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Capillary Action in Small Jets Impinging on Liquid Surfaces¹

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A Wave Phenomenon

THIS note will report and explain a curious wave phenomenon which occurs when a small jet impinges upon a liquid surface.

Fig. 1 shows the apparatus used to observe this phenomenon. The bottom of a plastic standpipe is a thin brass sheet with a carefully drilled, 0.026-in.-dia hole in the center. An overflowing martini glass provides a smooth and visually accessible surface for the water jet to strike. The jet speed, U , as it meets the surface, is computed from Bernoulli's equation using an appropriate velocity coefficient.³

Fig. 2 shows photographs at various values of the Weber number, We , of the flow, where $We = \rho U^2 D / \sigma$. The photograph at the highest discharge rate reveals fine horizontal striations. As the flow decreases the striations broaden into a well-defined wave pattern. As We became small these waves were seen to become so pronounced that they sometimes nipped off droplets and prevented them from merging with the surface. When this occurred, the droplets frequently rolled across the surface without sticking to it.

Analysis

The observed waves, although they "stand" in the reference frame of the observer, are actually originating at the point of collision and traveling upstream at a speed, U , relative to the jet. Rayleigh⁴ described the behavior of such waves on liquid cylinders in 1878 and 1879. He found that the frequency, ω , of the waves can be expressed as:

$$\omega^2 = \frac{I_1(kD/2)}{I_0(kD/2)} \left[\frac{\sigma}{\rho} k^3 - \frac{4\sigma k}{\rho D^2} \right] \quad (1)$$

Since the wave speed, ω/k , must equal the jet speed, U , we get immediately from equation (1):

$$We = \frac{I_1(kD/2)}{I_0(kD/2)} [kD - 4/kD] \quad (2)$$

This reduces to:

$$kD \approx We \text{ or } \lambda \approx 2\pi D / We \quad (3)$$

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⁴ See, e.g., Lamb, H., *Hydrodynamics*, 6th ed., Dover, New York, 1945, Articles 273 and 274.

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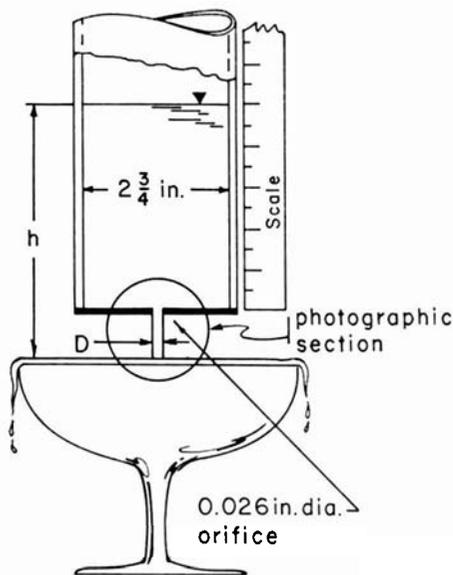


Fig. 1 Experimental apparatus

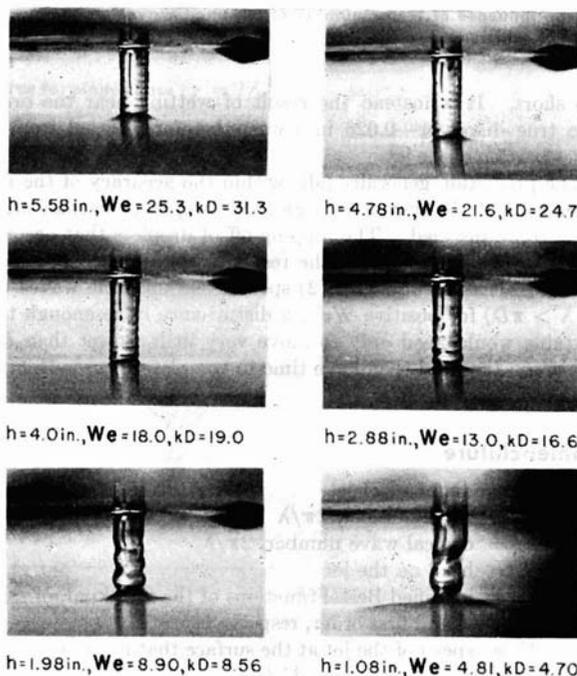


Fig. 2 Photographic results showing "standing" waves on water jets. The width of each picture is one fourth of a full-scale in.

for large kD (or small wavelengths).

Equation (1) also specifies the longest stable wave that can exist in a jet. This is the longest wave (or smallest k) for which ω is still real. It is obtained by setting $\omega = 0$ and solving equation (1) for the "critical" values of λ_c and k_c :

$$k_c D = 2 \text{ and } \lambda_c / D = \pi \quad (4)$$

Comparison of Analysis With Observations

Fig. 3 compares observed wave numbers, obtained from Fig. 2, with equation (2) and the asymptotic result, equation (3). The reader will note that the jets are a little wider at the top than at the liquid surface. This is not a gravity effect since the jets are

A Note on the Saint-Venant Theory of Failure

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SAINT-VENANT's theory of failure, or the maximum normal strain theory, is based upon the assumption that failure occurs at a point P under any loading when the maximum normal strain at P equals plus or minus the yield strain in a tensile test. The theory is of little use, except for materials with a high Poisson's ratio ν , such as plastics. This Note shows that serious error will result in use of this theory in its usual form for plane stress, unless it is modified for materials with $\nu > 1/3$.

In the mechanics of materials texts, the plane-stress yield curve for this theory is made-up of four lines, which result from setting the principal strains equal to plus and minus the axial test yield strains. In terms of stress, these lines have the equations

$$\frac{\sigma_1}{\sigma_{ys}} - \nu \frac{\sigma_2}{\sigma_{ys}} = \pm 1 \quad (1)$$

and

$$\frac{\sigma_2}{\sigma_{ys}} - \nu \frac{\sigma_1}{\sigma_{ys}} = \pm 1 \quad (2)$$

In (1) and (2), σ_1 and σ_2 are the principal stresses at P , and σ_{ys} is the yield stress in an axial test of the material. Plots of (1) and (2) give the usual yield curve for this theory, shown in Fig. 1.

If the normal strain in the perpendicular direction is also set equal to plus and minus the yield strain σ_{ys}/E (E being the elastic modulus) in an axial test, the pair of resulting equations may be written as

$$\frac{\sigma_1}{\sigma_{ys}} + \frac{\sigma_2}{\sigma_{ys}} = \mp \frac{1}{\nu} \quad (3)$$

Equations (3) form a family of straight lines with slopes of minus one, which intersect points A and B in Fig. 1 when $\nu = 1/3$. For $\nu < 1/3$, these lines lie outside the "diamond," and thus do not affect the yield criterion. However, when ν is such that $1/3 < \nu \leq 1/2$, the yield surface shape in Fig. 1 is in error. The lines of (3) require a more conservative yield curve, to prevent failure by normal straining in the transverse direction.

Two such modified curves are shown in Fig. 2. It is seen that condition (3) requires exclusion of a considerable portion of the usually depicted shape. The Saint-Venant theory curve now passes through the point (1,1) when $\nu = 0.5$, as do all other common theories of failure. It is noted that for this value of ν (which represents an incompressible material and which is used for plastics), the theory without the present modification can lead to far too liberal biaxial loadings.

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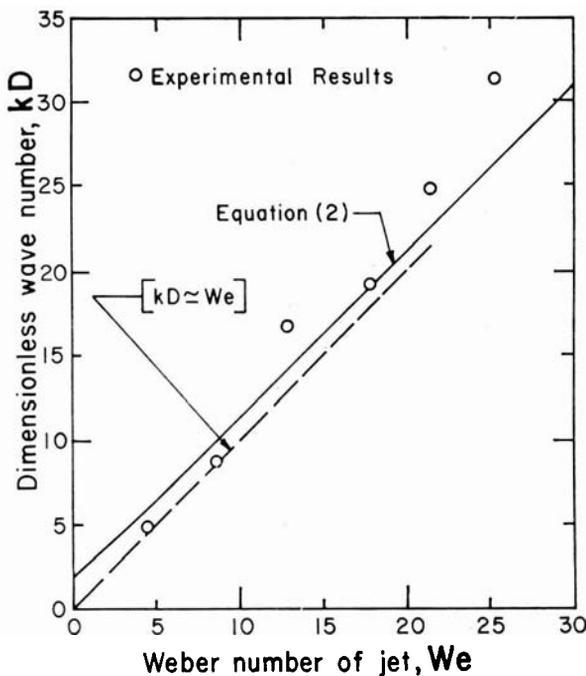


Fig. 3 Influence of Weber number of jet, upon waves emanating from point of collision

too short. It is instead the result of wetting near the orifice. The true diameter—0.026 in.—was, therefore, scaled from the lower portion of the jet.

The prediction generally falls within the accuracy of the measurements of the wavelength and the proposed mechanism appears to be justified. The nipping-off of droplets that occurs at very low We 's is probably the result of the growth of unstable waves. Although equation (2) specifies no unstable waves (i.e., no $\lambda > \pi D$) for positive We 's, a disturbance large enough to be unstable would need only to move very little slower than U at low We . It would then have time to complete its growth at low We but not at high We .

Nomenclature

- D = diameter of jet
- k = wave number, $2\pi/\lambda$
- k_c = critical wave number, $2\pi/\lambda_c$
- h = head on the jet
- I_0 and I_1 = modified Bessel functions of the first kind, of zeroth and first order, respectively
- U = speed of the jet at the surface that it strikes
- We = Weber number, $\rho U^2 D / \sigma$
- λ = wavelength of disturbance
- λ_c = critical, or longest stable, wavelength
- σ = surface tension
- ρ = density of liquid
- ω = frequency of disturbance