\frac{V_1\cos\theta_1 - U_1}{\cos\beta_1}\right)\cos\beta_2 \qquad 12.7$$

The force on the vanes is

$$F = \dot{m}\left[V_1\cos\theta_1 - U_2 - K_B\left(\frac{V_1\cos\theta_1 - U_1}{\cos\beta_1}\right)\cos\beta_2\right] \qquad 12.8$$

In an axial flow impulse turbine $U_1 = U_2$. Then eq. 12.8 becomes

$$F = \dot{m}\left[V_1\left(\cos\theta_1 - K_B\frac{\cos\theta_1\cos\beta_2}{\cos\beta_1}\right) - U\left(1 - K_B\frac{\cos\beta_2}{\cos\beta_1}\right)\right] \qquad 12.9$$

The head supplied at the nozzle inlet flange for the axial flow turbine is

$$\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g} = H_0 = \frac{p_1}{w} + z_1 + \frac{V_1^2}{2g} + K_N\frac{V_1^2}{2g} \qquad 12.10$$

In eq. 12.10 $z_0 = z_1$, p_1 is taken as the reference pressure of zero.

$$H_0 = \frac{V_1^2}{2g}(1+K_N)$$

or,

$$V_1 = \frac{1}{(1+K_N)^{1/2}}(2gH_0)^{1/2} \qquad 12.11$$

$(1+K_N)^{1/2}$ may be replaced by the usual form of the nozzle coefficient:

$$V_1 = C_N(2gH_0)^{1/2} \qquad 12.12$$

The power output of the turbine, neglecting bearing and windage losses is

$$P = FU \qquad 12.13$$

$$P = \dot{m} U \left[C_N (2gH_0)^{1/2} \left(\cos\theta_1 - K_B \frac{\cos\theta_1 \cos\beta_2}{\cos\beta_1} \right) - U \left(1 - K_B \frac{\cos\beta_2}{\cos\beta_1} \right) \right] \qquad 12.14$$

The MECHANICAL energy rate input at the inlet flange of the nozzle is

$$E = g \dot{m} H_0 \qquad 12.15$$

And the efficiency is

$$e = \frac{P}{E} \qquad 12.16$$

FACTOR OUT $\cos\theta_1$

Of interest is the best vane speed for a given head at the nozzle.

$$\frac{de}{dU} = \frac{C_N (2gH_0)^{1/2} \left(\cos\theta_1 - K_B \frac{\cos\theta_1 \cos\beta_2}{\cos\beta_1} \right) - 2U \left(1 - K_B \frac{\cos\beta_2}{\cos\beta_1} \right)}{g H_0}$$

Replacing H_0 by V_1 and setting $\frac{de}{dU} = 0$ for best speed

$$\frac{U}{V_1} = \frac{\cos\theta_1 - K_B \frac{\cos\theta_1 \cos\beta_2}{\cos\beta_1}}{2 \left(1 - K_B \frac{\cos\beta_2}{\cos\beta_1} \right)} = \frac{\cos\theta}{2} \qquad 12.17$$

With this velocity ratio the best efficiency is

$$e_{max} = \frac{\cos^2\theta}{2} \left(1 - K_B \frac{\cos\beta_2}{\cos\beta_1} \right) \qquad 12.18$$

The maximum possible efficiency will be obtained when θ_1 and β_1 are zero and $\beta_2 = 180°$. However, if β_2 is 180° the discharge fluid would remain in the path of the vanes and create losses which are not included in the development leading to eq. 12.18. β_2 is made as large as possible, approximately 165°.

If $\theta_1 = 0$, $\beta_1 = 0$, and $\beta_2 = 180°$ with ideal flow (no friction) making $K_B = 1$; $e_{max} = 1.0$ at $U/V_1 = \frac{1}{2}$.

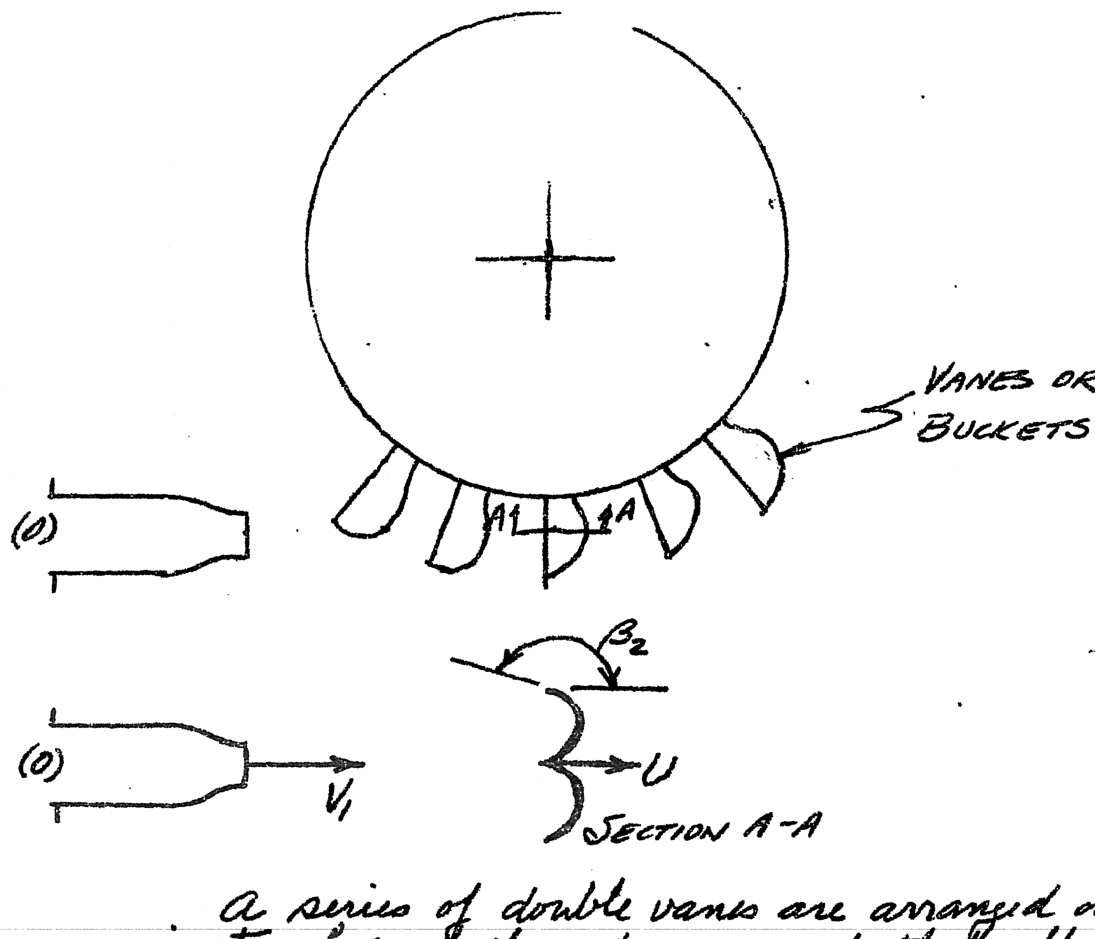
PERFECT FLUID GUIDANCE

Since eq. 12.17 for maximum efficiency was obtained by differentiating eq 12.16 with E constant at constant head, it follows that maximum power output occurs at the same velocity ratio as for maximum efficiency — the best possible combination.

If $\beta_1 = 180 - \beta_2$, Then $e_{max} = \frac{\cos^2\theta}{2}(1+K_B)$. This is a common design condition in steam turbines.

From eq. 12.9, maximum force is obtained when $U=0$ giving a high starting force or torque.

12.3 The hydraulic impulse turbine or Pelton wheel.



A series of double vanes are arranged on the perimeter of a wheel and so spaced that all the nozzle flow is turned by the vanes. This design gives the desirable condition of $\theta_1 = 0$ and $\beta_1 = 0$. The double vanes also produce a balanced axial flow and no axial thrust.

The development in Section 12.2 gives the performance for these special conditions.

From eq. 12.9 $F = \dot{m}\left[V_1(1-K_B\cos\beta_2) - U(1-K_B\cos\beta_2)\right]$

$$= \dot{m}(V_1-U)(1-K_B\cos\beta_2) \qquad 12.19$$

From eq. 12.13 $P = \dot{m}U\left[C_N(2gH_0)^{1/2} - U\right]\left[1-K_B\cos\beta_2\right]$ 12.20

From eq. 12.16 $e = \dfrac{\dot{m}U\left[C_N(2gH_0)^{1/2}-U\right]\left[1-K_B\cos\beta_2\right]}{g\,\dot{m}\,H_0}$ 12.21

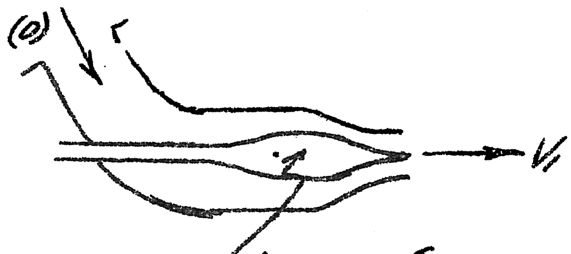
also by eq 12.12 $Q = C_N A_N (2gH_0)^{1/2}$ 12.22

From eq. 12.17, the best speed is $\dfrac{U}{V_1} = \dfrac{1}{2}$ 12.23

At this best speed $e_{max} = \dfrac{1-K_B\cos\beta_2}{2.}$ 12.24.

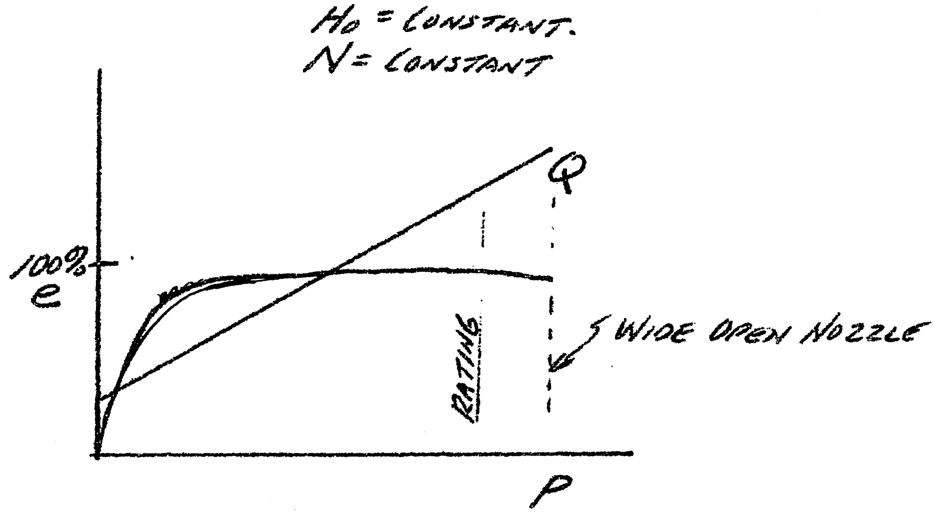
In a well-designed nozzle, C_N is approximately 0.99. For well-designed vanes, K_B is approximately 0.97. Maximum turbine efficiencies, including all losses between the nozzle inlet flange and the shaft power output are over 90%.

The usual application is with a constant head fluid supply and with constant turbine speed. Speed control is accomplished by changing the nozzle area.



NEEDLE CONTROL VALVE.

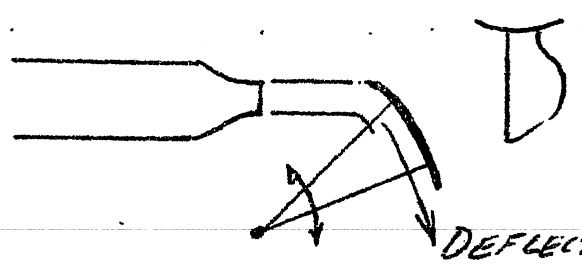
The needle valve is positioned by the speed control mechanism. The nozzle coefficient does not decrease much below its value at the wide open nozzle position until the flow is near shut-off. Thus, at constant head and constant turbine speed the efficiency remains high over all of the power output range of the turbine except at power outputs approaching zero.



IMPULSE TURBINE PERFORMANCE

12.4 Sudden load decrease. If the load on a turbine is suddenly removed, and if the governor does not act rapidly, the turbine speed could approach the the condition of $U = V_1$ or a speed of twice the normal speed.

If the needle valve acts rapidly to stop the flow, a water-hammer will result with pressures that may exceed bursting pressure of the nozzle or supply pipe. In applications where there is the possibility of a sudden removal of the load a jet deflector is incorporated in the control system. The control acts rapidly on the jet deflector to insert it in the nozzle stream. The needle valve THEN closes slowly to prevent excessive pressures in the supply line and to CONSERVE FLOW ENERGY. the DEFLECTOR RETRACTS from the STREAM



12.5 Model Laws. Model laws are obtained from eqs. 12.12, 12.13 and from $Q = A_N V_1$

From eq. 12.12 $$b_{V_1} = b_{C_N} b_g^{1/2} b_{H_0}^{1/2} \qquad 12.25$$

If C_N is independent of the Reynolds number and with the velocity referred to the wheel speed and pitch diameter

$$V_1 \propto ND \text{ and } b_{C_N} = 1.0$$

$$b_H = \frac{b_V^2}{b_g} = \frac{b_N^2 b_D^2}{b_g} \qquad 12.26$$

Since b_g = constant

$$b_H = b_N^2 b_D^2$$

or, $$H \propto N^2 D^2 \qquad 12.27$$

From eq. 12.13, using only the first term of the right side,

$$b_P = b_{\dot m} b_U b_{C_N} b_g^{1/2} b_{H_0}^{1/2} b_{\cos\theta_1} \qquad 12.28$$

$$b_{\dot m} = b_\rho b_Q = \frac{b_w}{b_g} b_Q = \frac{b_w}{b_g} b_N b_D^3 \qquad 12.29$$

$$b_U = b_N b_D \qquad 12.30$$

With geometrical similarity, which can be shown to be a necessary condition from the first and third terms of the right side of eq. 12.13,

$$b_{\cos\theta_1} = 1.0$$

also, from eq. 12.27 $b_H = b_N^2 b_D^2$

Then $$b_P = \frac{b_w b_N^3 b_D^5}{b_g^{1/2}} \qquad 12.31$$

or, $$P \propto w N^3 D^5 \text{ for } b_g = \text{constant}. \qquad 12.32$$

From 12.29 $b_Q = b_N b_D^3$

or, $$Q \propto ND^3 \qquad 12.33$$

The efficiency ratio is 1.0.

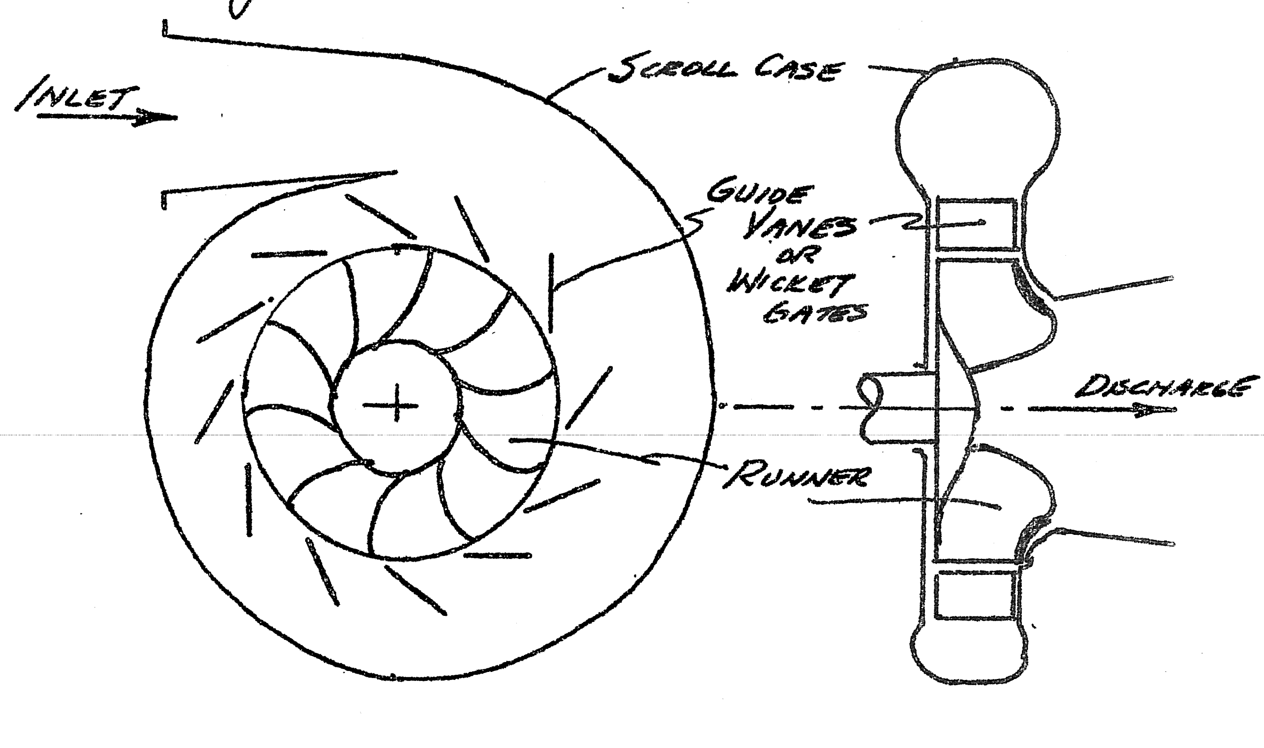
These model laws are the same as those found for centrifugal pumps, a conclusion which could have been reached without the developments of this section since the turbine vane dynamics are like that of the pump.

13.0 Radial Flow Turbines.

13.1 Description. The radial flow turbine runner has a series of vanes on which the fluid acts to impart a force on the vanes and a turning moment on the shaft to which the vanes are attached. The vanes are usually placed between a front and a back shroud plate. Turbine action is possible with either inward flow or outward flow but inward flow is preferred for reasons of regulation, flow containment and mechanical arrangement. The inward flow radial hydraulic turbine is also commonly called the Francis turbine.

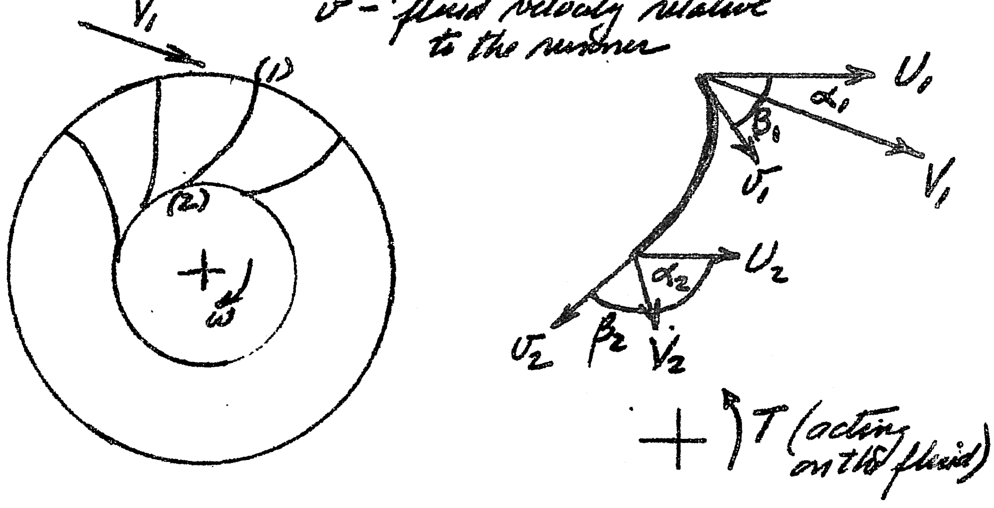
In the Francis turbine the fluid is introduced to the runner around the whole perimeter of the runner. The pipeline, or penstock, supply is distributed to the runner through a distribution channel or scroll case so shaped that the area continuously decreases from the inlet flange around to the tongue in order to maintain identical velocity patterns around the complete perimeter of the runner.

Regulation is accomplished by guide vanes between the scroll case and the runner. The guide vanes are adjustable with linkage between each guide vane pivot shaft and the master ring positioned by the control sensing device. Guide vanes are also called wicket gates.



13.2 Analysis of the runner for radial flow.

U – runner velocity
V – fluid absolute velocity
v – fluid velocity relative to the runner



Consider an impeller with a constant distance between the shrouds and with β_1 constant over b, the distance between the shrouds and with β_2 constant over b. All velocities are uniform at each section.

A moment balance around the axis of rotation gives

$$-T = \dot{m}\left[V_2 r_2 \cos\alpha_2 - V_1 r_1 \cos\alpha_1\right] \qquad 13.1$$

or

$$T = \dot{m}\left[V_1 r_1 \cos\alpha_1 - V_2 r_2 \cos\alpha_2\right] \qquad 13.2$$

The torque on the runner is equal to the torque acting on the fluid but opposite in sense. The power delivered by the runner is

$$P = T\omega = Q w H \qquad 13.3$$

where H is the decrease in fluid head from (1) to (2).

Then,
$$H = \frac{\omega r_1 V_1 \cos\alpha_1 - \omega r_2 V_2 \cos\alpha_2}{g} \qquad 13.4$$

also
$$P = Qw\left[\frac{\omega r_1 V_1 \cos\alpha_1 - \omega r_2 V_2 \cos\alpha_2}{g}\right] \qquad 13.5$$

α_1 is fixed by the guide vane setting, or by the flow rate and scroll case flow area if no guide vanes are present. V_1 is directly related to the flow rate.

V_2 and α_2 may be replaced by a function of the geometry and the flow rate.

From the discharge velocity pattern

$$V_2 \cos\alpha_2 = U_2 + v_2 \cos\beta_2 \qquad 13.6$$

Note that β_2 is greater than 90° making the cosine of β_2 negative.

also, for radial flow,

$$v_2 \sin\beta_2 = \frac{Q}{A_2} = \frac{Q}{2\pi r_2 b_2} \qquad 13.7$$

Then,

$$V_2 \cos\alpha_2 = U_2 + \frac{Q}{2\pi r_2 b_2} \cot\beta_2 \qquad 13.8$$

Eqs. 13.4 and 13.5 become

$$H = \frac{\omega r_1 V_1 \cos\alpha_1}{g} - \frac{\omega r_2}{g}\left(U_2 + \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)$$

$$= \frac{\omega r_1 V_1 \cos\alpha_1}{g} - \frac{(\omega r_2)^2}{g} - \frac{\omega Q}{2\pi b_2 g}\cot\beta_2 \qquad 13.9$$

$$P = Q\omega\left[\frac{\omega r_1 V_1 \cos\alpha_1}{g} - \frac{(\omega r_2)^2}{g} - \frac{\omega Q}{2\pi b_2 g}\cot\beta_2\right] \qquad 13.10$$

Eqs. 13.9 and 13.10 give the performance of the impeller with the assumption that the impeller efficiency is 100% at all values of Q, V_1 and α_1. The three variables Q, V_1 and α_1 are interrelated by

$$V_1 \sin\alpha_1 = \frac{Q}{2\pi r_1 b_1} \qquad 13.11$$

There is only one set of values, V_1 and α_1, which will give v_1 tangent to the vane at inlet. Values of V_1 and α_1 other than this one set give v_1 in directions other than β_1 with resulting "shock" losses at entry. These losses will be neglected in the following.

Inserting eq 13.11 into eqs. 13.9 and 13.10 gives

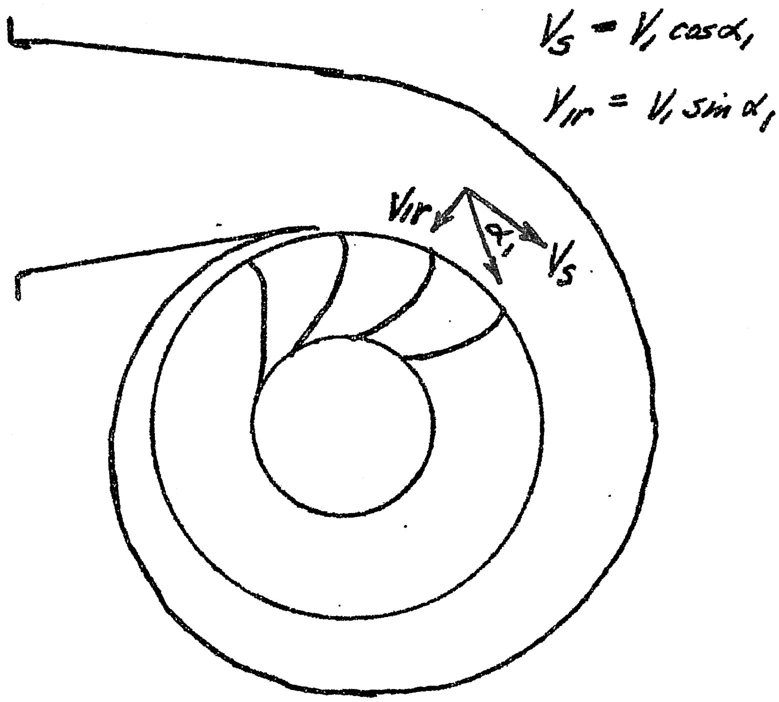
$$H = \frac{\omega Q}{2\pi g}\left(\frac{\cot\alpha_1}{b_1} - \frac{\cot\beta_2}{b_2}\right) - \frac{(\omega r_2)^2}{g} \qquad 13.12$$

$$P = \frac{Q\omega}{g}\left[\frac{\omega Q}{2\pi g}\left(\frac{\cot\alpha_1}{b_1} - \frac{\cot\beta_2}{b_2}\right) - \frac{(\omega r_2)^2}{g}\right] \qquad 13.13$$

Eqs 13.12 and 13.13 express H and P as functions of Q and α_1. The inlet angle, α_1, is determined by conditions in the scroll case.

13.3 Analysis of the scroll case.

Case I. The scroll case with no guide vanes.



The scroll case has a uniformly decreasing area to give a constant value of V_s at the design flow rate since V_{1r} and α_1 should be the same around the complete inlet to the runner.

At any flow rate other than the design flow rate

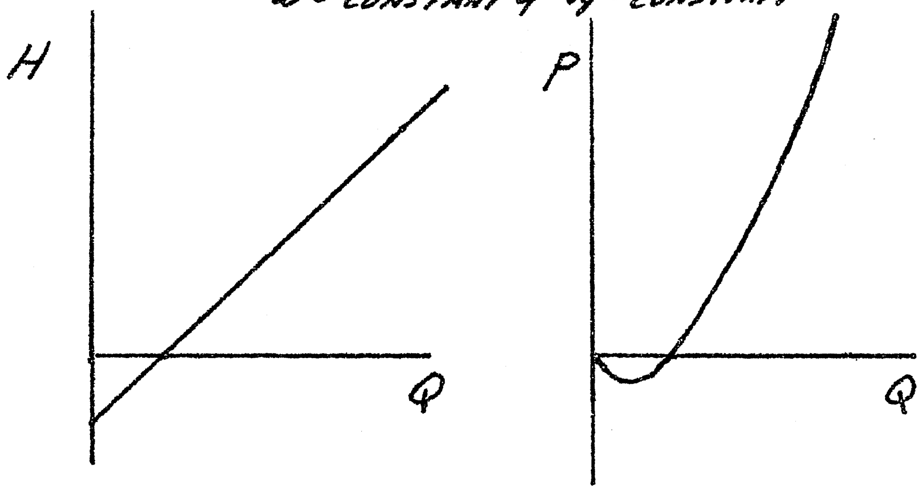
$$V_s' \propto V_s \quad \text{and} \quad V_{1r}' \propto V_{1r}$$

Then $\tan\alpha_1 = \dfrac{V_{1r}}{V_s} = \dfrac{V_{1r}'}{V_s'}$ = constant. α_1 is the same for all flow rates. Eqs. 13.12 and 13.13 are now in the form

$$H = K_1 Q - K_2 \qquad 13.14$$

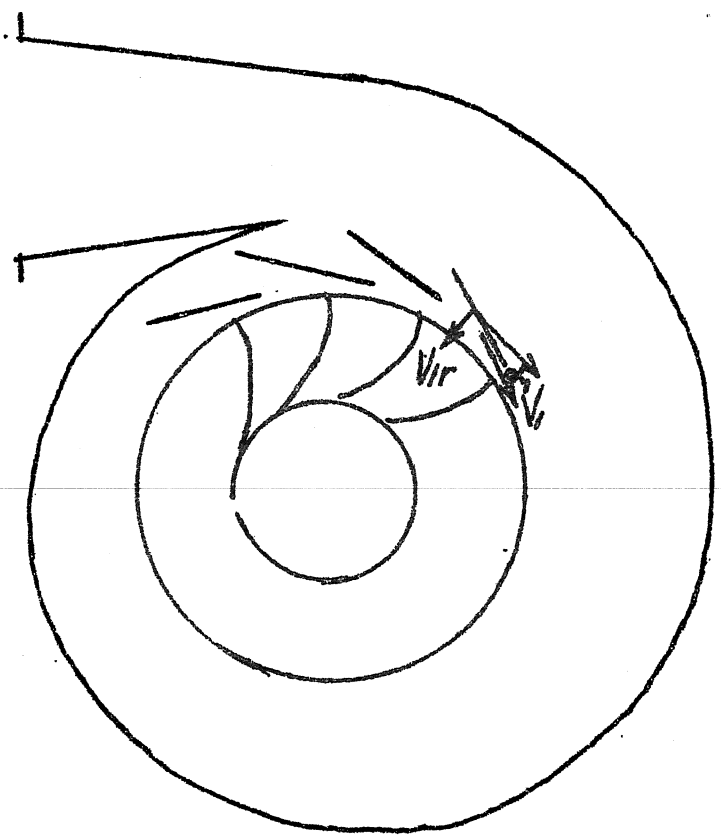
$$P = K_3 Q^2 - K_4 Q \qquad 13.15$$

ω = CONSTANT & α_1 = CONSTANT



In order to have variable power output depending upon output demand at constant speed, the head across the runner must vary as well as the flow rate. An external throttle device is needed if the supply head is constant. The throttle must be considered part of the turbine and all throttle losses charged to the turbine. There is a loss of mechanical energy in the flow except at the wide-open throttle position and an inefficient turbine except at the wide-open condition.

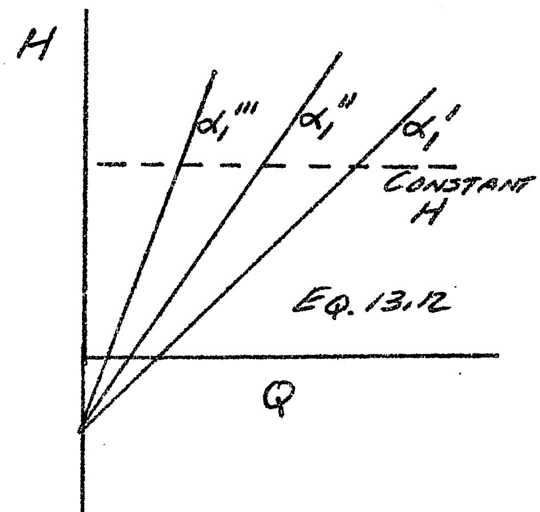
Case II. The scroll case with adjustable guide vanes.

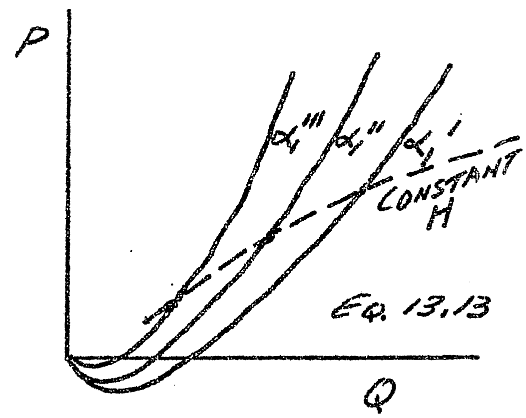


Q and α_1 are both determined by the guide vane setting. Changes in the flow rate are accomplished with changes in the flow area and with minor head losses. Eqs. 13.12 and 13.13 apply directly

$$H = \frac{\omega Q}{2\pi g}\left(\frac{\cot\alpha_1}{b_1} - \frac{\cot\beta_1}{b_2}\right) - \frac{(\omega r_2)^2}{g} \qquad 13.12$$

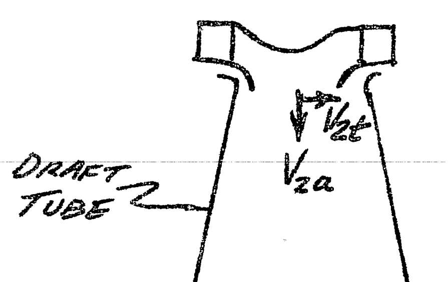
$$P = Q w\left[\frac{\omega Q}{2\pi g}\left(\frac{\cot\alpha_1}{b_1} - \frac{\cot\beta_2}{b_2}\right) - \frac{(\omega r_2)^2}{g}\right] \qquad 13.13$$





Variable power output at constant head and high efficiency is possible.

13.4 Analysis of the draft tube. The discharge flow from the runner carries one unavoidable loss — the velocity head of the flow. The loss is minimized by increasing the flow area in an expanding discharge conduit, or draft tube.



The tangential velocity of the flow leaving the impeller is always lost completely in internal dissipation or on final discharge from the draft tube.

The axial velocity of the draft tube flow is reduced in the expanding area to give, for maximum recovery of this velocity head, essentially zero velocity at (4) and zero axial velocity head at (4).

The unavoidable head loss is $V_{2t}^2/2g$. In the absence of friction and shock losses, the head supplied to the turbine is then the head used in the runner plus this tangential velocity head loss.

$$\left(\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g}\right) - z_5 = H + \frac{V_{2t}^2}{2g} \qquad 13.16$$

In eq 13.16, the location (0) is the inlet flange to the scroll case and the location (5) is the surface level of the discharge reservoir. Defining the left side of eq. 13.16 by h, the available head,

$$h = H + \frac{V_{2t}^2}{2g} \qquad 13.17$$

From eqs. 13.7 and 13.8

$$V_{2t} = V_2 \cos\alpha_2 = U_2 + \frac{Q}{2\pi r_2 b_2}\cot\beta_2$$

$$h = H + \frac{\left(U_2 + \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)^2}{2g} \qquad 13.18$$

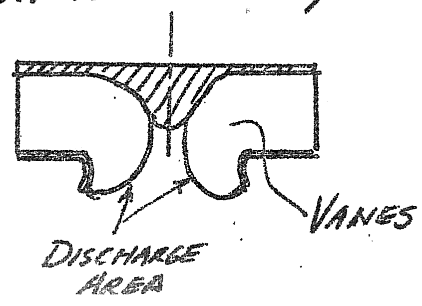
Maximum utilization of the available head occurs when

$$U_2 = -\frac{Q}{2\pi r_2 b_2}\cot\beta_2 \qquad 13.19$$

β_2 must be greater than 90°. The radial velocity leaving the impeller which becomes the axial velocity component in the draft tube should be as small as possible to effect the greatest recovery of the associated velocity head in the draft tube. Thus, β_2 should be as close to 180° as possible. For this condition

$$v_2 \simeq U_2 \qquad 13.20$$

Another feature which will produce a small radial velocity is a large area at the discharge of the impeller. This is accomplished by a mixed flow runner.



13.5 The complete turbine. As pointed out in Sections 13.3 and 13.4, the turbine includes all effects from the inlet flange of the scroll case (or from the inlet flange of the control valve on a turbine with no guide vanes) to the surface of the discharge reservoir.

The turbine efficiency is

$$e = \frac{P}{Qwh} \qquad 13.21$$

where $$h = \left(\frac{p_0}{w} + z_0 + \frac{V_0^2}{2g}\right) - z_5 \qquad 13.22$$

as shown in eq. 13.16

Then $$e = \frac{QwH}{Qwh} = \frac{H}{h} \qquad 13.23$$

Friction losses, bearing and seal losses, etc., have not been included in the analysis to this point. However, eqs. 13.21 and 13.22 apply to the complete turbine including all losses when P is the shaft power output. Eq. 13.23 applies only under the idealizations of the perfect runner with no friction or shock losses and a turbine in which the only loss is that due to the unavoidable tangential velocity at the runner discharge. Eq. 13.18 then applies, and

$$e = \frac{h - \dfrac{\left(U_2 + \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)^2}{2g}}{h}$$

$$= 1 - \frac{\left(U_2 + \frac{Q}{2\pi r_2 b_2}\cot\beta_2\right)^2}{2gh} \qquad 13.24$$

The maximum efficiency is 1.0 at $U_2 = -\frac{Q}{2\pi r_2 b_2}\cot\beta_2$

Dividing numerator and denominator of eq. 13.24 by U_2^2

$$e = 1 - \frac{\left(1 + \frac{Q}{2\pi r_2 b_2 U_2}\cot\beta_2\right)^2}{2gh/U_2^2} \qquad 13.25$$

The left side of eq. 13.28 is the total head at (a) above the vapor pressure head of the fluid. Eq. 13.28 shows that this is a function of the flow rate and the turbine geometry between the runner and the reference section (a) in the discharge throat.

For the flow between (a) and (5)

$$\frac{p_a}{w} + z_a + \frac{V_a^2}{2g} = \frac{p_5}{w} + h_{\ell_{a-4}} + \frac{V_4^2}{2g} + z_5 \qquad 13.29$$

$$z_a - z_5 = \frac{p_5}{w} - \left(\frac{p_a}{w} + \frac{V_a^2}{2g}\right) + h_{\ell_{a-4}} + \frac{V_4^2}{2g} \qquad 13.30$$

Define $\left(\frac{p_a}{w} + \frac{V_a^2}{2g}\right) - \frac{p_v}{w} = NPDH$ 13.31

where $NPDH$ = Net Positive Discharge Head when p_a is an <u>absolute</u> pressure.

Then, $z_a - z_5 = \frac{p_5}{w} - (NPDH) - \frac{p_v}{w} + h_{\ell_{a-4}} + \frac{V_4^2}{2g}$ 13.32

Eq. 13.32 gives the maximum elevation of the turbine above the discharge reservoir for cavitation free operation.

For Francis Turbines of good design

$$\frac{NPDH}{h} = 0.625\left(\frac{N_{sp}}{100}\right)^2 \text{ at maximum efficiency.} \qquad 13.33$$

N_{sp} is the specific speed based upon power output and will be defined later.

13.7 Model laws of radial flow turbines.

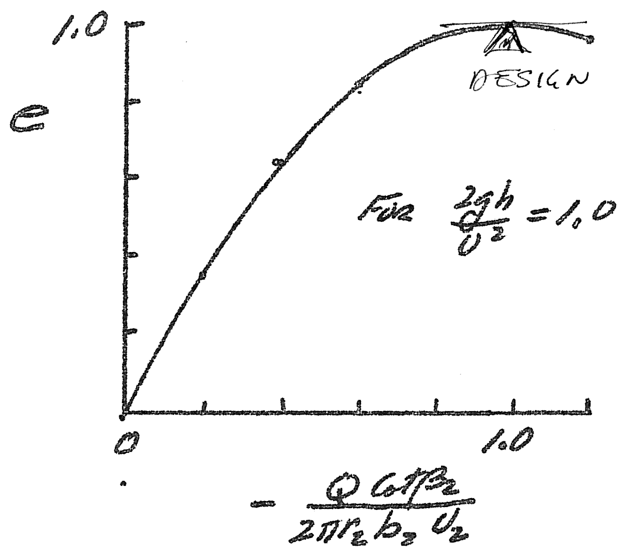
From eq. 13.18.

$$b_h = b_H = b_U^2 = \frac{b_U\, b_Q\, b_{\cot\beta_2}}{b_{r_2}\, b_{b_2}} = \frac{b_Q^2\, b_{\cot\beta_2}^2}{b_{r_2}^2\, b_{b_2}^2}$$

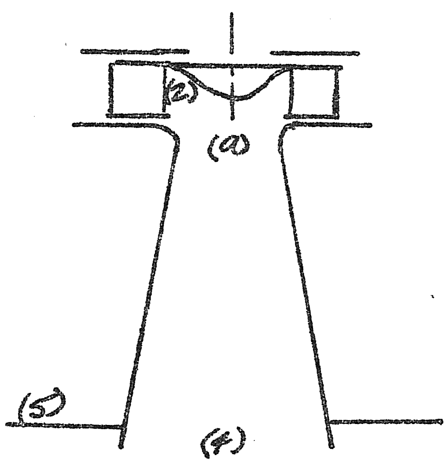
also, $b_U = b_N b_D$

$b_D = b_{r_2} = b_{b_2}$ for geometrical similarity

then $b_H = b_N^2 b_D^2$ 13.34



13.6 Cavitation. The vertical elevation of the turbine above the discharge reservoir is limited by cavitation considerations.



Between (2) and (a)

$$\frac{p_2}{w} + \frac{V_2^2}{2g} = \frac{p_a}{w} + \frac{V_a^2}{2g} + h_{e_{2-a}} \qquad 13.26$$

p_a is measured at the same elevation as the reference plane (2) of the runner discharge thereby eliminating elevation terms in eq. 13.26

At incipient cavitation $p_a = p_v$ where p_v is the vapor pressure of the liquid.

Then eq. 13.26 may be written

$$\frac{p_a}{w} + \frac{V_a^2}{2g} - \frac{p_v}{w} = \frac{V_2^2}{2g} - h_{e_{2-a}} \qquad 13.27$$

also $V_2 = V_a A_a / A_2$

and $h_{e_{2-a}} = K \frac{V_a^2}{2g}$

$$V_a = \frac{Q}{A_a}$$

Then $$\frac{p_a}{w} + \frac{V_a^2}{2g} - \frac{p_v}{w} = \frac{Q^2}{2g}\left[\frac{1}{A_2^2} - \frac{K}{A_a^2}\right] = f(Q, \text{geom}) \qquad 13.28$$

$b_{\cot\beta_2} = 1.0$ from geometrical similarity.

Then $\quad b_Q = b_U b_D^2 = b_N b_D^3 \qquad$ 13.35

The same model laws could be derived from eq 13.12.

From eq. 13.13.

$$b_P = \frac{b_Q^2 b_w b_\omega}{b_g b_{b_1}} b_{\cot\alpha_1} = \frac{b_Q^2 b_w b_\omega b_{\cot\beta_2}}{b_g b_{b_2}} = \frac{b_Q b_w b_\omega^2 b_{r_2}^2}{b_g}$$

As before, with $\quad b_{b_1} = b_{b_2} = b_{r_1} = b_{r_2} = b_D$

$$b_\omega = b_N$$

$$b_{\cot\alpha_1} = b_{\cot\beta_1} = 1$$

and $\quad b_g = 1.0$

$$b_P = \frac{b_Q^2 b_w b_N}{b_D} = \frac{b_N^2 b_D^6 b_w b_N}{b_D}$$

$$= b_w b_N^3 b_D^5 \qquad 13.36.$$

From eqs. 13.34, 13.35, and 13.36, the model laws are

$$\left.\begin{aligned} &Q \propto ND^3 \\ &H \propto N^2D^2 \\ &P \propto wN^3D^5 \end{aligned}\right\} \quad 13.37$$

Even though these model laws were developed from relations expressing performance of an ideal turbine they apply to the actual turbine as long as all shock and friction losses are proportional to the flow rate squared. In the usual hydraulic turbine sizes and speeds and with water this proportionality holds. The power losses in bearings, seals, and runner clearances do not model in the same ratios as the energy transfer to the runner. Thus, efficiencies of larger units, or the same units at higher speeds are greater than than those of smaller models or of prototypes at slower speeds.