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Laminar Film Condensation on Plane and Axisymmetric Bodies in Nonuniform Gravity[1]

Expressions are developed for the condensate film thickness and the local Nusselt number on arbitrary axisymmetric bodies, including vertical plates and cylinders. The expressions are the same as the Rohsenow-Nusselt expressions except that they are based on an "effective gravity" that corrects both for variable gravity and for the form of the body. The limitations on the expressions are: that radii of curvature greatly exceed the film thickness, that Prandtl numbers are never much less than unity, and that the ratio of sensible to latent heats is not large. These criteria include almost all practical situations. Several applications are developed.

Introduction

Contemporary schemes for augmenting heat transfer frequently call for condensation in nonuniform body force fields, or on various surfaces whose slopes change in a uniform gravity field. This is true, for example, in space applications, in certain heat pipe configurations, and for many bodies at earth-normal gravity. Our own interest in this problem was initially motivated by the need to design a reflux condenser for a centrifuge boiling experiment.

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The problem of laminar film condensation on a vertical plate in a constant gravity field was solved by Nusselt in 1916 [1][2] subject to two major assumptions. The first—that the temperature profile in the film is linear—was corrected by Rohsenow in 1956 [2]. Both this and the second assumption—that the inertia terms in the momentum equation could be neglected—were delimited by Sparrow and Gregg in 1959 [3]. The very complete treatment in [3] verified Rohsenow's modification of Nusselt's result[3]

$$\mathrm{Nu} = 0.707\left[g\,\frac{\rho_f(\rho_f-\rho_g)h_{fg}'x^3}{\mu k\Delta T}\right]^{1/4} \qquad (1)$$

for all Prandtl numbers on the order of unity or greater and for

[2] Numbers in brackets designate References at end of paper.

[3] Symbols not explained in context are defined in the Nomenclature section.

Nomenclature

$A = \mu k\Delta T/(\rho_f-\rho_g)\rho_f h_{fg}'$

C = arbitrary constant

c_p = specific heat of condensate

D = diameter of a horizontal cylinder or sphere

g = gravitational acceleration

g_{eff} = an effective g for nse in Nusselt's equations, defined by equation (10)

h = heat transfer coefficient

h_{fg} = latent heat of vaporization

$h_{fg}' = h_{fg}$ corrected to account sensible heat of subcooling in the film; equal to $h_{fg} + 0.68\,c_p\Delta T$

k = thermal conductivity of condensate (or of vapor in the film boiling problem)

Nu = local Nusselt number, hx/k or $h(\mu/\rho_f\omega^2)^{1/3}/k$

Nu_D = Nusselt number for a cylinder or a sphere, $(hD/k)_{\text{average}}$

R = radius of curvature of an axisymmetric body

X = distance from axis of rotation to top of plate in a rotational system

x = distance along plate from leading edge

y = distance normal to plate from surface

Γ_c = rate of mass flow of condensate per unit breadth

α = cone angle

ΔT = difference between saturation temperature and wall temperature

δ = condensate film thickness

μ = viscosity of condensate (or of vapor in the film boiling problem)

ρ_f, ρ_g = densities of liquid and saturated vapor, respectively

ω = angular velocity

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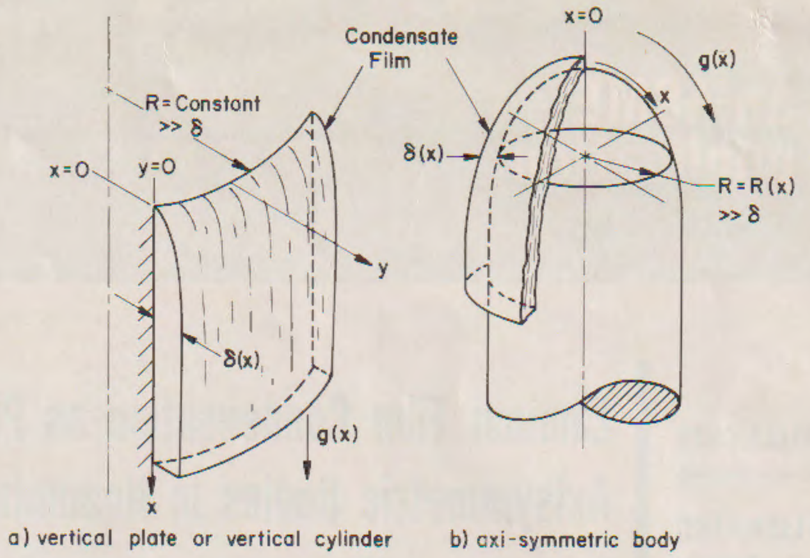

Fig. 1 Configurations under present consideration

$c_p\Delta T/h_{fg}$ less than the order of unity. The latent heat of vaporization corrected for sensible heat absorption in the film, h_{fg}', is

$$h_{fg}' = h_{fg}(1 + 0.68\, c_p\Delta T/h_{fg}) \tag{2}$$

which slightly exceeds Nusselt's earlier prediction.

The limitations imposed by [3] only exclude equation (1) from use with liquid metals. It will provide valid predictions of almost all other physically realistic situations. But liquid metal condensation, in turn, introduces grave complications related to accommodation coefficients and possible temperature decrements between the saturation temperature and the liquid-vapor interface. These difficulties exclude liquid metals from the present kind of treatment in any case.

With this limitation in mind we shall redevelop the Nusselt-Rohsenow analysis for a general gravity field, $g(x)$, where x is the distance from the leading edge of a plate, or nose of an axisymmetric body, as shown in Fig. 1. The "gravity" might actually be varying as it would in, say, a centrifugal field; or the slope of the surface might be curved so that the x coordinate is bent and the component of a gravity force might change with x; or both effects might be present. It would be convenient if Sparrow and Gregg's analysis could be adapted for $g = g(x)$; however, their similarity transformation generally breaks down when g varies arbitrarily with x.

We shall also allow the radius of curvature $R(x)$ of the body to vary. The vertical cylinder or plate (Fig. 1a, with R = constant or ∞) are special cases of the present situation. Our solution, like all of the previous condensation solutions, will not really be valid unless $x \gg \delta$. It will likewise be invalid unless $R \gg \delta$. Since these conditions are generally met while x and R are still much less than characteristic dimensions, their failure near $x = 0$ will introduce very minor error.

Sparrow and Gregg [4] adapted their method to one case of condensation in a nonuniform body force field on an axisymmetric body. They found that the condensing film on a disk rotating at ω rad/sec was of uniform thickness so that their similarity transformation became trivial. Their limiting value of Nusselt number for negligible inertia was independent of radius:

$$\mathrm{Nu} \equiv \frac{h}{k}\left(\frac{\mu}{\rho_f\omega}\right)^{1/2} = 0.904\left[\frac{(\rho_f - \rho_g)h_{fg}'\mu}{\rho_f k\Delta T}\right]^{1/4} \tag{3}$$

Since there is inherently great outward and Coriolis acceleration, the limitations on this solution are a little more stringent than on equation (1). It is generally accurate only for $c_p\Delta T/h_{fg} \leq 0.1$, and accurate for higher $c_p\Delta T/h_{fg}$ only if the Prandtl number stays just a little larger than unity.

Sparrow and Gregg also used their methods to treat a third case [5] which also will be accessible to our method, namely condensation on a horizontal pipe. Their no-inertia limit was

$$\mathrm{Nu}_D = C\left[g\,\frac{\rho_f(\rho_f - \rho_g)h_{fg}'D^3}{\mu k\Delta T}\right]^{1/4} \tag{4}$$

where C was 0.733. Nusselt had also done this problem in his original paper and obtained $C = 0.725$. Chen [6] subsequently corrected two of Sparrow and Gregg's analyses [3, 5] for vapor drag on the interface and obtained the presently accepted value of $C = 0.728$.

Analysis

Nusselt's local mass flow rate Γ_c lb$_m$/ft-hr is expressible in terms of the film thickness $\delta(x)$ without reference to the history of the film up to this location, as

$$\Gamma_c = [(\rho_f - \rho_g)\rho_f/3\mu]\delta^3 g(x) \tag{5}$$

Since the temperature distribution can be assumed linear,

$$h\Delta T = k\Delta T/\delta = \frac{h_{fg}'}{2\pi R(x)}\,\frac{d[\Gamma_c 2\pi R(x)]}{dx} \tag{6}$$

Substituting equation (5) in the right-hand equation of (6) gives

$$3A = \frac{3(\delta[gR]^{1/3})^3 d(\delta[gR]^{1/3})}{g^{1/3}R^{4/3}\,dx} \tag{7}$$

where $A \equiv \mu k\Delta T/(\rho_f - \rho_g)\rho_f h_{fg}'$. Integrating equation (7) subject to the boundary condition $\delta(x = 0) = 0$ gives

$$\frac{4A}{(gR)^{4/3}}\int_0^x g^{1/3}R^{4/3}\,dx = \delta^4 \tag{8}$$

Thus

$$\delta = \left[\frac{4\mu k\Delta T x}{(\rho_f - \rho_g)\rho_f h_{fg}' g_{\mathrm{eff}}}\right]^{1/4} \tag{9}$$

where

$$g_{\text{eff}} \equiv \frac{x(gR)^{4/3}}{\int_0^x g^{1/3} R^{4/3}\, dx} \tag{10}$$

Finally, we can write the local Nusselt number as

$$\text{Nu} = \frac{(k/\delta)x}{k} = \left[g_{\text{eff}} \frac{(\rho_f - \rho_g)\rho_f h_{fg}' x^3}{4\mu k \Delta T} \right]^{1/4} \tag{11}$$

With the exception of the use of an effective gravity, this is identical to equation (1). In terms of A, it can be written as

$$\text{Nu} = 0.707 \left[g_{\text{eff}} \frac{x^3}{A} \right]^{1/4} \tag{11a}$$

Examples of Applications

Equations (10) and (11) provide a convenient and versatile method for handling a wide variety of problems. We shall illustrate their use with several examples. Each example will be generally restricted to low $c_p \Delta T / h_{fg}$ or, for larger $c_p \Delta T / h_{fg}$, to Prandtl numbers on the high side of unity.

1 Vertical Flat Plate, Constant Gravity. In this case R is a constant which approaches infinity and equation (10) gives $g_{\text{eff}} = g$. Thus equation (11) reduces to equation (1).

2 Horizontal Cylinder. Again R is a constant approaching infinity, and the local x component of gravity, $g \sin (2x/D)$, should be used for $g(x)$ in equation (10). When g_{eff} is evaluated numerically, and the resulting value of h, obtained from equation (11), is averaged over the circumference of the cylinder, the result can be used to write a Nusselt number based on the diameter, D,

$$\text{Nu}_D = 0.729[gD^3/A]^{1/4} \tag{12}$$

which compares extremely well with the accepted value of $0.728[gD^3/A]^{1/4}$. Presumably Nusselt would also have obtained $C = 0.729$ had he had a digital computer to work with.

3 Upper Half of a Horizontal Cylinder. A similar integration halfway around the cylinder gives

$$\text{Nu}_D = 0.866[gD^3/A]^{1/4} \tag{13}$$

as compared with Nusselt's value of $0.861[gD^3/A]^{1/4}$.

4 Lower Half of a Horizontal Cylinder. If condensation begins at the middle of the tube and continues around the bottom, or if it simply continues around the bottom from the top, the results are fortuitously identical to three decimal places

$$\text{Nu}_D = 0.592[gD^3/A]^{1/4} \tag{14}$$

Nusselt obtained 0.589 for the numerical factor.

5 Rotating Horizontal Disk. In this case $g = \omega^2 x$ and $R = x$. Equation (10) gives $g_{\text{eff}} = 3\omega x/8$ and

$$\text{Nu} = 0.9036[(\mu/\rho_f)^2/A]^{1/4} \tag{15}$$

which agrees with Sparrow and Gregg's result.

6 Stationary Cone. Here we have a gravity of $g \cos (\alpha/2)$, where $R = x \sin (\alpha/2)$. This gives $g_{\text{eff}} = (7/3)\, g \cos (\alpha/2)$, and

$$\text{Nu} = 0.874[\cos (\alpha/2)]^{1/4}[gx^3/A]^{1/4} \tag{16}$$

7 Stationary Sphere. The gravity is $g \sin (2x/D)$ as it was for the horizontal cylinder but R is now $(D/2) \sin (2x/D)$. This, like the horizontal cylinder, requires a straightforward numerical integration. The result is

$$\text{Nu}_D = 0.785[gD^3/A]^{1/4} \tag{17}$$

8 Rotating Plate. The original motivation for undertaking this study was the problem of predicting condensation on a rotating plate aligned on a radial plane with its top located a distance X from the axis of rotation. In this case, R is a constant approaching infinity and $g = \omega^2 X + \omega^2 x$. Equation (10) gives

$$g_{\text{eff}} = \frac{4x\omega^2/3}{1 - [X/(X+x)]^{4/3}} \tag{18}$$

Accordingly the film thickness is

$$\delta = \left\{ \frac{3}{A\omega^2} [1 - (X/[X+x])^{4/3}] \right\}^{1/4} \tag{19}$$

When X is shrunk to zero, $\delta \rightarrow (3/A\omega^2)^{1/4}$ which (like δ for the rotating disk) is constant. The local Nusselt number in this case is

$$\text{Nu} \equiv \frac{h}{k} \left(\frac{\mu}{\rho_f \omega} \right)^{1/2} = 0.760[(\mu/\rho_f)^2/A]^{1/4} \tag{20}$$

which is of the same general form as the rotating disk result, with a smaller constant.

An Application to Film Boiling

Clearly a table of examples could be proliferated *ad nauseam*. The method is sufficiently simple that it is not necessary to do so, however. But an interesting experimental verification can be made for case 7 with the aid of data for film boiling on a sphere:

Several years ago Bromley [7] viewed film boiling on horizontal cylinders as the inverse of the condensation process and used equation (4), with a slightly modified value of h_{fg}', to describe it.[4] Since a portion of the vapor film at the top is inactive in the heat removal process owing to the bubble release process, C should be less than 0.728. Bromley's experiments showed it to be 0.62 or *85.2 percent* of the value for condensation.

We can also use equation (17) to predict the heat transfer during film boiling from a sphere. If we multiply it by the same correction factor of 85.2 percent, and change k and μ to the values for vapor, we get

$$\text{Nu}_D = 0.67[gD^3/A]^{1/4} \tag{21}$$

But this is *exactly* the expression that Frederking and Daniels [8] found would correlate their data for heat transfer during film boiling from spheres.[5]

Conclusion

The local Nusselt number for an axisymmetric body in a nonuniform gravity field is

$$\text{Nu} = 0.707[g_{\text{eff}} x^3/A]^{1/4}$$

where

$$g_{\text{eff}} = \frac{x(gR)^{4/3}}{\int_0^x g^{1/3} R^{4/3} dx}$$

These expressions are, practically speaking, valid for all but liquid metals applications.

References

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2 Rohsenow, W. M., "Heat Transfer and Temperature Distribution in Laminar Film Condensation," TRANS. ASME, Vol. 78, 1956, pp. 1645–48.

[4] The viscosity and thermal conductivity are now those for the *vapor* film instead of for saturated liquid.

[5] The prediction of Nu for film boiling is actually a little more complicated than we have indicated here. A recent study of film boiling from spheres by Hendricks and Baumeister [9] shows that the portion of the film not contributing strongly to heat transfer should actually vary slightly with A, and it shows that the liquid can exert significant traction on the film. However, it also shows that equation (21) gives generally good results.

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