JOHN H. LIENHARD
Professor of Mechanical Engineering, University of Kentucky, Lexington, Ky. Mem. ASME

JAMES B. DAY
Aerospace Engineer, USAF, Wright-Patterson AFB, Ohio; Formerly, Graduate Student, University of Kentucky, Lexington, Ky.

The Breakup of Superheated Liquid Jets

Observed average breakup lengths are presented for free orifice jets of superheated water and liquid nitrogen, and subcooled water. Dimensionless, semiempirical expressions are developed for both flashing, and aerodynamic and/or capillary, breakup, and verified with data. The distribution function for breakup length is predicted for the superheated case with the help of Boltzmann statistics.

Introduction

THE BREAKUP of liquid jets has been under fairly continuous scrutiny since Rayleigh [1][1] first explained the mechanism of capillary instability in 1878. These inquiries received considerable impetus forty years ago from attempts to improve diesel injection systems (see, e.g., [2]), and during the 1950's, by work with rocket injection systems (see, e.g., [3] and [4]). Summaries of work done on breakup as a result of capillary and aerodynamic instability are given by Huang [5] and by Grant and Middleman [6].

More recently, interest has turned toward another kind of jet breakup: the explosive flashing that results from the thermomechanical instability of a jet of highly superheated liquid. Brown and York [7] and Lienhard [8] described the spray forming capabilities of such jets, and Lienhard and Stephenson [9] gave a restrictive correlation of breakup lengths as a function of superheat. Flashing results in a very fine spray that is potentially useful in a wide variety of aerosol forming processes.

The aim of the present study is that of showing how to predict the breakup length of a given superheated jet, and its variability, under fairly general circumstances. This will require that we determine whether or not capillary or aerodynamic instability will give rise to breakup before superheat does, in any situation. Therefore, we shall begin by considering what has been done toward predicting the breakup of a "cold" jet.

Jet Breakup in the Absence of Superheat

In 1909, Niels Bohr [10] extended Rayleigh's analysis to include viscous effects, in a prize winning paper on the evaluation of surface tension, and Weber [11] went on to obtain the breakup length, L_b, for a viscous jet in 1931. His expression was

$$\frac{L_b}{D\sqrt{\mathrm{We}}} = \ln\frac{D}{2\delta}\left(1 + \frac{3\sqrt{\mathrm{We}}}{\mathrm{Re}}\right) \tag{1}$$

where D is the diameter of the jet, and We and Re are the Weber and Reynolds numbers. The symbol, δ, denotes the initial disturbance in the jet. For most cases of practical importance $\mathrm{Re} \gg \sqrt{\mathrm{We}}$ and equation (1) reduces to

$$\frac{L_b}{D\sqrt{\mathrm{We}}} \simeq \ln\frac{D}{2\delta} \tag{1a}$$

The term $\ln(D/2\delta)$ depends upon the initial disturbances, and these in turn are unknown. However, for most cases of jet efflux this term proves to be about 12 ± 1. This corresponds with a variability of a factor of $e^{\pm 1}$ in the initial disturbance.

When the velocity of efflux is high, aerodynamic forces override capillary forces and the breakup length begins to decrease with increasing velocity. The jet now breaks up by the growth of sinuous antisymmetric waves instead of the symmetric varicose waves that distinguish capillary breakup. Miesse [12] found that he could correlate data for many fluids in diesel injector nozzles operating in this range, using

$$\frac{L_b}{D\sqrt{\mathrm{We}}} = 540\ \mathrm{Re}^{-5/8} \tag{2}$$

This expression is restrictive in a variety of ways. It applies to jets in which there is considerable turbulence, and experience shows (see, e.g., [6]) that L_b decreases less rapidly with Re as turbulence increases. It can even begin to rise again with Re, at very large Re, when the jet is turbulent. Equation (2) also applies only for velocities above the transition point from capillary to aerodynamic breakup. Finally, it is limited to a single surrounding air density. Dumbrowski and Hooper [13] have shown that decreasing the air pressure around a water bell[2] stabilizes it, and vice versa.

If we consider that L_b depends upon the velocity, V, the liquid viscosity, μ, the densities of the liquid and of the surrounding air, ρ_f and ρ_a, the surface tension, σ, and D; then the Buckingham pi-theorem shows that four dimensionless groups are needed to characterize the process. Thus the general form of equation (2) would be

$$\frac{L_b}{D} = F(\mathrm{We}, \mathrm{Re}, \rho_a/\rho_f) \tag{3}$$

In the present study, we shall assume that L_b is always approximately proportional to $\sqrt{\mathrm{We}}$, as both equations (1a) and (2)

[1] Numbers in brackets designate References at end of paper.

Contributed by the Fluids Engineering Division and presented at the Winter Annual Meeting, Los Angeles, Calif., November 16–20, 1969, of THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS. Manuscript received at ASME Headquarters, July 31, 1969. Paper No. 69-WA/FE-19.

[2] A "water bell" is the spreading liquid sheet leaving the point of collision of two opposing coaxial jets. Its aerodynamic behavior was shown by Huang [5] to be strongly analogous to that of a jet.

ndicate it to be, and we shall work with only one value of ρ_a/ρ_f. This will be $\rho_a/\rho_f \simeq 0.0012$, which corresponds with any liquid whose specific gravity is close to unity, discharging into a standard atmosphere. We shall therefore attempt to correlate data for sharp-edged orifices using

$$\frac{L_b}{D\sqrt{\text{We}}} = F(\text{Re}) \tag{4}$$

over the entire range of efflux conditions.

Once data have been obtained to form this correlation, only a part of the breakup problem will have been completed. This breakup length will only apply if the jet does not first break up as a result of flashing. Our second objective will then be to predict the flashing breakup length and its variability.

Jet Breakup Under the Influence of Superheat

The Delay Time. We shall now redevelop some ideas from reference [9] in such a way as to provide necessary background and facilitate the subsequent development. The *delay time*, t_d, between the efflux of a jet and its breakup, will be used here instead of the breakup length because most of the prior work has been done in terms of time. The change is unimportant since $t_d = L_b/V$. The delay time is composed of two components: an idle time, t_{d1}, during which an unstable bubble nucleus in the fluid "dwells" before it begins rapid growth, and a time, t_{d2}, for rapid growth of the bubble up to the size at which it will fracture the jet.

The calculation of both these components of the delay time will make use of Dergarabedian's [14] bubble growth equation:

$$r\ddot{r} + \frac{3}{2}\dot{r}^2 - \frac{r-1}{r} + F(\text{physical properties, time}) = 0 \tag{5}$$

where r is a dimensionless form of the bubble radius, R,

$$r \equiv R/R_0 \tag{6}$$

and the independent time variable, τ, is

$$\tau \equiv t\left[\frac{(p_v - p_{\text{amb}})^3}{4\rho_f\sigma^2}\right]^{1/2} \tag{7}$$

The function, F, was ignored in Dergarabedian's original formulation, but added later by Forster and Zuber [15] and Plesset and Zwick [16] to account for the role of heat conduction in causing the bubble to grow.

These latter studies showed that after the bubble grows an order of magnitude beyond its unstable equilibrium radius, the inertia terms, $r\ddot{r} + (3/2)\dot{r}^2$, cease to be important. The asymptotic solution of the remaining equation applies through almost the entire growth of the bubble. This solution was given in [15] in terms of the specific heat, c_p, the latent heat, h_{fg}, the superheat, ΔT, the saturated liquid and vapor densities, ρ_f and ρ_g, and the thermal diffusivity, α, as

$$R = \left(\frac{c_p\Delta T}{h_{fg}}\right)\left(\frac{\rho_f}{\rho_g}\right)\sqrt{\pi\alpha t} \tag{8}$$

Photographic evidence indicates that a jet shatters when a bubble grows to about $R = D$. Therefore, we can approximate t_{d2} as

$$t_{d2} \simeq \frac{D^2}{\pi\alpha}\left(\frac{h_{fg}}{c_p\Delta T}\right)^2\left(\frac{\rho_g}{\rho_f}\right)^2 \tag{9}$$

The longer component of t_d is usually t_{d1}—the idle or dwell time.[3] To characterize this, let us consider the solution of equation (5) for small r. References [15] and [16] show that the function, F, can be neglected in this range, and reference [14] shows that equation (5), with the initial conditions, $r(0) = 1 + \epsilon$ and $\dot{r}(0) = 0$, admits the solution

$$\tau = \int_{1+\epsilon}^{r}\left(\frac{1}{3r^3} + \frac{2}{3} - \frac{1}{r}\right)^{-1/2} dr \tag{10}$$

This result is plotted in Fig. 1 for an initial perturbation, ϵ, equal to 0.01. Here we see that, depending upon the magnitude of ϵ, the bubble might grow very slowly indeed for a long time, before it picks up speed.

Reference [9] provided the basis upon which we wish to build a correlation for t_{d1}. There it was argued that the discharging liquid might contain "weak spots" that could be triggered into

[3] This is especially true in water. For this reason, t_{d2} was simply neglected in reference [9].

Nomenclature

A = cross sectional area of jet, $A = \pi D^2/4$
c_p = specific heat at constant pressure
D = diameter of jet—fully contracted
F = any unspecified function
Fo = Fourier number, $\text{Fo} \equiv \alpha t/D^2$
$f(x)$ = distribution function of x
ΔG = potential barrier to bubble nucleation, $\Delta G = \frac{4}{3}\pi\sigma R_0^2$
g_i = "degeneracy" of a random event
h_{fg} = latent heat of vaporization
J_0, J_1 = Bessel functions of the first kind of order zero and one
L_b = breakup length of a jet, $L_b \equiv Vt_d$
M_n = the roots of J_0
m = unspecified exponent, equation (27)
N = total number of breakup events
N_i = number of breakup events between t_{i-1} and t_i
Pr = Prandtl number, $\text{Pr} \equiv \mu/\rho_f\alpha$
p_{amb} = ambient pressure
p_v = vapor pressure at $T = T_0$
Δp = $p_v - p_{\text{amb}}$, the extent of superheat expressed in terms of pressure
R = bubble radius
Re = $\rho_f VD/\mu$
R_0 = equilibrium bubble nucleus radius, $R_0 = 2\sigma/\Delta p$
r = R/R_0
r_1 = perturbation radius, $r_1 \equiv r - 1$
T = temperature
$T_{\text{c.l.}}$ = temperature on the axis of a jet
T_0 = temperature of superheated jet at efflux
ΔT = the superheat, $\Delta T \equiv T_0 - T_{\text{sat}}$
t = time, or any random variable in the context of equation (24)
t_c = characteristic dwell time of a single bubble, $t_c < t_{d1}$
t_d = delay time, $t_d = t_{d1} + t_{d2}$
t_{d1} = dwell time required for a bubble to commence rapid growth after jet efflux
t_{d2} = time required for a bubble which has begun rapid growth to shatter the jet
$\overline{t_{d1}}$ = average dwell time
t_i = a particular value of the random variable, t
$t_{rm\beta}$ = the root mean βth moment of the distribution of events in t, defined by equation (26).
We = Weber number, $\text{We} = \rho_f V^2 D/\sigma$
y = radial coordinate, $y = 0$ on jet centerline
α = thermal diffusivity
β = number specifying a moment of a distribution. See equation (26)
δ = initial disturbance in a jet interface
ϵ = initial displacement of r from unity
μ = liquid viscosity
ρ_a = density of air surrounding jet
ρ_f = density of saturated liquid, $\rho_f \simeq$ density of jet at any temperature
ρ_g = density of saturated vapor
σ = surface tension between a saturated liquid and its vapor
τ = dimensionless time. See equation (7)
τ_c = τ for $t = t_c$
Φ = dimensionless dwell time. See equation (16)
ψ = dimensionless superheat. See equation (17)

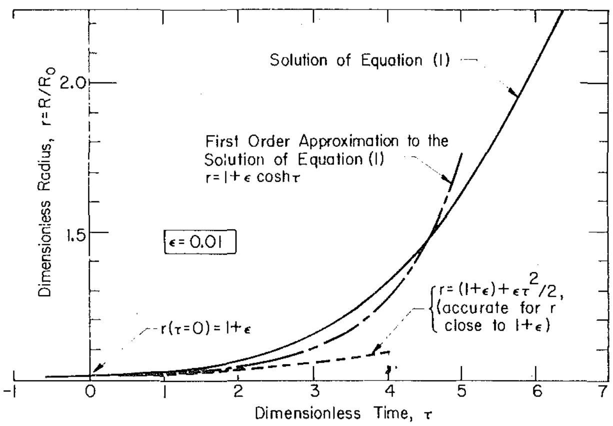


Fig. 1 Early growth of a vapor bubble when $r(\tau = 0) = 1.01$

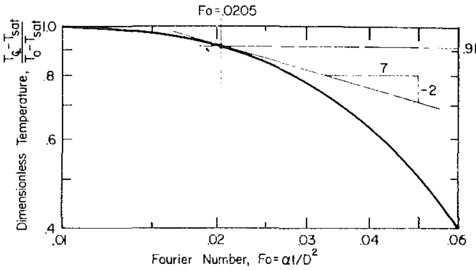


Fig. 2 Variation of jet centerline temperature with Fourier number

unstable nuclei by whatever "noise" might exist in the environment. The most effective source of noise is, in turn, that generated by other flashing bubbles. The energy required to trigger a nucleus in the liquid is Frenkel's [17] "potential barrier" to nucleation, ΔG. The potential barrier is the free energy of a bubble with respect to the surrounding liquid, and Frenkel found

$$\Delta G = \tfrac{4}{3}\pi\sigma R_0^2 \tag{11}$$

where R_0 is the radius of an unstable equilibrium nucleus. It is given by the force balance on a bubble as

$$R_0 = \frac{2\sigma}{p_v - p_{\text{amb}}} \tag{12}$$

where p_v is the vapor pressure corresponding with the temperature of the superheated liquid, and p_{amb} is the ambient pressure.

But, once a nucleus is formed, it must survive existing background noise for a characteristic time, t_c, before it has sufficient size for its growth to "run away." From equation (7),

$$t_c = \tau_c(\epsilon)\left[\frac{4\rho_f\sigma^2}{(p_v - p_{\text{amb}})^3}\right]^{1/2} \tag{13}$$

Fig. 1 indicates that for an initial disturbance, $\epsilon = 0.01$, τ_c would be about 2 or 3.

Without reproducing the formulation of the probability argument in reference [9], we can indicate how it went: The probability that there is a nucleus that will survive until rapid growth begins is proportional to jet area, A, inverse ΔG, and inverse t_c. The delay time in turn should be inversely proportional to the probability of survival. Thus we can write for the average, $\overline{t_{d1}}$, of t_{d1}:

$$\overline{t_{d1}} \sim \Delta G t_c/A \tag{14}$$

or, after substitution of equations (11), (12), and (13),

$$\overline{t_{d1}} \sim 1/A(p_v - p_{\text{amb}})^{7/2} \tag{15}$$

This dimensional result was verified with data obtained in flashing water jets. The data exhibited wide variability and emphasized that equation (15) gives the average of what is, in actuality, a broad distribution of t_{d1}. We shall therefore show: first, how to generalize equation (15) into an expression that will work for liquids other than water; and second, how to predict the distribution of t_{d1} about this average.

Dwell Time Correlation. The preceding discussion shows that $\overline{t_{d1}}$ should depend upon ρ_f, D, σ, and $(p_v - p_{\text{amb}})$. These comprise five variables in three dimensions—time, length, and force. The Buckingham Pi-Theorem shows that two dimensionless groups are needed to fully characterize the phenomenon. We shall choose these to give a dimensionless dwell time, Φ:

$$\Phi \equiv \frac{\Delta p^{5/2} D}{\sigma^2\sqrt{\rho_f}}\,\overline{t_{d1}} \tag{16}$$

and a dimensionless superheat, or dimensionless jet diameter,

$$\psi \equiv D\Delta p/\sigma \tag{17}$$

Using these expressions to eliminate t_{d1} and Δp from equation (15), and noting that $A = \frac{\pi}{4}D^2$, we obtain

$$\Phi\psi = \text{const} \tag{18}$$

Once the constant in equation (18) has been determined experimentally, the expression should predict the mean breakup length for any superheated liquid.

One tacit assumption has been made throughout these considerations, namely, that $\overline{t_{d1}}$ will be sufficiently short that cooling of the jet will be unimportant. Actually, the interface of the jet will assume the saturation temperature, T_{sat}, corresponding with p_{amb} as soon as it is formed. The validity of the assumption can be checked by solving the heat conduction equation. If the Péclet number, $VD/2\alpha$, is high (10 or more) axial conduction should be negligible and the problem becomes

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial T}{\partial y}\right); \quad \left\{\begin{array}{l} T(D/2, t) = T_{\text{sat}} \\ T(y, 0) = T_0 - T_{\text{sat}} \\ \left.\dfrac{\partial T}{\partial y}\right|_{y=0} = 0 \end{array}\right\} \tag{19}$$

where y is the radius of the jet and T_0 is the temperature of the emerging superheated liquid. The solution (see, e.g., [18]) is

$$\frac{T - T_{\text{sat}}}{T_0 - T_{\text{sat}}} = 2\sum_{n=1}^{\infty}\frac{J_0(M_n 2y/D)}{M_n J_1(M_n)}\exp\left(-4M_n^2\,\text{Fo}\right) \tag{20}$$

where the M_n's are roots of the Bessel function of the first kind of zeroth order, J_0, and the Fourier number, Fo, is

$$\text{Fo} \equiv \alpha t/D^2 \tag{21}$$

Fig. 2 shows the relation between the centerline temperature, $T_{c.l.}$, and Fo, computed from equation (20). One point on this curve is of great interest to us: Equation (15) shows that Δp is proportional to $t^{-2/7}$, but the Clausius-Clapeyron equation shows that, to a first approximation, Δp is proportional to $T - T_{\text{sat}}$. Therefore, if $T - T_{\text{sat}}$ decreases more rapidly than $t^{-2/7}$ or $\text{Fo}^{-2/7}$, the delay time will be increasing faster than time is passing. This point is reached at Fo = 0.0205 (see Fig. 2).

Thus cooling will protect the jet from flashing if the following criterion is met:

$$\overline{t_{d1}} \geq 0.0205\,D^2/\alpha \tag{22}$$

Nondimensionalization of this criterion with the help of equations (16) and (17) gives

$$\Phi \geq 0.0205\,\frac{\text{Pr Re}}{\sqrt{\text{We}}}\,\psi^{5/2} \tag{23}$$

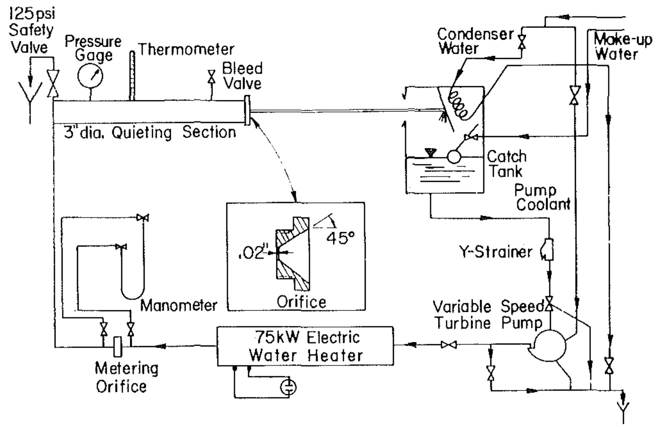


Fig. 3 Schematic diagram of water loop

where Pr is Prandtl number, $\mu\alpha/\rho_f$.

Equation (23) is a conservative criterion because considerable cooling has occurred before it is reached. It has been shown by Day [19] that the superheat energy has been reduced by about 32 percent, even though $T - T_{\mathrm{sat}}$ has dropped only 9 percent, at this point. However, since the breakup length is a random variable, equation (23) gives a *probable* limit and not an absolute limit. Thus cooling "hedges" against any such random occurrences of flashing as might occur in apparent violation of equation (23).

The Variability of the Dwell Time. It was shown by Lienhard and Meyer [20] that the generalized gamma distribution function, which contains most of the common distribution functions as special cases, can be obtained by the methods of statistical mechanics. The function is

$$t_{rm\beta}f(t) = \left[\frac{\beta}{\Gamma(m/\beta)}\left(\frac{m}{\beta}\right)^{m/\beta}\right]\left(\frac{t}{t_{rm\beta}}\right)^{m-1} \times \exp\left[-\frac{m}{\beta}\left(\frac{t}{t_{rm\beta}}\right)^{\beta}\right] \quad (24)$$

where the constants are explainable in terms of the constraints on the distribution. These are: (*a*) conservation of the events or elements that are being distributed—the dwell times in this case. If there are N_1 of event t_1, N_2 of event t_2, and N_i of event t_i, then the total number of events, N, is given as

$$\sum_{n=1}^{\infty} N_i = N \quad (25)$$

(*b*) A βth moment of the distribution is known. If $\beta = 1$, then t_{rm1} is a simple mean, $\bar{t}$. If $\beta = 2$, then t_{rm2} is the root-mean-square moment, t_{rms}, etc. Thus

$$\sum_{n=1}^{\infty} N_i t_i^{\beta} = (t_{rm\beta})^{\beta} N \quad (26)$$

(*c*) The degeneracy, g_i, or number of ways in which the event can occur at the *i*th level, is of the form

$$g_i \sim t_i^{m-1} \quad (27)$$

where m is a constant. Neither m nor β need be integral but both must be positive.

Equation (24) is the basis upon which it is possible to predict the variability of the dwell time, t_{d1}. The appropriate moment of the distribution is the simple mean given by equation (18); thus $\beta = 1$. The specification of m requires that we first consider the very early growth of a bubble. We can find this by solving for a small perturbation, $r_1(\tau)$, around the equilibrium radius, $r = 1$.

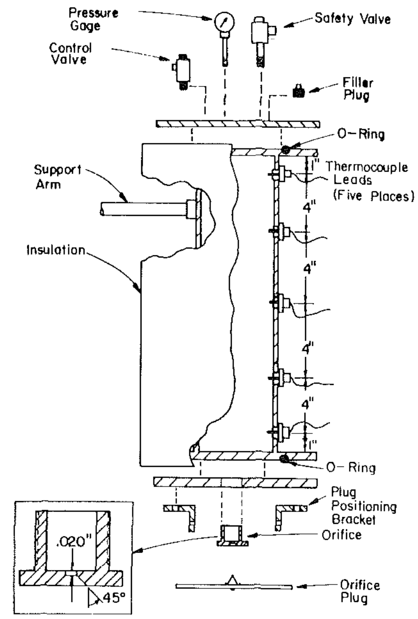


Fig. 4 Schematic diagram of blowdown tank

The substitution of $r = 1 + r_1$, $r_1 \ll 1$, in equation (5) with $F = 0$, gives

$$\ddot{r}_1 - r_1 = 0; \quad r_1(0) = \epsilon, \ \dot{r}_1(0) = 0 \quad (28)$$

after the elimination of terms on the order of r_1^2 or less. The solution of equation (28) is $r_1 = \epsilon \cosh \tau$, so

$$r = 1 + \epsilon \cosh \tau; \quad r - 1 \ll 1 \quad (29)$$

For very small τ, this can be approximated as

$$r = (1 + \epsilon) + \epsilon\tau^2/2; \quad r - 1 \ll 1, \quad \tau \ll 1 \quad (30)$$

These approximations are included in Fig. 1 to show how they compare with equation (10).

Equation (30) shows that as time passes an initial perturbation, ϵ, will slowly grow to a new effective perturbation $\epsilon(1 + \tau^2/2)$. This suggests that the likelihood of survival of a nucleus will generally increase as τ^2, so we shall assume

$$g_i \sim t_i^{m-1} = t_i^2$$

Thus the distribution function for t_{d1} should be equation (24) with $\beta = 1$ and $m = 3$, or

$$\overline{t_{d1}} f(t_{d1}) = \tfrac{27}{2}(t_{d1}/\overline{t_{d1}})^2 \exp(-3t_{d1}/\overline{t_{d1}}) \quad (31)$$

In the course of this discussion we have offered five predictive expressions, all of which require experimental verification or completion. These are: the correlation equation (4), equation (9) which anticipated that a jet will shatter when $R \simeq D$, equation (18) which requires an experimental constant, the criterion (23), and equation (31). In the next section we shall report data which will serve these ends and which will overlap the aerodynamic and flashing breakup regimes.

Experiments

Two kinds of apparatus have been developed for this study. One is the hot water loop shown in Fig. 3, the other is the liquid nitrogen blowdown apparatus shown in Fig. 4. These apparatus and our experiments are fully described in reference [19] and we shall only briefly describe the experiments here.

The hot water loop was used to deliver both cold and super-

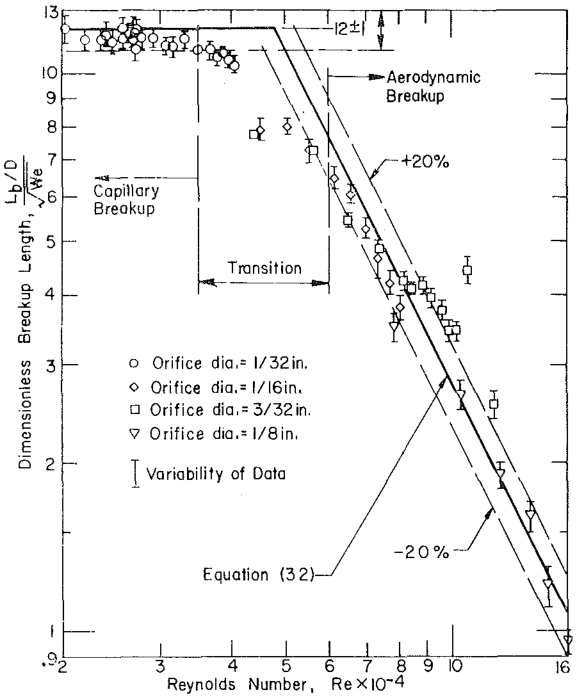


Fig. 5 Variation of dimensionless cold-water breakup length with Reynolds number

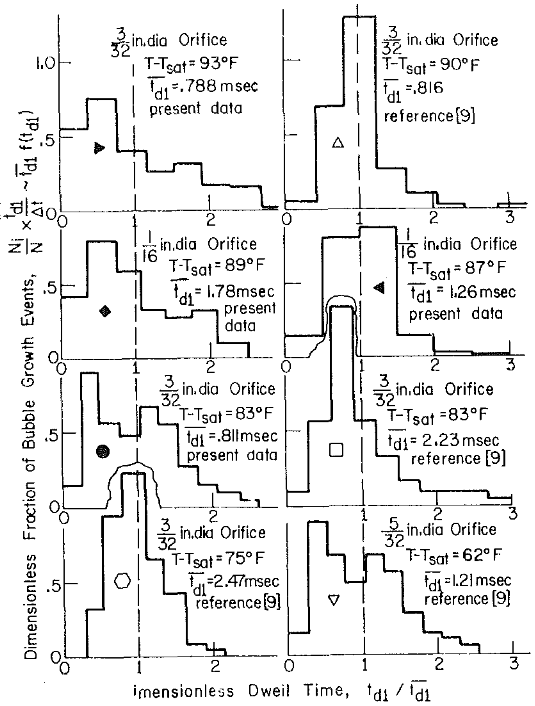


Fig. 6 Dwell time histograms for water. The symbols key to Fig. 7.

heated water to $^1/_{32}$, $^1/_{16}$, $^3/_{32}$, and $^1/_8$-in-dia sharp-edged orifices. The breakup lengths of cold jets, and their variability, were measured with the help of a strobe light, and the character of breakup was investigated with still photographs.

The superheated jets exhibited far greater variability in L_b, and far greater violence in breakup. They were photographed with a Hycam motion picture camera, from a distance, at about 8000 frames/sec. The jet velocity, V, was computed on the basis of the upstream gage pressure and a velocity coefficient. Care was taken to insure that the flow was substantially subcooled upstream during the superheated jet experiments.

The liquid nitrogen blowdown tests could only be done with superheated liquid since there was no way to bring the nitrogen below its saturation temperature in the pressure vessel. Both still and high-speed motion picture photography was used to observe jet breakup. Little could be done with strobe light observation because blowdown only lasted on the order of a minute. Blowdown was initiated by knocking away a spring-loaded brass plug, after taking care that no temperature stratification existed in the tank, and that the liquid was saturated. The pressure remained constant within the accuracy of the gage for almost the entire blowdown period. This is consistent with Swanson's [21] equation for the back-pressure, based on the conservation of energy, and with Swanson's observations during the blowdown of water in a similar apparatus.

The diameter, D, of the jet was computed by multiplying the orifice diameter by the square root of the coefficient of contraction. This in turn was obtained from a summary of coefficient of contraction data given by Huang [22] for small orifices. Data will be identified here on the basis of the orifice diameter, but all computations will be based upon the actual diameter, D, of the jet.

Results and Discussion

Cold Water Breakup. The observed breakup lengths of cold water jets are presented in dimensionless form in Fig. 5. The original data are tabulated in [19] and the nondimensionalization is that suggested by equation (4). All but one stray point fall within about ± 20 percent of

$$\frac{L_b}{D\sqrt{\mathrm{We}}} = 2.75 \times 10^{10}\,\mathrm{Re}^{-2} \qquad (32)$$

in the range of aerodynamic breakup. There is a transition region in the neighborhood of $\mathrm{Re} = 48{,}000$ (for $\rho_a/\rho_f \simeq 0.0012$); and below $\mathrm{Re} = 35{,}000$ equation (1a) is satisfied. In the present case in $(D/2\delta) \simeq 11.5 \pm .5$ which is typical of such data.

Equation (32) reveals a far stronger decrease of L_b with Re than did previous data for injectors and turbulent tube discharge.

The Dwell Time for Superheated Jets. A total of eleven motion picture records with useful dwell time data are available to us. Six movies of water jets were made and interpreted by Stephenson [23], and reported in [9]. One was made by Lienhard using Stephenson's water loop which resembled our own and is presented here as part of the present data.[4] Three were made with our water loop and one with our blowndown tank (see ref. [19]).

Of these records, eight provided enough bubble growth events —100 or more—to make a reasonable histogram. The histograms and other characteristics of these runs are presented in Fig. 6. The remaining three film records—one liquid nitrogen run, and two water runs from reference [9]—gave

Nitrogen, $^1/_{16}$ in-dia-orifice, $\Delta T = 14$ F, $\overline{t_{d1}} = 0.32$ msec

Water, $^3/_{32}$ in-dia-orifice, $\Delta T = 67$ F, $\overline{t_{d1}} = 5.17$ msec

Water, $^3/_{32}$ in-dia-orifice, $\Delta T = 60$ F, $\overline{t_{d1}} = 4.09$ msec

The histograms in Fig. 6 have been normalized to an area of unity to facilitate comparison with equation (31). This comparison is made in Fig. 7, where center points of all the histogram blocks are plotted together with the equation. The data scatter very consistently about the prediction with the exception of three points. These correspond with spikes in the histogram which result from the following phenomenon which we call "standing breakup": At certain preferred locations repeated flashing will occur as "weak spots"[5] flow into the disturbances

[4] Data from this film are designated with a black circle in Figs. 6 and 7.

[5] "Weak spot" is a term that has been used in the cavitation literature to identify a nucleation site, or imperfection, in the liquid that will be susceptible to nucleation.

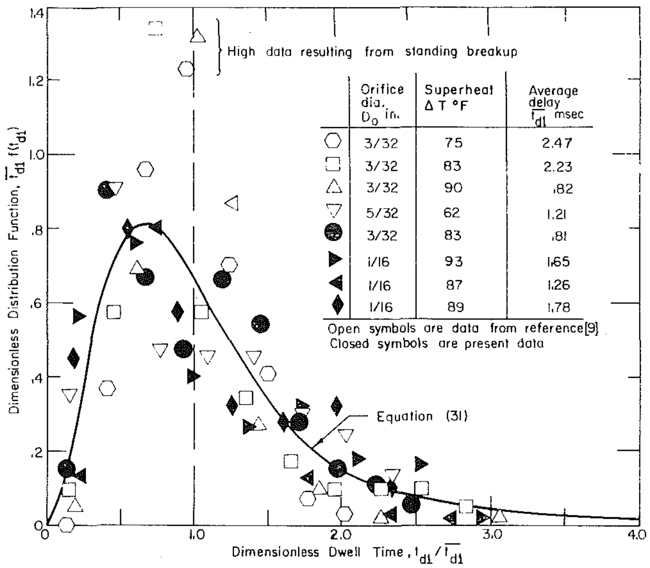


Fig. 7 Comparison of predicted and observed distributions of dwell times

created by the preceding bubble. Standing breakup is very near the mean t_{d1} when it occurs.

Equation (31) lies in the middle of the histogram points. The points scatter broadly because there are only 100 to 300 events on a single 100 ft reel of film exposed at 8000 frames per sec, in the cases reported. Presumably the $\overline{t_{d1}}$ data would scatter less if plenty of events were available at any condition. Equation (31) is therefore borne out in our results.

The eleven dwell times have been nondimensionalized in accordance with equations (16) and (17), and plotted in Fig. 8 on Φ versus ψ^{-1} coordinates. Since Φ must approach zero as $\frac{1}{\psi}$ approaches zero, the origin is also a legitimate point. The least-squares-fit straight line through these points is given by

$$\Phi\psi = 2.12 \times 10^{13} \tag{18a}$$

The correlation coefficient for these data is 0.762 which indicates acceptable correlation.

The data are repeated on ln Φ versus ln ψ coordinates and the protective cooling limit as given by equation (23) is also included. Here we see that the cooling time equals the dwell time at a time that exceeds the observed dwell time in every instance. Thus the cooling criterion is not violated in any instance. Equation (23) explains why Brown and York suggested that breakup would not occur before a certain fairly high superheat was obtained, and why Lienhard and Stephenson found no breakup in water jets below about 260 F. In these cases, $T - T_{sat}$ was low enough that t_{d1} approached this criterion.

Some Qualitative Results of the Photographic Observations. Fig. 10 shows four typical photographs from the present study and one from reference [9]. In one case a rectangular 1-in. marker is visible just below the jet. These photographs illustrate several of the phenomena we have been discussing.

Fig. 10(a) shows a condition of capillary breakup in a water jet. Fig. 10(b) shows an example of a very symmetrical bubble in the process of flashing in flashing in a water jet. This is about as large as bubbles ever grew during flashing and it is about four times the diameter of the jet. With any asymmetry in the location of the bubble, bursting would occur at a smaller diameter than this. The typical bubble would grow to about twice the jet diameter, or to $R = D$. Thus the assumption that bursting occurs at $R = D$ was a reasonable one to use in equation (9).

Fig. 10(c) shows what appears to be aerodynamic breakup in a liquid nitrogen jet. However, substitution of the parameters in the figure into equation (32) yields $L_b = 37.6$ in.—much longer than breakup actually required in this case. What then actually caused the jet to break up? Possibly moisture condensed onto the lip of the orifice, roughening it. We believe that it is more plausible that some bubble growth was taking place, even though no individual bubbles can be clearly identified. The physical properties of nitrogen are such that equation (8) predicts a much slower bubble growth rate than for water. Therefore, bubbles failed to perforate the nitrogen jets as they did the water jets. The wide uncertainty on the one nitrogen data point in Fig. 8 stems from the fact that the first appearance of bubbles had to be identified during this kind of "shredding" breakup—a situation that was unavoidable with our apparatus. The average flashing breakup length was only about 1.6 in this case. Fig 10(d) shows a liquid nitrogen jet at a higher superheat than in Fig. 10(c). Here there are obvious examples of flashing which acts to augment breakup. Fig. 10(e) is a picture of the flashing of a highly superheated water jet in the complete absence of either capillary or aerodynamic breakup (from reference [9]). The delay time here is shorter, by virtue of both higher superheat and larger cross-sectional area, than in Fig. 10(b). Several simultaneous flashing events are evident in Fig. 10(d) while only one occurs in Fig. 10(b).

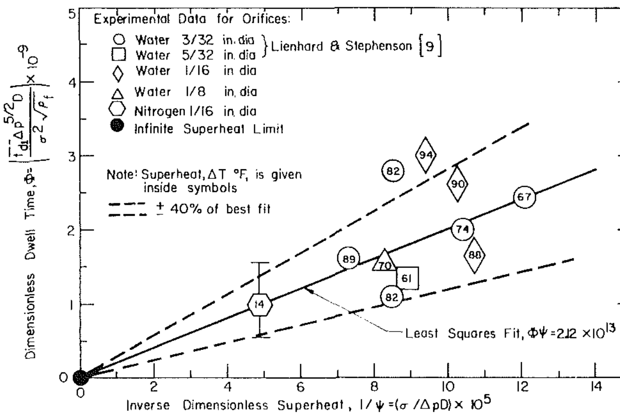


Fig. 8 Variation of dwell time with inverse superheat

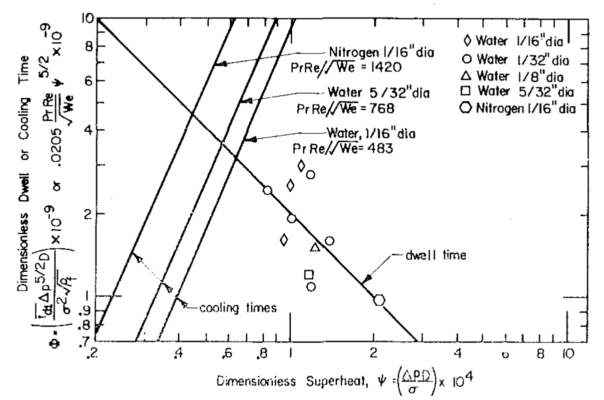
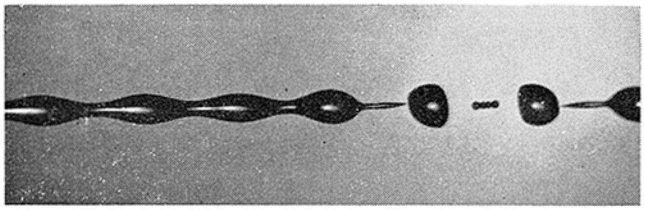


Fig. 9 Variation of dimensionless dwell time and dimensionless cooling time with superheat

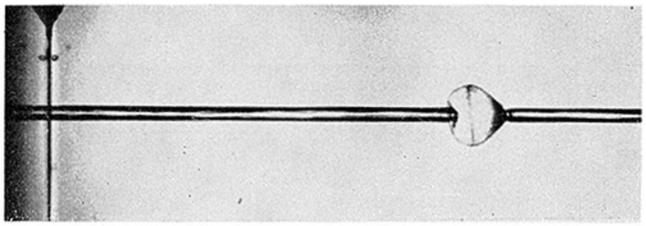
Summary

If flashing does not occur first, in a jet of superheated liquid leaving a sharp-edged orifice, the jet will either break up as a result of aerodynamic instability, in which case L_b will be

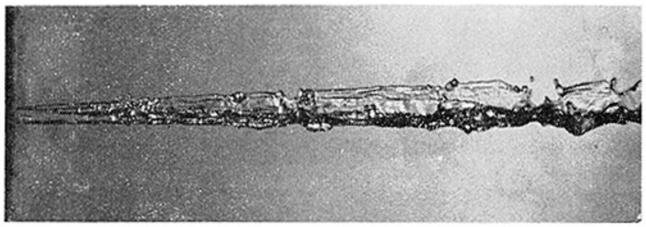
$$L_b = 2.75 \times 10^{10}\, D\, \sqrt{\mathrm{We}}/\mathrm{Re}^2; \qquad \mathrm{Re} > 48{,}000 \tag{32a}$$



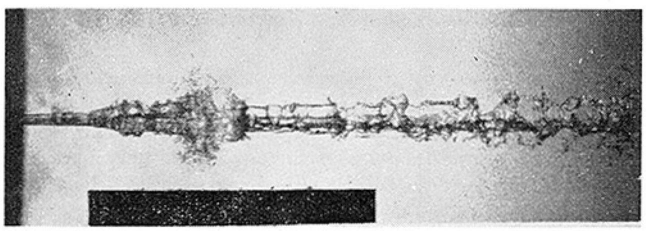
(a) 0.051-in-dia cold water jet; $V = 24$ fps; $Re = 3820$; $We = 845$



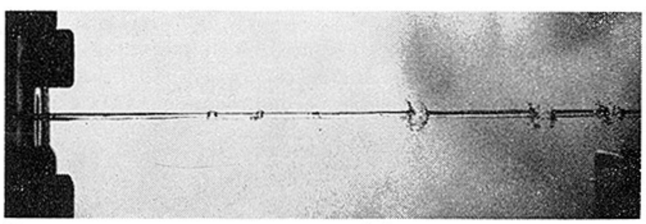
(b) 0.051-in-dia superheated water jet; $V = 108$ fps; $\Delta T = 70$ F; $Re = 170{,}000$; $We = 20{,}300$



(c) 0.051-in-dia superheated liquid nitrogen jet; $\Delta T = 6$ F; $V = 29.4$ fps; $Re = 63{,}300$; $We = 12{,}000$



(d) 0.051-in-dia superheated liquid nitrogen jet; $\Delta T = 10$ F; $V = 37.7$ fps; $Re = 81{,}900$; $We = 21{,}200$



(e) 0.079-in-dia superheated water jet [9]; $\Delta T = 79$ F; $V = 106$ fps; $Re = 272{,}000$; $We = 45{,}000$

Fig. 10 Examples of jet breakup under a variety of conditions

or it will break up as a result of capillary instability in which case

$$L_b = 11.5D\sqrt{\mathrm{We}}; \qquad \mathrm{Re} < 48{,}000 \tag{33}$$

with relatively little variability in either case. These results are restricted to $\rho_a/\rho_f \simeq 0.0012$ and they will give slightly high values in the transition range $35{,}000 < \mathrm{Re} < 60{,}000$.

Flashing will not occur at all if it has not occurred after a distance of $0.0205\ VD^2/\alpha$, or if

$$\Phi \geq 0.0205\,\frac{\mathrm{Pr\,Re}}{\sqrt{\mathrm{We}}}\,\psi^{5/2} \tag{23}$$

Flashing will occur in an average distance given by

$$L_b = V(\overline{t_{d1}} + t_{d2}) \tag{34}$$

where $\overline{t_{d1}}$ is obtainable from equation (18*a*), and t_{d2} is obtainable from equation (9). Actually L_b is a random variable. The extent of its variability is specified by the variability of t_{d1} and this in turn is given by equation (31).

In the nitrogen jets $t_{d2} > t_{d1}$; thus early nucleation was followed by slow bubble growth. This combination (possibly helped by other systematic complications) made breakup very hard to describe in this case.

References

1 Lord Rayleigh, "On the Instability of Jets," *Proc. London Math. Soc.*, Vol. 10, 1878, p. 7.

2 Schweitzer, P. H., "Mechanism of Disintegration of Liquid Jets," *Jour. App. Phys.*, Vol. 8, 1937, pp. 513–521.

3 Heidmann, M. F., Priem, R. J., and Humphrey, J. C., "A Study of Sprays Formed by Two Impinging Jets," NACA TN 3835, 1959.

4 Ingebo, R. D., "Drop-size Distributions for Impinging Jet Breakup in Airstreams Simulating the Velocity Conditions in Rocket Combustors," NACA TN 4222, 1958.

5 Huang, C. P., "Dynamics of Free Axisymmetric Liquid Sheets," College of Engineering Bulletin 306, Washington State University, Pullman, Aug. 1967.

6 Grant, R. P., and Middleman, S., "Newtonian Jet Stability," *AIChE Jour.*, Vol. 12, 1966, pp. 669–678.

7 Brown, R., and York, J. L., "Sprays Formed by Flashing Liquid Jets," *AIChE Jour.*, Vol. 8, 1962, p. 149.

8 Lienhard, J. H., "An Influence of Superheat Upon the Spray Configuration of Superheated Liquid Jets," Journal of Basic Engineering, Trans. ASME, Vol. 88, Series D, 1966, pp. 685–687.

9 Lienhard, J. H., and Stephenson, J. M., "Temperature and Scale Effects Upon Cavitation and Flashing in Free and Submerged Jets," Journal of Basic Engineering, Trans. ASME, Vol. 88, Series D, 1966, pp. 525–532.

10 Bohr, N., "Determination of the Surface-Tension of Water by Method of Jet Vibration," *Phil. Trans. Roy. Soc. London*, Series A, Vol. 209, 1909, p. 281.

11 Weber, C., "Zum Zerfall eines Flüssigkeitsstrahles," *Zeit. Ang. Math. Mech. (ZAMM)*, Vol. 2, 1931, pp. 136–154.

12 Miesse, C. C., "Correlation of Experimental Data on the Disintegration of Liquid Jets," *Ind. and Engr. Chem.*, Vol. 47, 1955, pp. 1690–1701.

13 Dumbrowski, N., and Hooper, P. C., "The Effect of Ambient Density on Drop Formation in Sprays," *Chem. Engr. Sci.*, Vol. 17, 1962, pp. 291–305.

14 Dergarabedian, P., "The Rate of Growth of Vapor Bubbles in Superheated Water," *Journal of Applied Mechanics*, Vol. 20, Trans. ASME, Vol. 75, 1953, p. 537.

15 Forster, H. K., and Zuber, N., "The Growth of a Vapor Bubble in a Superheated Liquid," *Jour. Applied Physics*, Vol. 25, 1954, p. 474.

16 Plesset, M. S., and Zwick, S. A., "The Growth of Vapor Bubbles in Superheated Liquids," *Jour. Applied Physics*, Vol. 25, 1954, p. 493.

17 Frenkel, J., *Kinetic Theory of Liquids*, Dover Publications, New York, N. Y., 1955, p. 374.

18 Carslaw, H. S., and Jaeger, J. C., *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, 1959, Section 7.6.

19 Day, J. B., "Combined Effects of Superheat and Dynamical Instability on the Breakup of Liquid Jets," MS thesis, University of Kentucky, June 1969.

20 Lienhard, J. H., and Meyer, P. L., "A Physical Basis for the Generalized Gamma Distribution," *Quar. App. Math.*, Vol. 25, 1967, pp. 330–334.

21 Swanson, G. T., "Unsteady Flow from a Pressurized Vessel," MS thesis, Washington State University, Pullman, 1967.

22 Huang, C. P., "The Behavior of Liquid Sheets Formed by the Collison of Jets," College of Engineering Bulletin No. 291, Washington State University, 1965.

23 Stephenson, M. J., "A Study of Cavitating and Flashing Flows," Washington State University, Institute of Technology Bulletin No. 290, 1965.