Iranian J. Sci. & Tech. Vol. 1, No. 2, September 1971

SIMILARITY OF VAPOR AND GAS BUBBLE GROWTH

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The bubble growth problems in both boiling and electrolysis are nondimensionalized in such a way as to make them identical. Nonuniformity of the initial temperature and the possibility of a measure of subcooling in the ambient liquid are considered. The nondimensionalization makes it possible to compare gas-bubble growth theory with vapor-bubble growth data. Such a comparison is made for the data of Perrais and Schrock.

INTRODUCTION

While boiling has been the subject of intensive study for twenty years owing to its importance in a host of compact heat transfer configurations, electrolysis has only more recently attracted the close attention of process engineers. One resason has been its importance in a variety of life support systems (see e.g. [1]). There is thus a large body of theoretical understanding of the component processes of boiling and somewhat less information related to gas producing electrolytic processes. This study is part of an ongoing project aimed at making use of the existing knowledge in either of these two areas to explain behavior in the other.

The present work will deal specifically with the similarity of the bubble growth problem in the two situations. We shall set the similarity up in a fairly general way and then apply it to the specific problem of a hemispherical bubble growing at a wall, the liquid above which may or may not be subcooled or undersaturated. The radius-time relationship in this case was predicted for vapor-bubbles by Skinner and Bankoff

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[2], and for gas-bubbles by Cheh and Tobias [3]. Cheh and Tobias adapted the work in [2] to the electrolysis bubble problem and they treated a different initial distributions of concentration in the liquid. Perrais and Schrock [4] observed the growth of vapor-bubbles on a wall under conditions corresponding with those described for gas bubbles in [3]. They compared their data with the theory in [2] although the initial temperature and concentration distributions did not really match. The comparison was not too good and there was necessarily some lingering doubt as to the extent to which the comparison was valid.

One of our present aims will be that of repeating this comparison using the electrolysis bubble theory, based on the correct initial conditions, to predict vapor-bubble growth.

The earliest vapor-bubble growth theories that took full account of the complex three-dimensional heat conduction problem around the bubble interface were those of Plesset and Zwick [5], and Forster and Zuber [6]. These investigators both found that after the very initial stages of growth, inertial and surface forces ceased to influence growth to any real extent, and that the heat transfer into the interface totally governed bubble growth. Since these investigators considered growth in an infinite superheated liquid their driving temperature difference was the difference between the ambient liquid temperature, $T_s + \Delta T$, and the saturation temperature at the interface, T_s.

The electrolysis counterpart to this problem--the growth of a bubble in a uniformly super-saturated liquid--was done by Epstein and Plesset [7]. Their work antedated [5] and [6] by four years and helped set the pattern for the subsequent vapor -bubble growth work.

Many other investigators have offered imporvements upon the approximations involved in [5] and [6]. One of the more notable was Scriven's [8] prediction of bubble growth when there is simultaneous heat and mass transfer. We shall avoid such complication in the present work since it would normally occur only in such a complex processes as boiling in a solution. Scriven's analysis can, however, be particularized to homogeneous bubble growth in a pure superheated liquid.

It would likewise be unnecessary for us to deal with more than two of the many works treating bubble growth at a wall--one [2] in boiling and the other [3] in electrolysis--since any others would involve the same dimensional considerations. We shall therefore proceed to set up a general non-dimensionalization and apply it to the cases we have discussed.

NON-DIMENSIONALIZATION OF BUBBLE-GROWTH EXPRESSIONS

The result of a bubble-growth theory is generally an expression relating the bubble radius, R, as a function of time, t, to the superheat, ΔT, or supersaturation, ΔC, as a parameter. We shall non-dimensionalize these variables as follows:

$$\mathcal{R} \equiv R/\ell, \qquad \tau \equiv \frac{\alpha t}{\ell^2} \text{ or } \frac{Dt}{\ell^2}$$

$$\text{Jakob number,} \quad J \equiv \frac{\rho_f c_f}{\rho_g h_{fg}} \Delta T \quad \text{or} \; \frac{\Delta C}{\rho_g} \tag{1}$$

where ℓ is a characteristic length. For a bubble growing in a non-uniform temperature field, ℓ would characterize the temperature field. Since there is no characteristic length for the asymptotic growth of a bubble in a uniform temperature field it is worthwhile to define a new dimensionless group to replace $\mathcal{R}$ and τ:

$$\Lambda \equiv R/\sqrt{(\alpha \text{ or } D)t} = \mathcal{R}/\sqrt{\tau} \tag{2}$$

Table 1 presents the predictions that we have discussed thus far for growth in a homogeneous temperature or concentration field, under these nondimensionalizations. Predictions by Cheh and Tobias are included in the table since they did the homogeneous problem as a special case. Scriven's prediction, even when it is restricted to a single component fluid, still takes account of sensible heat transfer which is ignored by the other authors. This refinement is ignored in Table 1 to facilitate comparisons. Both Scriven and Cheh-Tobias show that, as the fluid approaches saturation conditions, the growth equation changes form. Although for all other cases, Λ is proportional to the Jakob number, it varies as $\sqrt{J}$ near saturation, for either vapor or gas bubble growth.

The Buckingham Pi-Theorem tells us that for the 5 variables that define homogeneous bubble growth (R, t, ΔT, or ΔC, D or α, and $[\rho_g h_{fg}/\rho_f c_f]$ or ρ_g) in the 3 dimensions (length, time, temperature or concentration) there should be only two independent dimensionless groups. Thus $\Lambda = \Lambda\ (J)$ is the correct form for the bubble growth law. The important implication is that $R \sim \sqrt{t}$, is the only possible form for the bubble growth equation in a homogeneous medium.

The Skinner-Bankoff and Cheh-Tobias solutions both envision a wall with a time-dependent temperature or concentration gradient, $T = T_\infty + f(y,t)$ or $C = C_\infty + f(y,t)$, in the adjacent liquid as shown in Fig. 1. At some initial time, $t_i > 0$, a hemispherical bubble begins to grow on the wall. This bubble grows, carrying the gradient and feeding off it until it is depleted. Thereafter it begins to collapse unless T_∞ or C_∞ equals or exceeds

Table 1 Equations for Bubble Growth in a Homogeneous Medium

Source	Dimensional Equation	Dimensionless Equation $\mathcal{R}(J,\tau)$	Dimensionless Equation $\Lambda(J)$	Eq. No.
	vapor bubble growth			
Forster-Zuber [6]	$R_{FZ} = \rho_f c_f \sqrt{\pi\alpha t}\,\Delta T/\rho_g h_{fg}$	$\mathcal{R}_{FZ} = 1.772J\sqrt{\tau}$	$\Lambda_{FZ} = 1.772J$	(3)
Plesset-Zwick [5]	$R_{PZ} = 2\rho_f c_f \sqrt{3\alpha t}\,\Delta T/\sqrt{\pi}\rho_g h_{fg}$	$\mathcal{R}_{PZ} = 1.953J\sqrt{\tau}$	$\Lambda_{PZ} = 1.953J$	(4)
Scriven [8]	$R_S \simeq R_{PZ}$	$\mathcal{R}_S \cong 1.953J\sqrt{\tau}$	$\Lambda_S \sim 1.953J$	(5)
Scriven [8] for low ΔT	$R_{S/low} \simeq \sqrt{2\rho_f c_f \alpha_f t\,\Delta T/\rho_g h_{fg}}$	$\mathcal{R}_{S/low} \cong 1.414\sqrt{J\tau}$	$\Lambda_{S/low} \sim 1.414\sqrt{J}$	(6)
	gas bubble growth			
Epstein-Plesset [7]	$R_{EP} = 2\sqrt{Dt}\,\Delta C/\sqrt{\pi}\rho_g$	$\mathcal{R}_{EP} = 1.128\,J\sqrt{\tau}$	$\Lambda_{EP} = 1.128\,J$	(7)
Cheh-Tobias [3]	$R_{CT} = 2\sqrt{3Dt}\,\Delta C/\sqrt{\pi}\rho_g$	$\mathcal{R}_{CT} = 1.953\,J\sqrt{\tau}$	$\Lambda_{CT} = 1.953\,J$	(8)
Cheh-Tobias [3] for low ΔC	$R_{CT/low} = \sqrt{2Dt\,\Delta C/\rho_g}$	$\mathcal{R}_{CT/low} = 1.414\sqrt{J\tau}$	$\Lambda_{CT/low} = 1.414\sqrt{J}$	(9)

T_s or C_s. An additional dimensionless group, ω, must be defined to account for the subsaturation of the liquid in this case:

$$\omega \equiv \frac{T_w - T_\infty}{T_w - T_s} \quad \text{or} \quad \frac{C_w - C_\infty}{C_w - C_s} \tag{10}$$

Two new variables have now been added to the problem--T_∞ or C_∞ and ℓ--with no new dimensions. Accordingly there should be two more dimensionless groups in the problem and the bubble growth equation should now be of the form $\mathcal{R}=\mathcal{R}(\tau, J, \omega)$.

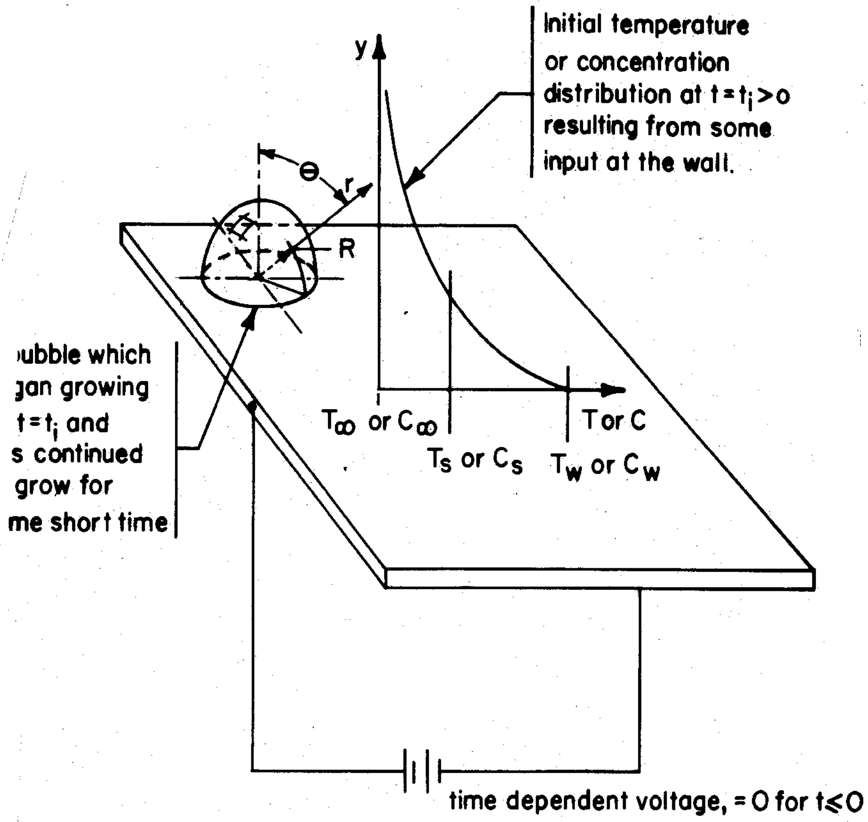

Fig. 1 Representation of an initial temperature or concentration distribution and the coordinate scheme.

THEORY FOR BUBBLE GROWTH AT A WALL

The Skinner-Bankoff and Cheh-Tobias solutions are, by and large, numerical ones since the problem of bubble growth at a wall is very complex. Skinner and Bankoff considered the convective terms in the energy equation and used a series method of solution. While Cheh included convection in a similar way in his thesis, Cheh and Tobias argued the terms were not important in the relatively slow gas-bubble growth problem and solved the simplified equations.

Vapor bubbles grow too rapidly to justify the neglect of inertia. Accordingly we shall have to use Cheh's more complete solution. Both Cheh and Skinner-Bankoff employed a perturbation method which implied that the domain of thermal influence around the bubble is thin. Accordingly the characteristic dimension, ℓ, does not really play a role in the problem, and there is one less dimensionless group in the growth equation. It turns out that the growth equation takes the form $\mathcal{R}(3J)^{1/3} = f(\omega, \tau[3J]^{4/3})$ under this assumption when the convective terms are retained, or $\mathcal{R}/J = f(\omega,\tau)$ when they are omitted.

Skinner and Bankoff considered the following initial distributions in the liquid:

1. linear distribution from wall to ambient conditions:

$$\left.\begin{aligned} T - T_\infty &= \Delta T(1 - y/\ell), \quad y/\ell \leqslant 1 \\ &= 0 \qquad\qquad\quad , y/\ell > 1 \end{aligned}\right\} \tag{11}$$

2. exponential distribution (the distance, ℓ, is the "period" in space of the exponential decay):

$$T - T_\infty = \Delta T \exp(-y/\ell) \tag{12}$$

3. modified exponential distribution (intended to approximate the situation during ongoing pool-boiling):

$$T - T_\infty = \Delta T \exp(-1/9[r/\ell]^6) \tag{13}$$

Of these, equation (13) was treated in the first Skinner-Bankoff paper as a spherically symmetric distribution. Equations (11) and (12) were treated in the second paper as azimuthally asymmetric or "axisymmetric". Thus equation (11) took the form $\Delta T(1-r\cos\theta/\ell)$, for example. The form $(T - T_\infty) = f(r, \theta, \ell)$ is, of course, far more realistic, since a hemispherical bubble growing in a unidimensional temperature field does not have a spherically symmetrical field around it.

Cheh and Tobias carried realism forward another step in their analysis by adding consideration of two situations that actually might give rise to bubble growth. Their initial distributions included:

1. uniform concentration (the results of this analysis were reported in equations (8) and (9), Table 1)

2. linear distribution (cf. equation (11))

$$C - C_\infty = \Delta C(1 - y/\ell), \quad y/\ell \leqslant 1$$
$$= 0 \qquad , y/\ell > 1 \tag{14}$$

3. exponential distribution (c.f. equation (12))

$$C - C_\infty = \Delta C \exp(-y/\ell) \tag{15}$$

4. constant potential distribution. If the concentration at the wall is set at C_w at time, $t = 0$, the well-known result for $C - C_\infty$ at $t = t_i$ is [9]:

$$C - C_\infty = \Delta C \operatorname{erfc}(y/2\sqrt{Dt_i}) \tag{16}$$

where $2\sqrt{Dt_i}$ can be regarded as a characteristic length.

5. constant current distribution. If the electric current at the wall is turned on at $t = 0$ and held constant we obtain from the constant heat flux solution [9]:

$$\frac{C - C_\infty}{\rho_g} = \frac{q_o}{D}\left[\frac{\exp(-y^2/\ell^2)}{\sqrt{\pi}} - y/\ell \, . \operatorname{erfc}\frac{y}{\ell}\right] \tag{17}$$

where ℓ is again $2\sqrt{Dt_i}$ and where q_o is the volume flux at the wall.

Cheh and Tobias' solution for these cases with the convection terms eliminated, is outlined in Table 2. This development shows how the dimensionless groups; $\mathcal{R}, \tau$, J, and ω evolve in the present problem. The general expression for the Jakob numbers (equations (25) and (26)), while they are more complicated, are analogous to those defined

Table 2 Analysis Leading to the Definition of Jacob Numbers for Bubble Growth in Nonuniform Potential Fields (Convective terms neglected)

MASS TRANSFER (Method of Cheh and Tobias)	HEAT TRANSFER (Vapor bubble growth)
1.) Start with the diffusion equation in spherical coordinates	1.) Start with the heat transfer equation in spherical coordinates
$\frac{\partial C}{\partial t} = \frac{D}{r^2} \frac{\partial}{\partial t}\left(r^2 \frac{\partial C}{\partial r}\right)$ (18)	$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$ (18)
2.) The boundary conditions for the spherically symmetric case are:	2.) The boundary conditions for the spherically symmetric case are:
a.) at $t = 0$, $r \geq 0$; $C = f(r)$ b.) at $r = R$, $t > 0$; $C = C_s$ (19)	a.) at $t = 0$, $r \geq 0$; $T = f(r)$ b.) at $r = R$, $t \geq 0$; $T = T_s$ (19)
c.) at $r = R$, $t > 0$; a mass balance gives	c.) at $r = R$, $t > 0$; a mass balance gives
$\rho_g \frac{dR}{dt} = D\left(\frac{\partial C}{\partial r}\right)_{r=R}$ (20)	$\rho_g \frac{dR}{dt} = \frac{k}{h_{fg}}\left(\frac{\partial T}{\partial r}\right)_{r=R}$ (20)

TABLE 2 (CONTINUED)

3.) Carslaw and Jaeger [9] give the solution to equation (18) as

$$C = \frac{1}{2r\sqrt{\pi D t}} \int_R^\infty r f(r') \left\{ \exp\left[-(r-r')^2/4Dt \right] - \exp\left[-(r+r'-2R)^2/4Dt \right] \right\} dr' + C_s \frac{R}{r} \operatorname{erfc} \frac{r-R}{2\sqrt{Dt}} \tag{21}$$

4.) Differentiating equation (21) w.r. to r, putting r = R and using equation (20) we get

$$\rho_g \frac{dR}{dt} = D\left[\frac{1}{2R\sqrt{\pi D t}} \int_R^\infty r' f(r') \left(\frac{r'-R}{Dt} \right) \times \exp\left\{ -(r'-R)^2/4Dt \right\} dr' - C_s \left(\frac{1}{R} + \frac{1}{\sqrt{\pi D t}} \right) \right] \tag{22}$$

5.) To non-dimensionalize equation (22) we need two additional definitions:

$$m \equiv \frac{r-R}{\ell} \quad \text{and} \quad g(m) \equiv \frac{f(m) - C_s}{f(0) - C_s} \tag{23}$$

3.) Carslaw and Jaeger [9] give the solution to equation (18) as

$$T = \frac{1}{2r\sqrt{\pi \alpha t}} \int_R^\infty r' f(r') \left\{ \exp\left[-(r-r')^2/4\alpha t \right] - \exp\left[-(r+r'-2R)^2/4\alpha t \right] \right\} dr' + T_s \frac{R}{r} \operatorname{erfc} \frac{r-R}{2\sqrt{\alpha t}} \tag{21}$$

4.) Differentiating equation (21) w.r. to r, putting r = R and using equation (20) we get

$$\rho_g \frac{dR}{dt} = \frac{k}{h_{fg}} \left[\frac{1}{2R\sqrt{\pi \alpha t}} \int_R^\infty r' f(r') \left(\frac{r'-R}{\alpha t} \right) \times \exp\left\{ -(r'-R^2)/4\alpha t \right\} dr' - T_s \left(\frac{1}{R} + \frac{1}{\sqrt{\pi \alpha t}} \right) \right] \tag{22}$$

5.) To non-dimensionalize equation (22) we need two additional definitions:

$$m \equiv \frac{r-R}{\ell} \quad \text{and} \quad g(m) \equiv \frac{f(m) - T_s}{f(0) - T_s} \tag{23}$$

TABLE 2 (CONTINUED)

Mass transfer	Heat transfer
6.) We finally get $$\frac{d\mathcal{R}}{d\tau} = \frac{J}{2\sqrt{\mathcal{R}\pi\tau^3}} \int_0^\infty \left[(m + \mathcal{R}) \times g(m)\, m \exp(-m^2/4\tau) \right] dm \qquad (24)$$	6.) We finally get $$\frac{d\mathcal{R}}{d\tau} = \frac{J}{2\sqrt{\mathcal{R}\pi\tau^3}} \int_0^\infty \left[(m + \mathcal{R}) \times g(m)\, m \exp(-m^2/4\tau) \right] dm \qquad (24)$$
7.) Where J, the Jacob number for mass transfer, is: $$J \equiv \frac{f(0) - C_s}{\rho_g} \qquad (25)$$	7.) Where J, the Jacob number for heat transfer, is: $$J \equiv \frac{\rho_f c_f \left[f(0) - T_s \right]}{h_{fg} \rho_g} \qquad (25)$$

The boundary conditions 2.) are given for spherical symmetry when in fact our major interest is in cases in which there is azimuthal asymmetry, and only axi-symmetry. In this case Cheh and Tobias obtained:

Mass transfer	Heat transfer
8.) $$J \equiv \frac{1}{\rho_g} \int_0^\pi \left[f(0, \theta) - C_s \right] \sin\theta\, d\theta \qquad (26)$$	8.) $$J \equiv \frac{\rho_f c_f}{\rho_g h_{fg}} \int_0^\pi \left[f(0, \theta) - T_s \right] \sin\theta\, d\theta \qquad (26)$$
9.) $$g(m) \equiv \frac{\int_0^\pi \left[f(m, \theta) - C_s \right] \sin\theta\, d\theta}{\int_0^\pi \left[f(0, \theta) - C_s \right] \sin\theta\, d\theta} \qquad (27)$$	9.) $$g(m) \equiv \frac{\int_0^\pi \left[f(m, \theta) - T_s \right] \sin\theta\, d\theta}{\int_0^\pi \left[f(0, \theta) - T_s \right] \sin\theta\, d\theta} \qquad (27)$$

where the symbol $f(0,\theta)$ is used to denote f for $m = 0$ and $R \to 0$.

in equation (1) for growth in a homogeneous fluid.

The very important fact that emerges from Tables 1 and 2 is that, under the non-dimensionalization

$$\mathcal{R} = \mathcal{R}(\tau, J, \omega) \tag{28}$$

which we anticipated earlier on the basis of the Pi-Theorem, the bubble growth problems in boiling and electrolysis are identical to one another. If the liquid is initially homogeneous then equation (28) reduces, as we have noted, to

$$\Lambda = \Lambda(J) \tag{29}$$

and if the thin thermal boundary layer assumption is used the complete problem reduces to

$$\mathcal{R}/(3J)^{1/3} = f(\omega, \tau(3J)^{4/3}) \tag{30}$$

APPLICATION TO DATA OF PERRAIS AND SCHROCK

Perrais and Schrock subjected a 1 mil platinum heater, comparable to that shown in Fig. 1, to two types of transient heat flux, they were:

1. step function:

$$q = 0, \qquad t < 0$$
$$q = \text{constant}, \qquad t \geqslant 0 \tag{31}$$

for which the temperature at time, t_i, is [10]

$$T - T_\infty = \frac{2q\sqrt{at_i}}{k}\left[\frac{\exp(-y^2/4at_i)}{\sqrt{\pi}} - \frac{y}{2\sqrt{at_i}} \operatorname{erfc} \frac{y}{2\sqrt{at_i}}\right] \tag{32}$$

or

$$\frac{(T - T_\infty)\rho_f c_f}{\rho_g h_{fg}} = \frac{q_o \ell}{a}\left[\frac{\exp(-y^2/\ell^2)}{\sqrt{\pi}} - \frac{y}{\ell} \operatorname{erfc} \frac{y}{\ell}\right] \tag{32a}$$

which is identical in form to equation (17). Equation (32) is valid only if the heater material does not have sufficient thermal capacity to borrow heavily from the heat transfer to the liquid. We shall restrict our examples to cases in which the heater had low thermal capacity, and in which the initial temperature distribution, $T - T_{\infty}$, is matched by equation (32) within 4%

2. exponential function:
$$\left.\begin{array}{ll} q = 0 & t < 0 \\ q = \text{constant.} \exp(t/t_o), & t \geqslant 0 \end{array}\right\} \qquad (33)$$

Of these two cases, the first corresponds with Cheh and Tobias' 5th case for which the theory is complete. We have therefore selected 7 bubbles, representing a wide conditions for which Perrais and Schrock provide measurements, and plotted $\mathcal{R}$ data for these bubbles against dimensionless time in Fig. 2. We have then obtained J and ω for the boiling data and included Cheh's prediction for these values in the figure. The prediction sketched in the figure is the one which includes the convective terms.

The evaluation of ω simply follows equation (10) and presents no problem. The Jacob number computed from equation (26) is the same as that given by equation (1).

The seven cases are arranged into two groups of comparable Jacob numbers and increasing ω. We note that in Figures 2a to 2d, J is on the order of 30, and as ω increases from 1.21 to 1.92 the accuracy of the prediction decreases. Figures 2e to 2g correspond with high Jacob numbers (on the order of a hundred) and ω's that range from 1.20 to 2.44. For the higher Jacob numbers the prediction deteriorates, and with increasing ω it also becomes quite poor.

While the predictions are more accurate at low J and ω, they fail rather badly in prediction the onset of collapse. Conversely, while the high J and ω predictions are less accurate, they generally more accurately predict the time at which the radius maximizes.

The comparison between theory and data is thus not very satisfactory. However, this comparison is not, in itself, the important matter. The comparison is actually some improvement over that which Perrais and Schrock obtained when they compared their data with Skinner and Bankoff's vapor bubble growth theory for an exponential initial temperature distribution.

To provide good comparison with data, a theory would have to account for influences that no theorist has been able to describe very completely. The major one is the influence of a microlayer of liquid under the growing bubble. Possibly the presence of a no-slip condition on the liquid at the meridional plane (y = 0) also influences the results. While the former effect has been widely documented in the literature, we can really only conjecture that the no-slip condition induces some sort of secondary vorticity that hastens bubble collapse in certain cases.

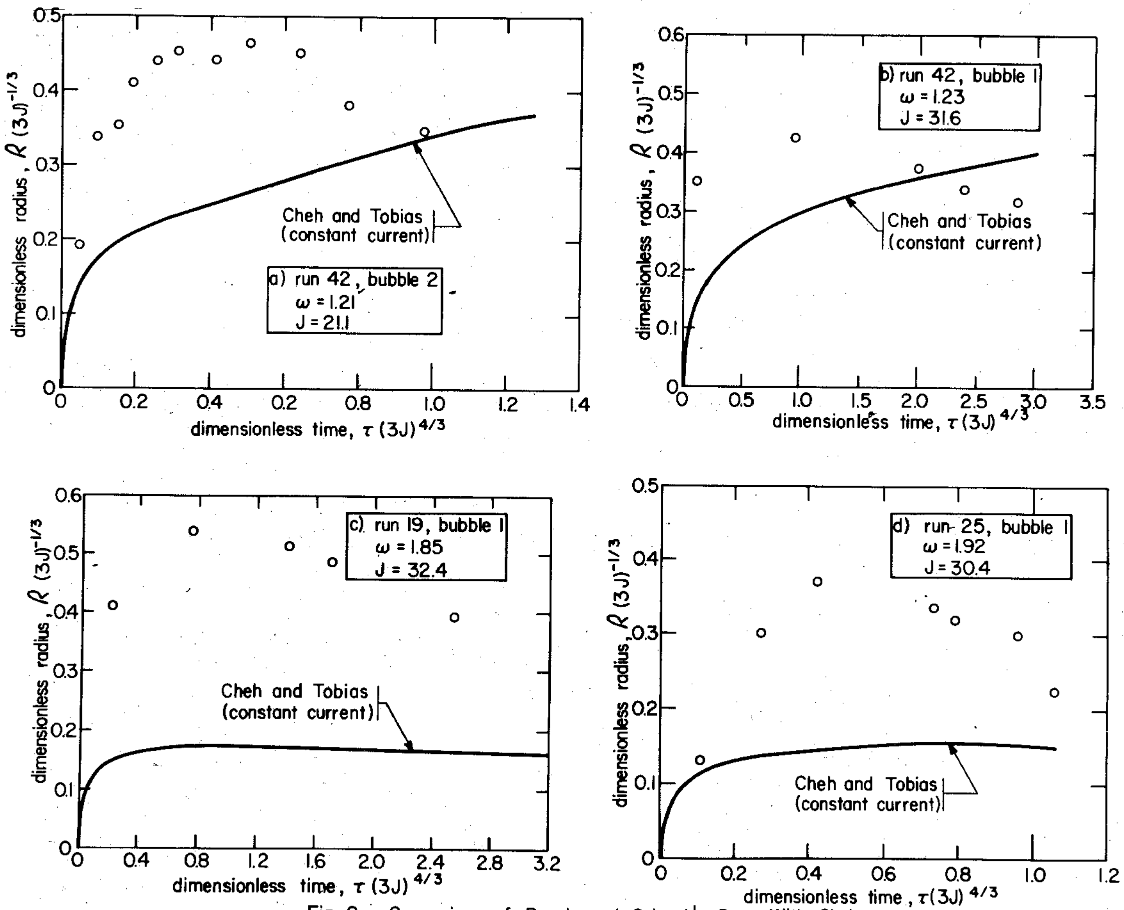

Fig. 2 Comparison of Perrais and Schrock's Data With Cheh and Tobias' Theory.

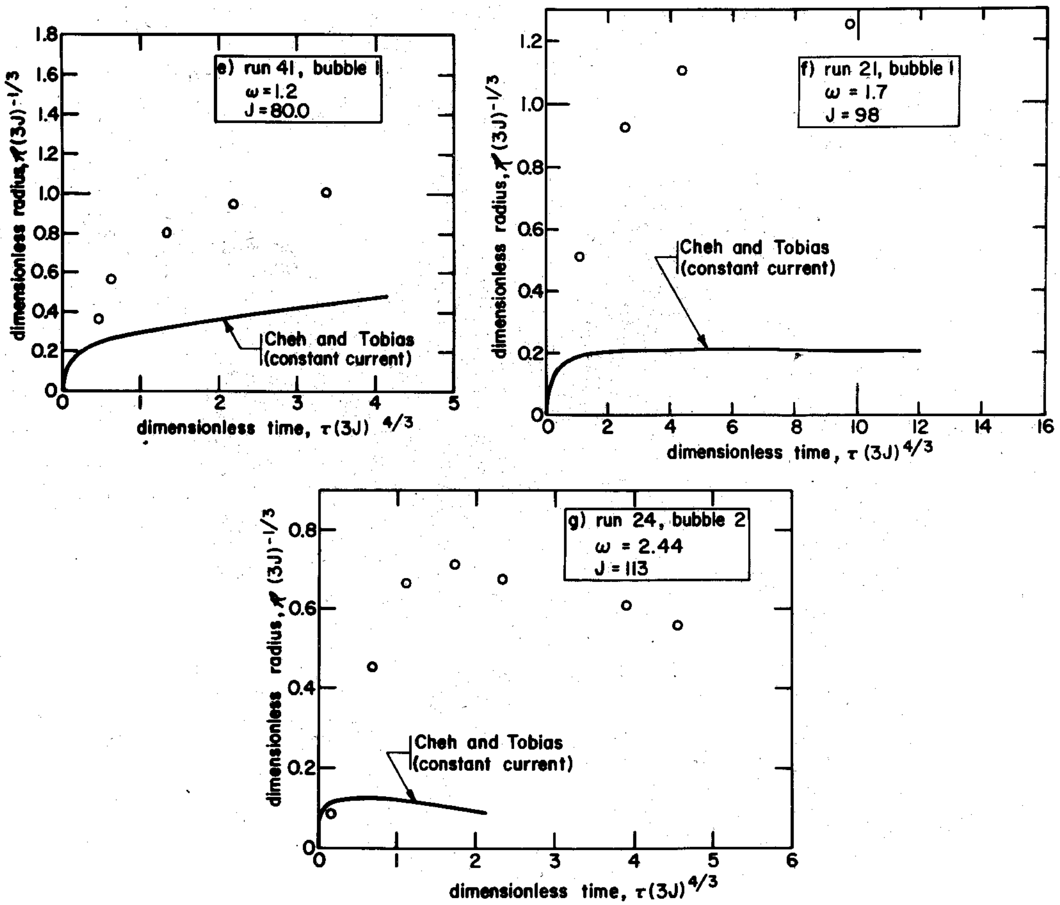
e) run 41, bubble 1
ω = 1.2
J = 80.0
Cheh and Tobias
(constant current)
dimensionless radius, $r(3J)^{-1/3}$
dimensionless time, $\tau(3J)^{4/3}$
f) run 21, bubble 1
ω = 1.7
J = 98
Cheh and Tobias
(constant current)
dimensionless radius, $r(3J)^{-1/3}$
dimensionless time, $\tau(3J)^{4/3}$
g) run 24, bubble 2
ω = 2.44
J = 113
Cheh and Tobias
(constant current)
dimensionless radius, $r(3J)^{-1/3}$
dimensionless time, $\tau(3J)^{4/3}$

Fig. 2 Cont.

Hospeti and Mesler [11] recently showed that, for the growth of hemispherical vapor bubbles during boiling on a heavy metal plate, heat transfer through the microlayer was the major contributor to growth. We have computed the energy stored in Perrais and Schrock's one mil heater and found it more than sufficient to account for the entire growth of the bubble the only limitation being the resistance of the intervering micro-layer itself. Thus it appears that for vapor bubble growth on a wall the Skinner-Bankoff and Cheh-Tobias models are simply inadequate.

CONCLUSIONS

1. Both vapor and gas bubble growth can be characterized by the same functional equation

$$\mathcal{R} = \mathcal{R}(\tau, J, \omega)$$

In the limiting case of bubble growth in a homogeneous medium this reduces to:

$$\Lambda = \Lambda(J)$$

and, for practical purposes, the Jakob number can be absorbed into $\mathcal{R}$ and τ :

$$\mathcal{R}/(3J)^{1/3} \simeq f(\omega, \tau(3J)^{4/3})$$

2. The electrolysis bubble growth theory of Cheh and Tobias and the vapor bubble growth experiments of Perrais and Schrock can accurately be compared with one another.

3. The theory of Cheh and Tobias is based upon simplifications which significantly limit its applicability to vapor bubble growth at a wall. This is undoubtedly true of the Skinner-Bankoff theory as well. It is possible that either theory might still be used to describe gas bubble growth at a wall.

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NOMENCLATURE

C	concentration of a dissolved gas
C_s	concentration of a dissolved gas at saturation
c	specific heat
D	mass diffusivity of a dissolved gas in a liquid
$f(m)$	initial temperature or concentration distribution, see Table 2
$g(m)$	dimensionless initial temperature or concentration, see Table 2
J	$\rho_f c_f (T_w - T_s)/\rho_g h_{fg}$, the Jacob number for heat transfer, or $(C_w - C_s)/\rho_g$, the Jacob number for mass transfer, in spherically symmetric geometries. Defined by equations (26) for axisymetric geometries.
k	thermal conductivity of liquid
ℓ	length characterizing the temperature or concentration distribution
m	a dimensionless radial coordinate, $(r - R)/\ell$
q	heat flux
q_o	constant rate of formation of gas (equal to the heat flux divided by $\rho_g h_{fg}$ or

	$\rho_f c_f \Delta T/J$ for a vapor bubble).
R	radius of a bubble
r	radial coordinate
$\mathcal{R}$	a dimensionless bubble radius, R/ℓ
T	temperature
T_s	temperature at saturation
t	time
t_o	period of a transient pulse
y	distance normal to the wall
α	thermal diffusivity of the liquid
ΔT	wall superheat, $T_w - T_s$, for inhomogeneous growth. Liquid superheat, $T_\infty - T_s$, for homogeneous growth
ΔC	wall supersaturation, $C_w - C_s$, for inhomogeneous growth. Liquid supersaturation, $C_\infty - C_s$, for homogeneous growth.
θ	angle between the axis and any other line passing through the center of the bubble
Λ	$\mathcal{R}/\sqrt{\tau}$
ρ	density
τ	a dimensionless time or "Fourier number", $D\tau/\ell^2$ or $\alpha\tau/\ell^2$
ω	a dimensionless temperature or concentration, defined by equations (10)

General Subscripts

CT	refers to work done by Cheh and Tobias
EP	refers to work done by Epstein and Plesset
FZ	refers to work done by Forster and Zuber
f	refers to properties in the liquid phase
g	refers to properties in the gaseous phase or saturated vapor phase
i	refers to conditions at inception of a bubble
PZ	refers to work done by Plesset and Zwick
S	refers to work done by Scriven
w	refers to quantities at the wall
∞	refers to quantities in the liquid bulk
/low	refers to behavior in the low ΔC or ΔT limit

The manuscript of the paper was submitted on April 1, 1971